WOCOMAL

Varsity Meet #3

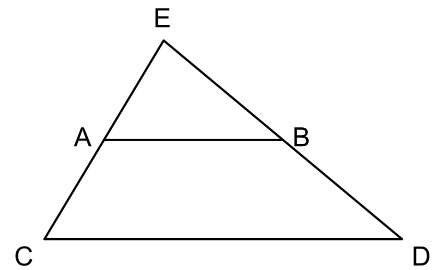
February 5, 2003

WoCoMaL

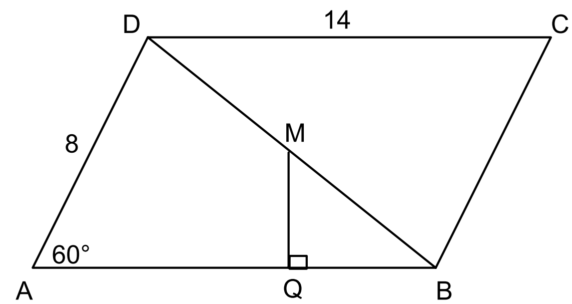
February 5, 2003 Varsity Meet#3

ROUND#1: **Similarity & The Pythagorean Theorem**

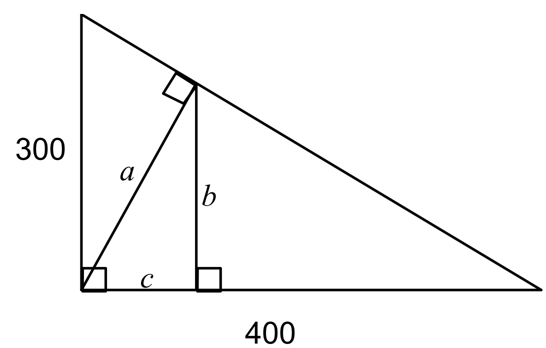
1. In ΔCDE, $\overline{AB} \parallel \overline{CD}$, as shown. If $AB = 8,\ CD = 18,\ and\ AE = 4$, compute AC.



2. $ABCD$ is a parallelogram with sides of lengths 8 and 14 and a 60° angle, as shown. If M is the midpoint of diagonal $\overline{BD}$ and $\overline{MQ} \perp \overline{AB}$ at Q, find MQ.



3. A large right triangle has legs of lengths 300 feet and 400 feet. Perpendiculars are drawn forming a smaller right triangle, as shown. Find its perimeter: $a + b + c$.



Answer here: 1. (1 pt.) $AC =$ ______

2. (2 pts.) __________

3. (3 pts.) _______ feet

Shepherd Hill, Saint John's, Assabet Valley

WoCoMaL

February 5, 2003 Varsity Meet#3

ROUND#2: **Algebra 1**

1. If the average of 12 numbers is 18 and the average of 18 other numbers is 12, find the average of all the numbers.

2. There are four consecutive odd integers having the property that the sum of the second and twice the fourth is forty-eight more than the third. Find the simple sum of the four integers.

3. The area of a rectangle is 6 square inches. One side has length $3\sqrt{2}+2\sqrt{3}$ inches. Compute the exact perimeter.

Answer here: 1. (1 pt.) __________

2. (2 pts.) _________

3. (3 pts.) _________ inches

Tantasqua, Holy Name, Westborough

February 5, 2003 Varsity Meet#3

ROUND#3: **Functions** - **<< No Calculators >>**

1. If $f(x)=3x^2-4$ and $g(x)=2x+C$, find C so that the equation $g(f(x))=0$ has real roots ± 2 .

2. The function f is defined by
$$f(2)=1$$
$$f(n^2)=f(n)+2 \quad \text{for } n \geq 2$$
Find $f(256)$.

3. Suppose f^{-1} is the inverse of $f(x)=\dfrac{-x}{x+2}$.
Find all values of n for which $f(n)=f^{-1}(n)$.

Answer here: 1. (1 pt.) __________

2. (2 pts.) _________

3. (3 pts.) _________

North, Algonquin, Burncoat & Hopedale

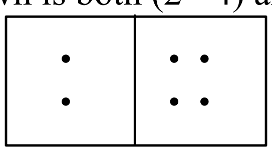
WoCoMaL

February 5, 2003 Varsity Meet#3

ROUND#4: **Combinatorics**

1. A seven digit number is formed in which the digits of $\{1, 2, 3, 4, 5, 6, 7\}$ are each used exactly once. How many numbers have only even digits in the first three positions ?

