Worcester County Mathematics League

Freshman Meet 3 – March 26, 2008

Round 1: Graphing on a Number Line

1

NO CALCULATOR ALLOWED

Carefully draw the graph of each of the solution sets for the following inequalities on the corresponding number line provided below. Please specify all endpoints on your graph.

1. $8 > 5 - 3x \geq -13$

2. $\frac{1}{3}(4x-1) - \frac{3}{2}(2x+1) \leq \frac{19}{6}$

3. $9 - 4x < |2x+3|$

<u>ANSWERS</u>

(1 pt.) 1.

(2 pts.) 2.

(3 pts.) 3.

Doherty, Saint John's, West Boylston

Worcester County Mathematics League

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Round 2: Operations on Polynomials

2

All answers must be in simplest exact form

NO CALCULATOR ALLOWED

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1. What polynomial must be added to $3x - xy - y$ to obtain $7x - 3xy$ as a sum?

2. Simplify the following expression to a single polynomial:

$$6a - \left[(3a+1) + \left(3a - 6 - (a+2) - 3a\right) - a\right]$$

3. Factor the following polynomial completely over the integers:

$$12x^3y^2 - 15x^2y^3 - 18xy^4$$

<u>ANSWERS</u>

(1 pt.) 1.____________________________________

(2 pts.) 2.____________________________________

(3 pts.) 3.____________________________________

Shrewsbury, Auburn, Quaboag

WORCESTER COUNTY MATHEMATICS LEAGUE

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Round 3: Techniques of Counting and Probability

3

All answers must be in simplest exact form

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1. In a club with 10 members, how many ways can a committee of three members be selected?

2. A family of five is going to line up for a picture in a row of five chairs. In how many ways can the family be seated if the mother and father must sit next to each other?

3. You and your best friend are in all of the same classes. Today, in your literature class of 20 students, five students are randomly selected to work with the laptop computers. Find the probability that neither you nor your best friend will be selected. Give your answer as a fraction reduced to lowest terms.

ANSWERS

(1 pt.) 1.________________________

(2 pts.) 2.________________________

(3 pts.) 3.________________________

South, Auburn, Quaboag

Worcester County Mathematics League

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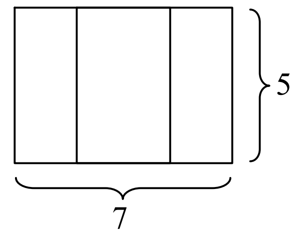
Round 4: Perimeter, Area and Volume

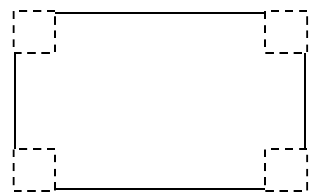
4

All answers must be in simplest exact form <u>except for #3</u>
<u>The diagrams are not drawn to scale</u>

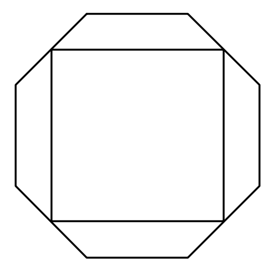
1. Two 5 inch by 5 inch squares overlap to form a 5 inch by 7 inch rectangle as shown in the figure to the right. Find the area, in square inches, of the region in which the two squares overlap.



2. A rectangular open box (which has no top) is constructed by starting with a rectangular sheet of metal 10 inches by 14 inches. Identical square pieces are removed from the corners so that when the sides are folded the box will be 1½ inches deep (see the illustration to the right). Find the volume of the box (in cubic inches).



3. A square is constructed by joining the midpoints of alternate sides of a regular octagon, as pictured to the right. Each side of the octagon has length 10. Find the area of the square to the *nearest whole number*.



<u>ANSWERS</u>

(1 pt.) 1.__________________________ square inches

(2 pts.) 2.__________________________ cubic inches

(3 pts.) 3.__________________________

South, St. John's, Doherty

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TEAM ROUND

All answers must be in <u>simplest exact form</u> (3 pts. each)

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1. The length of a rectangular field is 12 meters greater than twice its width. It cost \$1,228.50 to put a fence around the field at a cost of \$5.25 per meter. Find the width of the field in meters.

2. One solution of the equation $2x^2 - 7x + p = 0$ is $x = 2$, where p is a constant. Find the other solution of the equation.

3. At a recent concert Kim bought four t-shirts and 6 bumper stickers for \$90. Marc bought six t-shirts and 4 bumper stickers and spent \$30 more than Kim. Find the cost of a single t-shirt (in dollars).

4. In a 3-mile race, Samantha can beat Sydney by ½ mile and Erica can beat Sydney by one-third of a mile. In a 5-mile race, by what fraction of a mile could Samantha beat Erica? Assume that no matter the distance, the rates that each of the runners run are constant.

