Worcester County Mathematics League

Freshman Meet 3 – March 7, 2007

Round 1: Graphing on a Number Line

1

NO CALCULATOR ALLOWED

Carefully draw the graph of each of the solution sets for the following inequalities on the corresponding number line provided below. Please specify all endpoints on your graph.

1. $7 \geq 3 - 2x \geq 1$

2. $|1 - (2 - x)| < 4$

3. $x^3 > 4x$

ANSWERS

(1 pt.) 1.

(2 pts.) 2.

(3 pts.) 3.

Shrewsbury, Notre Dame, QSC

WORCESTER COUNTY MATHEMATICS LEAGUE

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Round 2: Operations on Polynomials

2

All answers must be in simplest exact form
NO CALCULATOR ALLOWED

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1. Subtract $3x^2 - y^2$ from the sum of $5xy - 2x^2 + y^2$ and $3y^2 - 4xy$. Express your answer as a single polynomial in terms of *x* and *y*. Do NOT factor your answer.

2. Simplify the following expression to a single polynomial:
$$(x+y)(x^2 - xy + y^2) - (x-y)(x^2 + xy + y^2)$$

3. If $A * B = (A-B)^2$ and $A \Delta B = A^2 - B^2$, then evaluate the following expression to a single polynomial in terms of *x*. Do NOT factor your answer.
$$\left[(x^2 - 3x + 1) * (x^2 - 4x + 2)\right] \Delta (x+1)^2$$

ANSWERS

(1 pt.) 1.__________________________

(2 pts.) 2.__________________________

(3 pts.) 3.__________________________

Shepherd Hill, Doherty, Worcester Acad.

WORCESTER COUNTY MATHEMATICS LEAGUE

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Round 3: Techniques of Counting and Probability

3

All answers must be in simplest exact form

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1. The president, vice president, secretary, and treasurer of the math team are to be seated in a row of 4 chairs for a yearbook picture. How many different seating arrangements are possible?

2. How many ways can six card players be seated around a circular table?

3. Suppose that you have a pair of brown socks and a pair of black socks in your dresser drawer. Besides their color, the socks are identical. On a dark morning, without being able to see anything, you reach into the drawer and randomly pull out 2 socks. What is the probability that you pull out a matching pair of socks? Express your answer as a fraction, reduced to lowest terms.

ANSWERS

(1 pt.) 1.________________________

(2 pts.) 2.________________________

(3 pts.) 3.________________________

Shepherd Hill, W. Boylston, Algonquin

WORCESTER COUNTY MATHEMATICS LEAGUE

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Round 4: Perimeter, Area and Volume

4

All answers must be in simplest exact form <u>except for #3</u>

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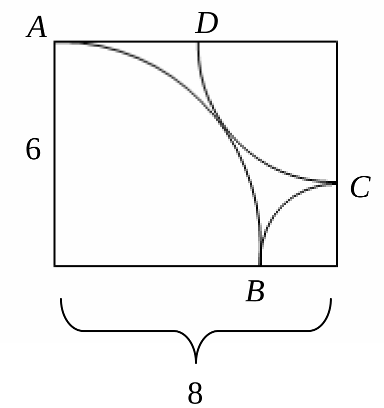
1. The side of one square is 3 cm longer than the side of a second square. The area of the second square is 51 sq. cm. less than the area of the first square. Find the length of each side of the first square in centimeters.

2. Suppose that $\ell(PQ)$ represents the length of circular arc PQ. In the diagram, if the circular arcs are centered at three of the vertices of the 6 by 8 rectangle and are tangent to each other as shown, find $\ell(AB) + \ell(BC) + \ell(CD)$ in terms of π.



3. An aquarium, shaped as a rectangular solid, is 2 feet long, 11 inches wide and is filled with water to a depth of 13 inches. If 1 cubic inch of water weighs 0.0361 pounds, find the weight of the water in the aquarium to the *nearest whole pound*.

<u>ANSWERS</u>

(1 pt.) 1._________________________ cm

(2 pts.) 2._________________________

(3 pts.) 3._________________________pounds

Shepherd Hill, Worcester Acad., Bartlett

WORCESTER COUNTY MATHEMATICS LEAGUE

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TEAM ROUND

All answers must be in <u>simplest exact form</u> (3 pts. each)

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1. A runner completes a quarter-mile lap in exactly 99 seconds. Assuming that she runs at a constant rate, how many minutes will it take her to run $3\frac{1}{3}$ miles?

2. What is the smallest number in base-10 which is greater than 10 and for which the sums of the digits in base-4 and base-6 are equal?

3. On five tests (on which scores could vary from 0 to 100 points), Chris had an average of exactly 88. Find the lowest score Chris could have received on one of these tests.

4. When n is divided by 3, the remainder is 2. What is the remainder when $5n$ is divided by 3?

5. The height of a rectangular solid is 6 inches and the solid has a total surface area of 264 square inches. If the width of the solid's base is 2 inches less than its length, find the volume of the solid in cubic inches.

6. On the space provided on the answer sheet, graph the solution set of:
$$\frac{x}{x-1} \geq 0$$

7. What is the smallest total number of marbles which can be placed in 8 boxes if each box is to contain a different positive number of marbles?

8. Factor the following expression as the product of two trinomials:
$$x^2 + 2xy - 16z^2 + y^2$$

Douglas, Worc. Acad., Burncoat, Algonquin, QSC (5, 6), Hudson, Auburn

WORCESTER COUNTY MATHEMATICS LEAGUE

All answers must be in simplest exact form!

