Worcester County Mathematics League

WOCOMAL Varsity Meet #3

Coaches' Booklet

February 1, 2006

February 1, 2006 **WOCOMAL Varsity Meet**

Round 1: Similarity and Pythagorean Theorem

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1. A person 6 feet tall standing near a streetlight casts a 10-foot shadow on the ground. If the base of the streetlight is 45 feet from the tip of the shadow, what is the height, in feet, of the streetlight?

2. The lengths of the sides of the acute angled $\triangle ABC$ are a = x + 1, b = x, and c = x – 1. If BD $\perp$ AC, then determine the value of (CD – DA).

3. Through a point P inside the $\triangle ABC$, a line is drawn parallel to the base AB, dividing the triangle into two equal areas. If the altitude to AB has length 1, then determine the distance from P to AB.

ANSWERS

(1 pt.) 1. _______________

(2 pts.) 2. _______________

(3 pts.) 3. _______________

Auburn, Bancroft, Mass Academy

February 1, 2006 **WOCOMAL Varsity Meet**

Round 2: Algebra 1 (open)

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1. Find the largest of four consecutive odd integers such that the sum of the first and the fourth is 27 less than three times the first integer.

2. If the line that passes through the points A(2,-1) and B(3,-2) is parallel to the line $ax - by + c = 0$, then determine the value of $a^2 - b^2$.

3. If a straight line intersects the horizontal axis at j and intersects the vertical axis at $\frac{1}{j}$, at what point (x,y) does it intersect the line y = x ?

ANSWERS

(1 pt.) 1. _______________

(2 pts.) 2. _______________

(3 pts.) 3. _______________

Leicester, Mass Academy. Tantasqua

February 1, 2006 **WOCOMAL Varsity Meet**

Round 3: Functions (NO CALCULATORS)

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1. If f(x) = 3x – 7, determine $f^{-1}(x)$ and write your answer as a **single** fraction.

2. A function f satisfies f(x+y) = 4f(x) • f(y) for all real numbers x and y.
If f(3) = 32, determine the value of f(1).

3. Consider the following ordered pairs in the table below. Determine the polynomial function of smallest degree, which passes through these points.

X	F(x)
0	1
1	-3
2	-1
3	7
4	21
5	41
6	67

ANSWERS

(1 pt.) 1. $f^{-1}(x) =$ ____________________________

(2 pts.) 2. ____________________________________

(3 pts.) 3. F(x)= ______________________________

Assabet Valley, Bartlett, Mass Academy

February 1, 2006 **WOCOMAL Varsity Meet**

Round 4: Combinatorics

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM EXCEPT WHERE NOTED.

1. How many possibilities are there for the "win, place, and show" (i.e. 1^{st}, 2^{nd}, and 3^{rd}) positions in a horse race with 12 horses, if all order of finish are possible, all horses do finish, and there are no ties?

2. How many different four-person committees can be formed from a group of 5 males and three females, if the committee must be equally divided between males and females?

3. A basketball player makes 73% of all free throws. Assuming independence, if the player shoots five free throws during practice, what is the probability that she will make **exactly** three of the free throws? **Round to the nearest percent.**

ANSWERS

(1 pt.) 1. _______________

(2 pts.) 2. _______________

(3 pts.) 3. _______________

Athol, Blackstone Valley, Tantasqua

February 1, 2006 **WOCOMAL Varsity Meet**

Round 5: Analytic Geometry (NO CALCULATORS)

1. Consider the circle whose equation is $x^2 + y^2 + 10x - 4y - 7 = 0$. If the center is at (h,k) and the radius = r, then determine the value of $(h + k + r)$.

2. What is the standard form, i.e. $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$, of the equation of the ellipse with a major axis of length 6 and foci at (-2,0) and (2,0)?

3. Suppose all the points on the curve $x^2 + y^2 - 10x = 0$ are reflected about the line $y = x + 3$. In the form $x^2 + y^2 + Cx + Dy + F = 0$, find the locus of the reflected points. Write the sum $(C + D + F)$ as your answer.

