

WORCESTER COUNTY MATHEMATICS LEAGUE

Freshman Meet 2 – January 11, 2006

Round 1: Algebraic Word Problems

1

All answers must be in simplest exact form

1. If a certain number is added to the numerator and denominator of $\frac{7}{9}$, the result is $\frac{5}{6}$.
Find the number.
2. How many gallons of a 20% alcohol solution must be mixed with a 50% alcohol solution to get 9 gallons of a 40% alcohol solution?
3. John, by himself, can mow a lawn in 90 minutes. On the other hand, Pete can do it alone in 60 minutes. If Pete joins John 10 minutes after John started mowing, how many total minutes are needed to mow the lawn?

ANSWERS

(1 pt.) 1. _____

(2 pts.) 2. _____ gallons

(3 pts.) 3. _____ minutes

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Round 2: Number Theory



All answers must be in simplest exact form

NO CALCULATOR ALLOWED

1. Find the greatest common factor of 56, 70 and 84.

2. In base 12, the digits are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A and B. Write the number 560_{10} in base 12. (Note: the subscript indicates the number's base.)

3. Find the sum of the two different smallest counting numbers which have exactly six positive factors (note: the factors of a number include 1 and the number).

ANSWERS

(1 pt.) 1. _____

(2 pts.) 2. _____

(3 pts.) 3. _____

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Round 3: Operations on Fractions, Decimals, Percents
and Percentage Word Problems

3

All answers must be in simplest exact form

NO CALCULATOR ALLOWED

1. Simplify the following fraction: $\frac{3\frac{2}{3} - 2\frac{1}{2}}{\frac{3}{4} + \frac{5}{6}}$
2. A class of 20 students averaged 66% on an examination. Another class of 30 students averaged 56% on the same exam. Find the average percentage on the exam for all 50 students combined.
3. A jar contains only quarters, nickels and pennies. Six of the coins are quarters, 25 percent of the coins are nickels and $\frac{5}{8}$ of the coins are pennies. How many coins are in the jar?

ANSWERS

(1 pt.) 1. _____

(2 pts.) 2. _____%

(3 pts.) 3. _____ coins

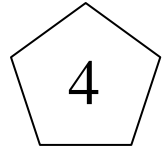
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Round 4: Set Theory

All answers must be in simplest exact form

NOTE: S' indicates the complement of the set S

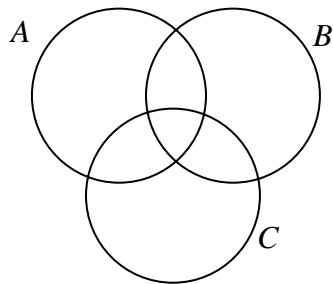


- Using the Venn diagram in the answer section below, shade the region represented by the set $(A \cup B) \cap C$.
- After the math meet, 20 teachers ate at Mac's Place. Three of the teachers had only a burger; three had a burger and fries; one had fries and a shake; five teachers had only a shake; four had a burger and a shake; two teachers had a burger a shake and fries. How many teachers ate only fries?
- Let $A = \{\text{even, non-negative whole numbers}\}$, $B = \{\text{even integers less than 12}\}$, $C = \{\text{integers that are not multiples of three}\}$, and the universal set $U = \{\text{all integers}\}$. Find $(C' \cap A) \cap B$.

ANSWERS

(1 pt.)

1.



(2 pts.)

2. _____

(3 pts.)

3. _____

WORCESTER COUNTY MATHEMATICS LEAGUE
Freshman Meet 2 – January 11, 2006
TEAM ROUND

All answers must *either* be in simplest exact form or as decimals rounded correctly to at least three decimal places! (3 pts. each)