2. A domino set contains all number pairs from double zero (0 - 0) to double six (6 - 6), with each number pair occurring exactly once. For example, the domino shown is both (2 - 4) and (4 - 2). How many dominoes are in a set ?



3. The Universalist Church choir contains 7 women and 4 men. This year the choir may select 3 members to attend the State Choral Convention. If one attendee must be a man, in how many different ways can they make their selection ?

Answer here: 1. (1 pt.) __________

2. (2 pts.) _________

3. (3 pts.) _________

Auburn, Hudson, Burncoat

WoCoMaL

February 5, 2003 Varsity Meet#3

ROUND#5: **Analytic Geometry (Lines & Conics)** - **<< No Calculators >>**

1. The graphs of $2y + x + 3 = 0$ and $3y + Ax + 2 = 0$ are perpendicular lines. Solve for A .

2. In the form $Ax + By = C$, simplified, write the equation of the asymptote to the graph of $9x^2 - 4y^2 - 18x - 27 = 0$ which has positive slope.

3. If $x^3 - 2x^2 + 20x - 12 = A(x-2)^3 + B(x-2)^2 + C(x-2) + D$, compute $A + C$.

Answer here: 1. (1 pt.) __________

2. (2 pts.) _________

3. (3 pts.) _________

Doherty, Bancroft, Hudson

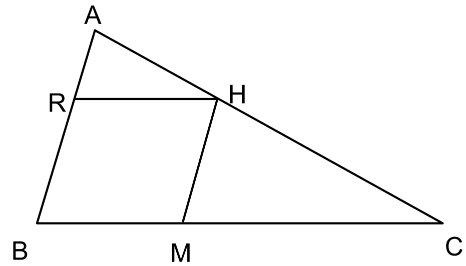
WoCoMaL

February 5, 2003 Varsity Meet#3

Answers must be **simplified** and **exact** or rounded to **three decimal** places.

Team Round:

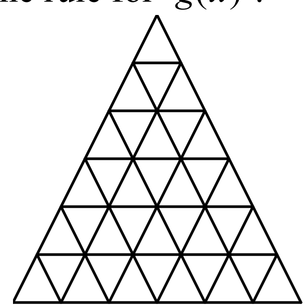
1. In the diagram, *RHMB* is a rhombus and ΔABC has $AB = 10$ and $BC = 20$. How long is the side of the rhombus ?



2. Euclid travels between two points A and B at the rate of 2 minutes per mile and then returns the same distance at 2 miles per minute. Find his average speed, in miles per hour, for the whole trip.

3. If $f(x) = 3x - 2$ and $g(f(x)) = 9x^2 - 12x + 8$, find the rule for $g(x)$.

4. How many triangles <u>similar</u> to  can be found embedded in this figure ?



5. If the center of the hyperbola $xy + 5x - 4y = 0$ is translated to the origin, through what minimal distance is it moved ?

6. Twenty-five of 100 people at a gathering are smokers. A group of four people is randomly selected. What is the probability that exactly three are non-smokers ?

7. 201,200 in base 3 is 2243 in base ***b*** . Find ***b*** .

8. A 25-foot ladder leans against a vertical wall such that the base of the ladder is 20 feet from the wall. The ladder slides until the base is 24 feet from the wall. Locate the point X at which the two ladder positions crossed. How far is X above the ground ?

9. Compute the area of the quadrilateral with consecutive vertices at $(4, 5)$, $(-4, 6)$, $(-5, -8)$, and $(6, 3)$.

Hudson, Burncoat, Auburn, Quaboag, Worcester Acad., Hudson, Auburn, Tantasqua, Shrewsbury

WoCoMaL

February 5, 2003 Team Round Varsity Meet#3

2 Points Each

Answers must be **simplified** and **exact**
or rounded to **three decimal** places.

Answers here ↓ :

1.________________________

2.________________________

3.________________________

4.________________________

5.________________________

6.________________________

7.________________________

8.________________________

9.________________________

School:_______________________________

Team#: ______

Players' Names ↓ :

#1:_______________________

#2:_______________________

#3:_______________________

#4:_______________________

#5:_______________________

WOCOMAL Answers Varsity Meet #3 February 5, 2003

R#1:
1. 5
2. $2\sqrt{3} \approx 3.464$
3. 576 feet

R#2:
1. $14\frac{2}{5} = 14.4 = \frac{72}{5}$
2. 88
3. $12\sqrt{2}$

R#3:
1. $C = -16$
2. 7
3. 0 and -3
   [Need both.]