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ANSWER SHEET – TEAM ROUND

All answers must be in simplest exact form (3 pts. each)

1.________________________________ minutes

2.________________________________

3.________________________________

4. _______________________________

5. _______________________________ cubic inches

6.

7.________________________________

8.________________________________

WORCESTER COUNTY MATHEMATICS LEAGUE

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ANSWERS

Round 1

1. 

2. 

3. 

Round 2

1. $-5x^2 + xy + 5y^2$ (or equivalent)

2. $2y^3$

3. $-8x^3 - 8x$ (or equivalent, not factored)

Round 3

1. 24

2. 120

3. $\frac{1}{3}$

Round 4

1. 10

2. 6π

3. 124

Team Round

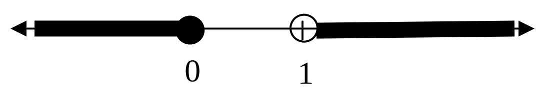
1. 22

2. 18

3. 40

4. 1

5. 288

6. 

7. 36

8. $(x + y - 4z)(x + y + 4z)$
(or equivalent)

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BRIEF SOLUTIONS

Round 1

1. $7 \geq 3-2x \geq 1 \Rightarrow 4 \geq -2x \geq -2 \Rightarrow -2 \leq x \leq 1$.

2. $|1-(2-x)| < 4 \Rightarrow -4 < 1-(2-x) < 4 \Rightarrow -5 < -2+x < 3 \Rightarrow -3 < x < 5$

3. $x^3 > 4x \Rightarrow x^3 - 4x > 0 \Rightarrow x(x^2-4) > 0 \Rightarrow x(x+2)(x-2) > 0 \Rightarrow -2 < x < 0$ or $x > 2$.

Round 2

1. The sum is $4y^2 + xy - 2x^2$. So, $4y^2 + xy - 2x^2 - (3x^2 - y^2) = 5y^2 + xy - 5x^2$

2. One way, using the sum and difference of cubes factorization:
$(x+y)(x^2-xy+y^2) - (x-y)(x^2+xy+y^2) = x^3 + y^3 - (x^3 - y^3) = 2y^3$

3. $\left[(x^2-3x+1) * (x^2-4x+2)\right]\Delta(x+1)^2 = (x-1)^2\Delta(x+1)^2 = (x-1)^4 - (x+1)^4$
$= x^4 - 4x^3 + 6x^2 - 4x + 1 - (x^4 + 4x^3 + 6x^2 + 4x + 1) = -8x^3 - 8x$

Round 3

1. The number of arrangements is simply $4! = 24$.

2. This is an example of a circular permutation. The number of ways to arrange n distinct objects along a fixed circle is $(n-1)!$. The solution is $(6-1)! = 5! = 120$.

3. The sample space is ${}_4C_2 = 6$. There are 2 successes: both socks being brown and both socks being black. Therefore, the probability is $\frac{2}{6} = \frac{1}{3}$.

Round 4

1. Let x be the side of the second square. Then the side of the first square is $x + 3$. So, the area of the first square is $(x+3)^2 = x^2 + 6x + 9$ and the area of the second square is x^2. Thus, $x^2 + 6x + 9 - x^2 = 51$
$\Rightarrow 6x + 9 = 51 \Rightarrow x = 7$.

2. Each of the arcs are quarter circles. $\ell(AB) = \frac{1}{4}\cdot 2\pi \cdot 6 = 3\pi$, $\ell(BC) = \frac{1}{4}\cdot 2\pi\cdot 2 = \pi$, and $\ell(CD) = \frac{1}{4}\cdot 2\pi \cdot 4 = 2\pi$. Therefore, the sum is $3\pi + \pi + 2\pi = 6\pi$.

3. The volume of the water is $24 \cdot 11 \cdot 13 = 3,432$ cubic inches. Therefore, the weight is $3,432 \times 0.0361 \approx 124$ pounds.

Team Round

1. She completes 1 mile in $99 \times 4 = 396$ seconds. She runs $3\frac{1}{3}$ miles in $3\frac{1}{3} \times 396$ seconds = 1,320 seconds = 22 minutes.

2. One way: start to catalogue the numbers starting with 11 base-10

Base-10 #	Base-4 #	Sum of Digits	Base-6 #	Sum of digits
11	23	5	15	6
12	30	3	20	2
13	31	4	21	3
14	32	5	22	4
15	33	6	23	5
16	100	1	24	6
17	101	2	25	7
18	102	3	30	3

3. The sum of the scores is $88 \times 5 = 440$. Assuming that he scored 100 on 4 tests, he could have only scored a 40 on the fifth test.

4. $n = 3q + 2 \Rightarrow 5n = 15q + 10 \Rightarrow 5n = 3(5q + 3) + 1 \Rightarrow$ the remainder is 1.

5. Let the base edges be x and $x - 2$. Then the surface area is $2 \cdot 6x + 2 \cdot 6(x - 2) + 2 \cdot x(x - 2) = 2x^2 + 20x - 24$. Thus, $2x^2 + 20x - 24 = 264 \Rightarrow x^2 + 10x - 144 = 0 \Rightarrow (x - 8)(x + 18) = 0 \Rightarrow x = 8$. So, the dimensions are 8, 6 and 6 and the volume is $8 \cdot 6 \cdot 6 = 288$.

6. $\frac{x}{x-1} \geq 0 \Rightarrow x \leq 0$ or $x > 1$.

7. Let one box contain 1 marble, then the others must contain 2, 3, 4, 5, 6, 7, and 8. The sum is 36.

8. Re-order the expression and then factor: $x^2 + 2xy + y^2 - 16z^2 = (x + y)^2 - 16z^2 = (x + y - 4z)(x + y + 4z)$.