5. A quadrilateral has 3 sides of lengths 5.5, 6.5 and 7.5 meters. The length of the fourth side (in meters) is a positive integer. How many possible lengths can the fourth side have?

6. On the space provided on the answer sheet, graph the solution set of:
$$|2x| \geq |x+3|$$

7. Some time ago, Steven bought some stock from the coffee company *Sunbanks* for \$1,000. The price of the stock doubled every year for the next n years after his purchase. In the following year, the stock price fell by 99%. Nevertheless the stock was still worth more than the \$1,000 that he paid for it. What is the smallest whole number of years n for which this is possible?

8. There are five marbles in a bag, which other than their color, are identical. There are 2 red marbles, 1 blue marble, 1 green marble and 1 black marble. How many blue marbles must be added to the bag so that the probability of randomly drawing a blue marble followed by a red marble (without replacement) is $\frac{5}{36}$?

St. Peter-Marian, St. John's, Assabet Valley, Worcester Academy, QSC (5, 7), Bromfield (6, 8)

Worcester County Mathematics League

All answers must be in simplest exact form!

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ANSWER SHEET – TEAM ROUND

All answers must be in simplest exact form (3 pts. each)

1.________________________________ meters

2.________________________________

3.________________________________

4. _______________________________ of a mile

5. _______________________________

6. ←――――――――――――――――――――――――――――――――→

7.________________________________ years

8.________________________________

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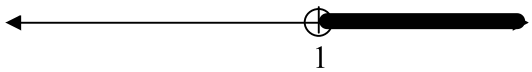
ANSWERS

Round 1

1. 

2. 

3. 

Round 2

1. $4x - 2xy + y$ (or equivalent)

2. $5a + 7$ (or equivalent)

3. $3xy^2(4x+3y)(x-2y)$ (or equivalent)

Round 3

1. 120

2. 48

3. $\dfrac{21}{38}$ (only)

Round 4

1. 15

2. $115.5 = 115\tfrac{1}{2} = \dfrac{231}{2}$

3. 291

Team Round

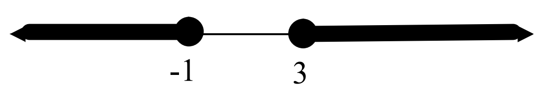
1. 35

2. $1.5 = 1\tfrac{1}{2} = \dfrac{3}{2}$

3. 18 (or \$18)

4. $\dfrac{5}{16} = 0.3125$

5. 19

6. 

7. 7

8. 4

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BRIEF SOLUTIONS

Round 1

1. $8 > 5 - 3x \ge -13 \Rightarrow 3 > -3x \ge -18 \Rightarrow -1 < x \le 6$

2. First, multiply by 6 to clear the fractions: $6 \cdot \left(\frac{1}{3}(4x-1) - \frac{3}{2}(2x+1) \le \frac{19}{6} \right) \Rightarrow 2(4x-1) - 9(2x+1) \le 19$

   Next, solve $\Rightarrow 8x - 2 - 18x - 9 \le 19 \Rightarrow -10x \le 30 \Rightarrow x \ge -3$.

3. The inequality represents a disjunction: $-(9-4x) > 2x+3$ OR $9-4x < 2x+3$. In the first case, we have $-9+4x > 2x+3 \Rightarrow 2x > 12 \Rightarrow x > 6$. In the second case, we have $-6x < -6 \Rightarrow x > 1$. The solution set is the union of these two sets $\Rightarrow x > 1$.

Round 2

1. Call the desired polynomial P. We have $3x - xy - y + P = 7x - 3xy \Rightarrow P = 4x + y - 2xy$.

2. $6a - [(3a+1) + (3a - 6 - (a+2) - 3a) - a] = 6a - [(3a+1) + (-6 - a - 2) - a)] = 6a - (a - 7) = 5a + 7$

3. First remove the GCF and then factor the remaining trinomial: $12x^3y^2 - 15x^2y^3 - 18xy^4$
   $= 3xy^2(4x^2 - 5xy - 6y^2) = 3xy^2(4x+3y)(x-2y)$

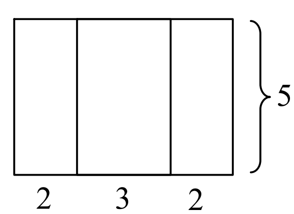
Round 3

1. This is simply a combination: ${}_{10}C_3 = \frac{10!}{7!3!} = 120$.

2. One way: first assume that Mom is to the left of Dad. Then there are 4 possibilities for their location: MF_ _ _, _MF_ _, _ _MF_, and _ _ _MF. In these arrangements there are 6 possibilities for the remaining 3 members of the family. This leads to $4 \cdot 6 = 24$ arrangements where Mom is on the left of Dad. Therefore, since Mom could also be on the right of Dad, there are a total of $2 \cdot 24 = 48$ arrangements.