R#4:
1. 144
2. 28
3. 130

R#5:
1. -6
2. $3x - 2y = 3$
   [or -3x + 2y = -3]
3. 25

Team:
1. $\frac{20}{3} = 6\frac{2}{3} = 6.\overline{6} \approx 6.667$
2. 48 mph
3. $g(x) = x^2 + 4$
   [or just $x^2 + 4$]
4. 78
5. $\sqrt{41} \approx 6.403$
6. $\frac{67,525}{156,849} \approx 0.431$
7. 6
8. $\frac{21}{11} = 1\frac{10}{11} = 1.\overline{90} \approx 1.909$
9. $78\frac{1}{2} = 78.5$

WoCoMaL

V3 - Solutions — Feb. 5, 2003

Round#1 1. $\Delta CDE \sim \Delta ABE$; so, $\frac{CD}{AB} = \frac{CE}{AE}$ or $\frac{18}{8} = \frac{AC+4}{4}$. $AC = 5$.

2. Draw $\overline{DX} \perp \overline{AB}$ and $\therefore$ parallel to $\overline{MQ}$. ΔADX is a 30°-60°-90°Δ with long leg $DX = 4\sqrt{3}$. Now, $\Delta MQB \sim \Delta DXB$ with ratio of similarity of 1 to 2. $\therefore$ $MQ = 2\sqrt{3}$.

3. ***a*** is the long leg of a 3,4,5 rt. Δ having hyp. 300. $\therefore$ $\frac{a}{4} = \frac{300}{5}$ and $a = 240$.
The perimeter of the largest Δ is 1200, and the hyp. of the a,b,c Δ is 240.
$\therefore$ $\frac{a+b+c}{1200} = \frac{240}{500}$ and $a + b + c = 576$

Round#2 1. The sum of all 30 numbers is $12 \times 18 + 18 \times 12 = 432$. So, ave. = 14.4 .

2. Use x, x+2, x+4, x+6, we have $x + 2 + 2(x + 6) = 48 + x + 4$. So, x = 19 and sum = 4x+12 = 88.

3. The other side is $\frac{6}{3\sqrt{2}+2\sqrt{3}} = \frac{6}{3\sqrt{2}+2\sqrt{3}} \cdot \frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} = \frac{6(3\sqrt{2}-2\sqrt{3})}{18-12} = 3\sqrt{2} - 2\sqrt{3}$.
So, $p = 2[(3\sqrt{2} + 2\sqrt{3}) + (3\sqrt{2} - 2\sqrt{3})] = 12\sqrt{2}$

Round#3 1. Set $g(f(x)) = 2 \cdot f(x) + C = 2(3x^2 - 4) + C = 0$. Substitute either +2 or -2, and C = -16 pops out.

2. If $n^2 = 256$, then $n = 16$, etc.
So, $f(256) = f(16) + 2 = [f(4) + 2] + 2 = [\{f(2) + 2\} + 2] + 2 = 1 + 2 + 2 + 2 = 7$.

3. First find $f^{-1}(x)$ by the usual interchanging the roles of x and y. $f^{-1}(x) = \frac{-2x}{x+1}$.
Now solve $\frac{-n}{n+2} = \frac{-2n}{n+1}$ to obtain $n = 0$ and $n = -3$.

Round#4 1. The 3 evens are 1st; the 4 odds are last. The answer is $(3\,!) \times (4\,!) = 6 \times 24 = 144$.

2. Just list the possibilities, and notice a pyramidal pattern with rows containing 7+6+5+4+3+2+1 elements. Answer is 28.

   06, 16, 26, 36, 46, 56, 66
   05, 15, 25, 35, 45, 55
   04, 14, 24, 34, 44
   etc.

3. They may select (1 m & 2 w) or (2 m & 1 w) or (3 m). Using combinations, the total count is ${}_4C_1 \times {}_7C_2 + {}_4C_2 \times {}_7C_1 + {}_4C_3 = 4 \times 21 + 6 \times 7 + 4 = 130$.

Round#5 1. If perpendicular, then the product of slopes $-\frac{1}{2} \cdot -\frac{A}{3} = -1$ or A = - 6.