ANSWERS

(1 pt.) 1. _______________

(2 pts.) 2. _______________

(3 pts.) 3. _______________

Athol, Blackstone Valley, Bromfield

February 1, 2006 **WOCOMAL Varsity Meet**

TEAM ROUND

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM AND WRITTEN ON THE SEPARATE TEAM ANSWER SHEET. **(2 points each)**

1. What is the minimum number of students required in a class to guarantee that at least six students will receive the same grade if the five possible grades are A,B,C,D,F ?

2. Determine the sum of the coordinates of the points where the line meets the circle, given the following equations: $x^2 + y^2 = 25$ and $3x - y = 5$.

3. In the right triangle ABC, leg AC is 50% longer than leg BC. Let D be the midpoint of side BC, and DE $\perp$ AB. What is the ratio of the area of $\triangle DBE$ to the area of $\triangle ABC$?

4. After distributing textbooks to his classes, Mr. Yurek had five Algebra 1, four Geometry, and four Algebra 2 textbooks remaining. In how many ways could he arrange these textbooks, upright left to right, on his shelf?

5. A 70 foot pole stands vertically in a plane supported by three 490 foot wires, all attached to the top of the pole, pulled taut, and anchored to three equally spaced points in the plane. How many feet apart are any two of these anchor points?

6. A cubic block with dimensions n by n by n is made up of a collection of 1 by 1 by 1 unit cubes. What is the smallest value of n so that if the outer two layers of unit cubes are removed from the block, more than half the original unit cubes will still remain?

7. A non-constant polynomial function f(x) satisfies: $f(-3) = f(-1) = f(2) = f(4) = 5$. What is the smallest possible degree of f(x) ?

8 If the circles $x^2 + (y-6)^2 = r^2$ and $(x-8)^2 + y^2 = 25$ are internally tangent, determine the value of r.

9. How many different positive integers divide 10! ?

Assabet Valley, Athol, Blackstone Valley, Leicester, Shrewsbury, Tantasqua

February 1, 2006 **WOCOMAL Varsity Meet ANSWERS**

Round 1: Similarity and Pythagorean Theorem

1. **(1 pt.)** **27 feet or 27**

2. **(2 pts.)** **4**

3. **(3 pts.)** $\frac{2-\sqrt{2}}{2}$ **or** $1-\frac{\sqrt{2}}{2}$

Round 2: Algebra 1-open

1. **(1 pt.)** **39**

2. **(2 pts.)** **0**

3. **(3 pts.)** $\left(\frac{j}{1+j^2}, \frac{j}{1+j^2}\right)$

Round 3: Functions

1. **(1 pt.)** $f^{-1}(x) = \frac{x+7}{3}$

2. **(2 pts.)** $\sqrt[3]{2}$ **or** $2^{\frac{1}{3}}$

3. **(3 pts.)** $F(x) = 3x^2 - 7x + 1$

Round 4: Combinatorics

1. **(1 pt.)** **1320**

2. **(2 pts.)** **30**

3. **(3 pts.)** **28%**

Round 5: Analytic Geometry

1. **(1 pt.)** **3**

2. **(2 pts.)** $\frac{x^2}{9} + \frac{y^2}{5} = 1$

   or $\mathbf{5x^2 + 9y^2 = 45}$

3. **(3 pts.)** **38**

February 1, 2006

TEAM ROUND (2 pts. Each)

1. **26**

2. **2**

3. $\frac{1}{13}$ **or 1:13**

4. **90,090**

5. **840**

6. **20**

7. **4**

8. **15**

9. **270**

February 1, 2006 **WOCOMAL Varsity Meet**

TEAM ROUND

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM AND ON THIS SEPARATE TEAM ANSWER SHEET. (2 points each)

1. ____________________________

2. ____________________________

3. ____________________________

4. ____________________________

5. ____________________________

6. ____________________________

7. ____________________________

8. ____________________________

9. ____________________________

February 1, 2006 **WOCOMAL Varsity Meet**

TEAM ROUND **School: ________________________**

Team #:________________________

Team Members:

1. ______________________________________
2. ______________________________________
3. ______________________________________
4. ______________________________________
5. ______________________________________