

# WORCESTER COUNTY MATHEMATICS LEAGUE

Freshman Meet 3 - March 25, 2009

Round 1: Graphing on a Number Line

1

**NO CALCULATOR ALLOWED**

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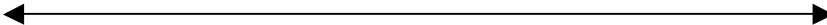
Draw the graph of each of the following inequalities on the corresponding number line provided below. Please specify all endpoints on your graph.


1.  $3(4 - x) + x < 10$


2.  $x + 8 \leq 3x + 2$  or  $2x - 8 < 3x - 2$

3.  $-|2x - 3| \geq -1$

## ANSWERS

(1 pt.) 1. 

(2 pts.) 2. 

(3 pts.) 3. 

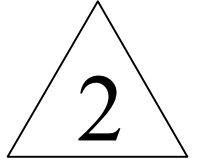
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Round 2: Operations on Polynomials

All answers must be in simplest exact form

**NO CALCULATOR ALLOWED**



1. Simplify the following expression to a single polynomial:

$$12x - \left(10x + \left[3 - 6x - (4 + x) - 17\right] + 5x\right) - 19$$

2. Factor the following expression as the product of two polynomials with integer coefficients:

$$y^n(y^2 + 2) - y^{n+1}(y + 2)$$

3. After  $P(x) = (5 - x)^2(x - 5) - (x - 5)^2(5 - x)$  is expanded and simplified as a single polynomial, compute the sum of the coefficients of  $P(x)$ .

## ANSWERS

(1 pt.) 1. \_\_\_\_\_

(2 pts.) 2. \_\_\_\_\_

(3 pts.) 3. \_\_\_\_\_

# WORCESTER COUNTY MATHEMATICS LEAGUE

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Round 3: Techniques of Counting and Probability

All answers must be in simplest exact form

**NO CALCULATOR ALLOWED**

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1. How many odd three-digit numbers can be formed if the tens' digit must be four? (Repetition of digits is allowed and zero cannot be the hundreds' digit.)
2. From a group consisting of 5 boys and 3 girls, two people are to be randomly selected to serve on a student advisory committee. Find the probability that one boy and one girl are selected. Express your answer as a fraction reduced to lowest terms.
3. A standard, fair six-sided die is rolled five times. Find the probability that an even number is obtained on exactly two of the rolls. Express your answer as a fraction reduced to lowest terms.

## ANSWERS

(1 pt.) 1. \_\_\_\_\_

(2 pts.) 2. \_\_\_\_\_

(3 pts.) 3. \_\_\_\_\_

# WORCESTER COUNTY MATHEMATICS LEAGUE

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Round 4: Perimeter, Area and Volume

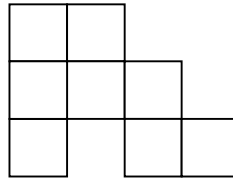
4

All answers must be in simplest exact form

**NO CALCULATOR ALLOWED**

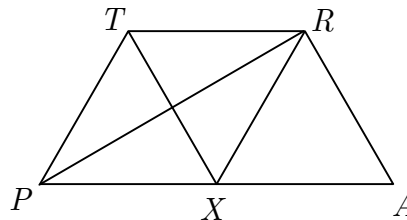
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1. The shape below consists of congruent squares. If the perimeter of the entire shape is 64 centimeters, find its area (in square centimeters).



2. A right rectangular solid has dimensions 5 inches, 12 inches and 7 inches. If congruent 1-inch by 1-inch by 1-inch cubes are removed from each vertex of the solid, find the number of square inches in the surface area of the resulting solid.

3. The diagram below shows trapezoid  $TRAP$  with bases  $TR$  and  $PA$ . If  $\triangle TPX$  and  $\triangle RXA$  are equilateral triangles with sides of length 10, find the area of triangle  $TPR$ . Please express your answer in simple radical form.



## ANSWERS

(1 pt.) 1. \_\_\_\_\_ square centimeters

(2 pts.) 2. \_\_\_\_\_ square inches

(3 pts.) 3. \_\_\_\_\_

# WORCESTER COUNTY MATHEMATICS LEAGUE

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## TEAM ROUND

All answers must *either* be in simplest exact form *or* as decimals rounded correctly to at least three decimal places! (3 pts. each)

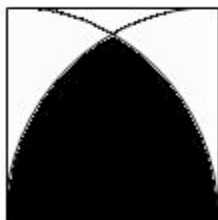
**APPROVED CALCULATORS ALLOWED**

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1. The number 135 is called a *digi-number* because it has the property of being divisible by each of its digits. How many digi-numbers are there between 10 and 50?
2. A club consists of 5 men and 5 women. An activity committee of three people is chosen at random. What is the probability that the committee has more men than women?
3. Factor the following expression as the product of three polynomials whose coefficients are integers:  $18a^2x + 12a^2y - 45abx - 30aby$
4. A geometry class took a test and the average score was 74. If 1 more student had taken the test and earned a score of 100, the class average would have been 75. How many students are in the class?
5. Each interior angle of a certain regular  $n$ -gon has a measure of  $150^\circ$ . Find the value of  $n$ .
6. On the space provided on the answer sheet, graph the solution set of:

$$|x| \leq 2x + 2$$

7. The Fibonacci sequence 1, 1, 2, 3, 5, 8, 13, ... begins with two 1's and each term afterwards is the sum of its two predecessors. Which of the ten digits (0, 1, 2, 3, ... 8, 9) is the last to appear in the units' position of a number in the Fibonacci sequence?
8. The diagram below shows a square and two quarter-circles whose centers are two of the vertices of the square. If the sides of the square have length 6, the area of the shaded region can be written as  $a\pi + b\sqrt{3}$ , where  $a$  and  $b$  are integers. Find the sum  $a + b$ .



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## ANSWER SHEET - TEAM ROUND

All answers must *either* be in simplest exact form *or* as decimals rounded correctly to at least three decimal places! (3 pts. each)

1. \_\_\_\_\_

2. \_\_\_\_\_

3. \_\_\_\_\_

4. \_\_\_\_\_ students

5. \_\_\_\_\_

6. 

7. \_\_\_\_\_

8. \_\_\_\_\_

# WORCESTER COUNTY MATHEMATICS LEAGUE

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## ANSWERS

### Round 1



### Round 2

1.  $4x - 1$

2.  $2y^n(1 - y)$  or  $-2y^n(y - 1)$  or equivalent

3.  $-128$

### Round 3

1. 45

2.  $\frac{15}{28}$  (only)

3.  $\frac{5}{16}$  (only)

### Round 4

1. 128

2. 358

3.  $25\sqrt{3}$

### Team Round

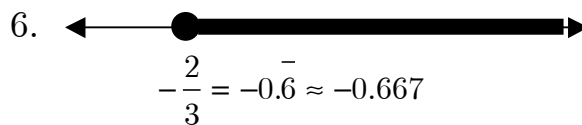
1. 9

2.  $\frac{1}{2} = 0.5 = 50\%$

3.  $3a(3x + 2y)(2a - 5b)$   
or equivalent, accounting for  
the commutativity of  
multiplication and addition

4. 25

5. 12



7. 6

8. 3