3. The size of the sample space is the number of ways to select 5 students from a group of 20 or ${}_{20}C_5 = 15,504$. Next, there are ${}_{18}C_5 = 8,568$ ways to select the group with you and your friend <u>not</u> being one the five selected. Therefore, the probability is $\frac{8,568}{15,504} = \frac{21}{38}$.

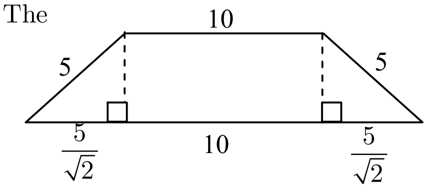
Round 4



1. The bottom side of the figure must be divided as the diagram to the right shows. Therefore, the area of the middle rectangle is $3 \cdot 5 = 15$

2. The dimensions of the box are 1.5, $10 - 2(1.5) = 7$, and $14 - 2(1.5) = 11$. Therefore, its volume is $1.5 \times 7 \times 11 = 115.5$.

3. The regions between the octagon and square are isosceles trapezoids whose "upper base" has length 10 and whose legs have length 5. Also, the angles of the trapezoid are $135°$ and $45°$. We are interested in finding the length of the lower base of the trapezoid (this is the side of the square). So, we draw altitudes in the trapezoid and use the 45-45-90 triangles that are created. See the diagram. The length of the lower base is $\frac{10}{\sqrt{2}}+10$. So, the area of the square is $\left(\frac{10}{\sqrt{2}}+10\right)^2 = 150+100\sqrt{2} \approx 291$ (let the calculator do the work!)



Team Round

1. Let w be the width of the field. Then, $5.25(6w+24) = 1,228.50 \Rightarrow 31.5w + 126 = 1,228.50 \Rightarrow w = 35$.

2. If $x = 2$ is a solution, then $2 \cdot 2^2 - 7 \cdot 2 + p = 0 \Rightarrow p = 6$. So, the equation is $2x^2 - 7x + 6 = 0$ $\Rightarrow (x-2)(2x-3) = 0 \Rightarrow x = \frac{3}{2}$.

3. Let $T =$ the cost of a t-shirt and $B =$ the cost of a bumper sticker. The given information gives us two equations: $4T + 6B = 90$ and $6T + 4B = 120$. Solving these equations simultaneously yields $B = 3$ and $T = 18$.

4. Assume that it takes Samantha 60 minutes to complete the 3-mile race. Then in 60 minutes Sydney ran 2.5 miles. So, Sydney can run is 1 mile in 24 minutes. In the second race (between Erica and Sydney), Sydney runs $2\frac{2}{3}$ miles. She needs $24 \cdot 2\frac{2}{3} = 64$ minutes to do this. Hence, Erica can run 3 miles in 64 minutes or 1 mile every $21\frac{1}{3}$ minutes. Now, Samantha will finish a five-mile race in 100 minutes. So, in 100 minutes, Erica will have completed $\frac{100}{21\frac{1}{3}} = 4.6875$ miles; and thus Samantha will beat Erica by $5 - 4.6785 = 0.3125$ miles.

5. The "quadrilateral inequality theorem" states that any three sides of a quadrilateral must sum to more than the length of the fourth side. The fourth side of the quadrilateral can be any of 1, 2, 3, …,19 $< 5.5 + 6.5 + 7.5 = 19.5$. There are 19 possibilities.

6. A clever way(?): square both sides. $4x^2 \geq x^2 + 6x + 9 \Rightarrow 3x^2 - 6x - 9 \geq 0 \Rightarrow 3(x^2 - 2x - 3) \geq 0$ $\Rightarrow 3(x-3)(x+1) \geq 0 \Rightarrow x \geq 3$ or $x \leq -1$.

7. The given information can be translated into the following inequality: $0.01(2^n \cdot 1,000) > 1,000$. We then solve for 2^n: $0.01(2^n \cdot 1,000) > 1,000 \Rightarrow 2^n \cdot 1,000 > 100,000 \Rightarrow 2^n > 1,000$. The least whole number satisfying this inequality is $n = 7$.

8. Let B represent the number of blue marbles that need to be added to the bag. We need $\frac{B+1}{B+5} \cdot \frac{2}{B+4} = \frac{5}{36}$. Solving gives us: $72(B+1) = 5(B+5)(B+4) \Rightarrow 72 + 72B = 5B^2 + 45B + 100 \Rightarrow 5B^2 - 27B + 28 = 0$ $\Rightarrow (5B-7)(B-4) = 0 \Rightarrow B = 4$ (we ignore the other solution because we need a whole number of marbles).