

# Worcester County Mathematics League

Varsity Meet 2 – December 1, 2010

Round 1: Fractions, Decimals and Percents

All answers must be in simplest exact form in the answer section

**NO CALCULATOR ALLOWED**

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1

1. Simplify:  $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5}$ .

2. Flor-Mart was selling a product at price A. Price A was reduced by 20%, thus creating the new price B. Price B was then increased to price C. Price C represented a 10% reduction from price A. By what percent was price B increased to reach price C?

3. The number  $0.\overline{362} = 0.36222\dots$  is an example of a repeating decimal. Express the sum  $\frac{5}{21} + \frac{1}{42} + \frac{5}{12} + \frac{1}{63}$  as a repeating decimal.

## ANSWERS

(1 pt.) 1. \_\_\_\_\_

(2 pts.) 2. \_\_\_\_\_%

(3 pts.) 3. \_\_\_\_\_

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Round 2: Algebra 1 - Open



All answers must be in simplest exact form in the answer section

**NO CALCULATOR ALLOWED**

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1. Suppose that  $a$ ,  $b$  and  $c$  are positive integers such that  $ab = 18$ ,  $bc = 24$  and  $ac = 48$ . Find the sum  $a + b + c$ .
  
2. How many integers are in the solution set of the inequality  $|2x + 2| - x < 6$ ?
  
3. The last will and testament of an eccentric millionaire reads as follows: “I leave  $\frac{4}{17}$  of my estate to my son,  $\frac{7}{13}$  of the remainder to my wife,  $\frac{2}{3}$  of what then remains to my daughter, and the remaining two million dollars to my dog.” What was the total amount of the estate (in dollars)?

## ANSWERS

(1 pt.) 1. \_\_\_\_\_

(2 pts.) 2. \_\_\_\_\_

(3 pts.) 3. \$ \_\_\_\_\_

# Worcester County Mathematics League

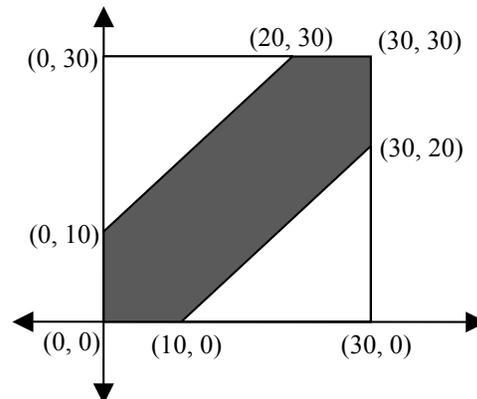
Varsity Meet 2 – December 1, 2010  
Round 3: Parallel Lines and Polygons



All answers must be placed in the answer section at the bottom  
NOTE: The diagrams are not drawn to scale.

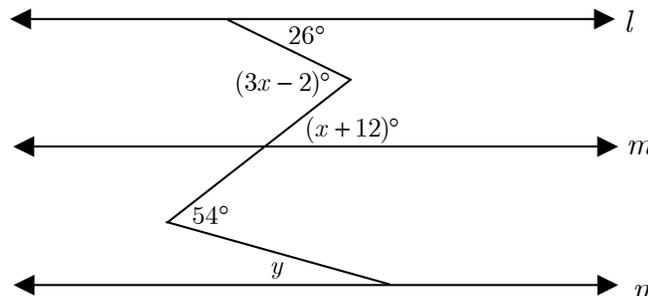
**NO CALCULATOR ALLOWED**

1. Find the area of the shaded region as defined in the diagram to the right.



2. In an isosceles triangle,  $\frac{1}{2}$  of the measure of the vertex angle is  $\frac{1}{7}$  of the measure of an exterior angle of the triangle at its base. Find the measure (in degrees) of the largest interior angle of the triangle.

3. In the diagram to the right,  $l \parallel m \parallel n$  along with the given angle measures. Find the measure of the angle marked  $y$  (in degrees).



## ANSWERS

(1 pt.) 1. \_\_\_\_\_

(2 pts.) 2. \_\_\_\_\_

(3 pts.) 3. \_\_\_\_\_

Worcester County Mathematics League  
Varsity Meet 2 – December 1, 2010  
Round 4: Sequences and Series

4

All answers must be in simplest exact form in the answer section

**NO CALCULATOR ALLOWED**

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1. Find the sum of the first twenty terms of the arithmetic sequence  
-24, -18, -12, -6,...

2. Express  $\sum_{k=1}^n 2k + 5$  in terms of  $n$ .

3. Let the sequence  $a_1, a_2, a_3, \dots, a_n, \dots$  be defined by  $a_1 = 2$  and  $a_{n+1} = a_n + 2n$ .  
Find the value of  $a_{100}$ .