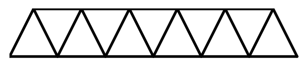
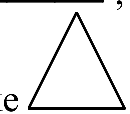
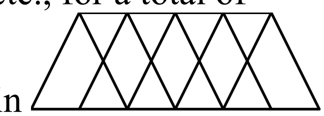
2. Rewrite the hyperbola as $\frac{(x-1)^2}{4} - \frac{y^2}{9} = 1$, and the ans. is $\frac{x-1}{2} - \frac{y}{3} = 0$ or $3x - 2y = 3$.

3. If $x = 3$, then $A + B + C + D = 57$. If $x = 1$, then $-A + B - C + D = 7$ or $A - B + C - D = -7$. Add the 1st and this last to obtain $2A + 2C = 50$ and $A + C = 25$.

Team 1. Suppose $BR = RH = s$. Then $AR = 10 - s$. But $\Delta ARH \sim \Delta ABC$ and $\frac{AR}{AB} = \frac{RH}{BC}$ or $\frac{10-s}{10} = \frac{s}{20}$, from which $s = \frac{20}{3}$.

2. 2 min / mile = 1 mile / 2 min = 30 mph ; 2 miles / min = 120 mph.
So, total time = $\dfrac{D}{30} + \dfrac{D}{120} = \dfrac{2D}{r}$, where r is average rate for whole trip.
The D's cancel. Then $\frac{1}{30} + \frac{1}{120} = \frac{5}{120} = \frac{1}{24} = \frac{2}{r}$ and $r = 48mph$.

3. You must convert what is going into g to "x". Do it by stuffing $f^{-1}(x)$ into $g(f(x))$. Since $f^{-1}(x) = \frac{x+2}{3}$, $g(x) = g\left(f\left(f^{-1}(x)\right)\right) = 9\left(\frac{x+2}{3}\right)^2 - 12\left(\frac{x+2}{3}\right) + 8 = x^2 + 4$.

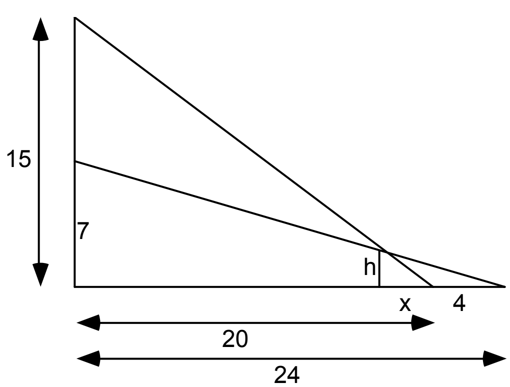
4. There are 11 like  in the bottom row , etc., for a total of $11 + 9 + 7 + 5 + 3 + 1 = 36$ of this size. There are 8 like  in .
Keep counting to obtain $36 + 21 + 11 + 6 + 3 + 1 = 78$ of increasing sizes.

5. This hyperbola can be rewritten in the form $(x-4)(y+5) = -20$. We want to change it to $XY = -20$ using $X = x - 4$ and $Y = y + 5$. So, it is moved 4 hor. and 5 vert. for a minimal translation of $\sqrt{4^2 + 5^2} = \sqrt{41}$ units.

6. The number of ways to select 1 smoker and 3 non-s is ${}_{25}C_1 \times {}_{75}C_3 = 1,688,125$. The number of ways to select any 4 is ${}_{100}C_4 = 3,921,225$. The reduced ratio is $\frac{67,525}{156,849}$.

7. The expanded form of 201,200 (base 3) = 2243 (base b) is $2 \times 3^5 + 1 \times 3^3 + 2 \times 3^2 = 2 \times b^3 + 2 \times b^2 + 4 \times b^1 + 3 \times b^0$. Thus we need solve $531 = 2b^3 + 2b^2 + 4b + 3$ for $b \geq 5$. Answer is $b = 6$.

8.



From this figure, two equations: $\frac{x}{h} = \frac{20}{15} = \frac{4}{3}$ and $\frac{x+4}{h} = \frac{24}{7}$. Subtract to obtain $\frac{4}{h} = \frac{24}{7} - \frac{4}{3} = \frac{44}{21}$. So, $h = \frac{21}{11}$.

9. Surround by a rectangle. From its area = $11 \times 14 = 154$, subtract the areas of four right triangles and one rectangle. So, $\alpha(Quad) = 154 - 7 - 4 - 2 - 2 - 60.5 = 78.5$