ANSWERS

(1 pt.) 1. \_\_\_\_\_

(2 pts.) 2. \_\_\_\_\_

(3 pts.) 3. \_\_\_\_\_

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Round 5: Matrices & Simultaneous Equations



All answers must be in simplest exact form in the answer section

**NO CALCULATOR ALLOWED**

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1. Let  $A = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}$  and  $B = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$ . Find  $(A+B)^2$ .

2. Find the numerical value of  $y$  so that the following two determinant equations are true:

$$\begin{vmatrix} x & 3 \\ y & 4 \end{vmatrix} = \begin{vmatrix} 4 & 5 \\ 1 & -2 \end{vmatrix} \quad \text{and} \quad \begin{vmatrix} -x & 5 \\ 1 & -1 \end{vmatrix} = \begin{vmatrix} 5 & -1 \\ y & x \end{vmatrix}$$

3. Let  $X = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$  and  $Y = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}$ . If  $A^T$  represents the transpose of matrix  $A$  and  $A^{-1}$

represents the inverse of matrix  $A$ , find the matrix representing  $(X \cdot Y^{-1})^T - (Y \cdot X^{-1})^T$ .

## ANSWERS

(1 pt.) 1.  $\begin{bmatrix} \_ & \_ \\ \_ & \_ \end{bmatrix}$

(2 pts.) 2. \_\_\_\_\_

(3 pts.) 3.  $\begin{bmatrix} \_ & \_ \\ \_ & \_ \end{bmatrix}$

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## TEAM ROUND

All answers must *either* be in simplest exact form *or* as decimals rounded correctly to at least three decimal places, unless stated otherwise (2 pts. each)

### **APPROVED CALCULATORS ALLOWED**

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1. Draw an isosceles trapezoid along with its diagonals such that the diagonals of the trapezoid are perpendicular. If the smallest angle in your diagram is  $15^\circ$ , find the measure of the largest angle in your diagram (in degrees).
2. One morning at a pond there was a flock of ducks. One-fifth of the ducks flew away, and half of those ducks returned to the pond. Then, one-third of the ducks that remained on the pond flew away. Then, one-sixth of the remaining ducks flew away, and then after that 15 more ducks flew away, leaving 40 ducks on the pond. How many ducks were on the pond at the beginning of the morning?
3. In what positive number bases less than 12 is the number 2101 a perfect square?
4. A survey of calculus students showed that exactly 23.2% were math majors. What is the minimum number of students who could have been surveyed?
5. In an increasing sequence of four positive integers, the first three terms form an arithmetic progression, the last three terms form a geometric progression and the first and fourth terms differ by 30. Find the sum of the four terms.
6. Consider the matrix  $A = \begin{bmatrix} 6 & x \\ 2 & y \end{bmatrix}$ . If the determinant of  $A$  is 8 and the inverse of  $A$  is  $A^{-1} = \begin{bmatrix} \frac{1}{2} & -1 \\ -\frac{1}{4} & \frac{3}{4} \end{bmatrix}$ , determine the numerical value of the product  $xy$ .
7. Suppose a person can walk down a down-escalator in 10 seconds, and walk up the same down-escalator in 40 seconds. Assuming that the person and the escalator both move at constant rates, how many seconds would it take a person to go down the escalator while standing on it?
8. Which point on the line  $y = \frac{1}{2}x - 3$  is closest to the point  $(3, 3)$ ?
9. At a gathering of  $N$  people, everyone shook hands with everyone else exactly once. If there were 120 total handshakes, what is  $N$ ?

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## ANSWERS

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### Round 1

1.  $\frac{4}{5} = 0.8$

2.  $12.5 = 12\frac{1}{2} = \frac{25}{2}$

3.  $0.\overline{694} = 0.6944444\dots$

### Round 2

1. 17

2. 6

3. 17,000,000 or 17 million  
(accept either answer)

### Round 3

1. 500

2. 75 (or  $75^\circ$ )

3. 22 (or  $22^\circ$ )

### Round 4

1. 660

2.  $n^2 + 6n = n(n + 6)$

3. 9902

### Round 5

1.  $\begin{bmatrix} 7 & 9 \\ 6 & 10 \end{bmatrix}$

2. 2

3.  $\begin{bmatrix} -3 & -2 \\ 6 & 3 \end{bmatrix}$

### Team Round

1. 120 (or  $120^\circ$ )

2. 110

3. 3 and 8 (need both, either order)

4. 125

5. 129

6. 32

7.  $\frac{80}{3} = 26\frac{2}{3} = 26.\overline{6} \approx 26.667$

8.  $(4.8, -0.6) = \left(4\frac{4}{5}, -\frac{3}{5}\right) = \left(\frac{24}{5}, -\frac{3}{5}\right)$

9. 16