1. Thirty-three and a third percent of what positive number is 75% of the number's reciprocal?
2. A number is 2 less than 3 times its square root. Find all such numbers.
3. One hundred and fifty-six pennies are to be divided among three math teachers in the ratio $\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$. How many pennies will the teacher receiving the fewest get?
4. Find the value of b so that the point $(2\frac{1}{2}, 5)$ lies on the line described by the equation $y = bx - \frac{5}{2}$.
5. How many whole numbers from 100 to 1000 contain the digit 6 exactly twice?
6. Let sets R and S be subsets of $U = \{1, 2, 3, 4, 5, 6\}$. Specify S by a roster if $R = \{2, 4, 6\}$, $R \cap S = \{2, 6\}$, and $R \cup S = \{1, 2, 3, 4, 6\}$.
7. The perimeter of a rectangle is 74 and the area is 336. If the length of the rectangle is greater than its width, find the length.
8. Find the number of integers between 1 and 100 (inclusive) which are not divisible by 2, 3, 5 or 7.

WORCESTER COUNTY MATHEMATICS LEAGUE

All answers must be in simplest exact form!

Freshman Meet 2 – January 11, 2006

ANSWER SHEET – TEAM ROUND

All answers must *either* be in simplest exact form *or* as decimals rounded correctly to at least three decimal places! (3 pts. each)

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

7. _____

8. _____

WORCESTER COUNTY MATHEMATICS LEAGUE

Freshman Meet 2 – January 11, 2006

ANSWERS

Round 1

1. 3
2. 3
3. 42


Round 2

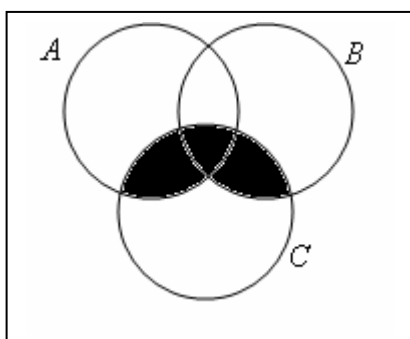
1. 14
2. $3A8$ or $3A8_{12}$
3. 30

Round 3

1. $\frac{14}{19}$
2. 60
3. 48

Round 4

1. 
2. 2
3. $\{0, 6\}$



Team Round

1. $1.5 = 1\frac{1}{2} = \frac{3}{2}$
2. 1 and 4 (need both)
3. 36
4. 3
5. 26
6. $\{1, 2, 3, 6\}$ (any order)
7. 21
8. 22

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BRIEF SOLUTIONS

Round 1

1. Let x be the number. Then, $\frac{7+x}{9+x} = \frac{5}{6} \Rightarrow 42 + 6x = 45 + 5x \Rightarrow x = 3$.
2. Let x be the amount of 20% solution required. Then, $0.2x + 0.5(9-x) = 9(0.4) \Rightarrow x = 3$.
3. John's rate is $\frac{1}{90}$ and Pete's rate is $\frac{1}{60}$. Let x be the number of minutes required. Then,
$$\frac{1}{60}(x-10) + \frac{1}{90}x = 1 \Rightarrow x = 42.$$

Round 2

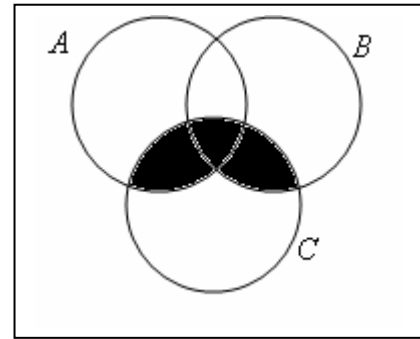
1. "Prime factorize" 56, 70 and 84: $56 = 2^3 \cdot 7$, $70 = 2 \cdot 5 \cdot 7$, $84 = 2^2 \cdot 3 \cdot 7$. Therefore, the GCF is $2 \cdot 7 = 14$.
2. $560 = 3 \cdot 12^2 + 10 \cdot 12 + 8 \Rightarrow 560_{10} = 3A8_{12}$
3. The two counting numbers with 6 factors are $12 = 2^2 \cdot 3$ and $18 = 2 \cdot 3^2$. The sum is 30.

Round 3

1.
$$\frac{\frac{11}{3} - \frac{5}{2}}{\frac{3}{4} + \frac{5}{6}} = \frac{44-30}{9+10} = \frac{14}{19}$$
2. The sum of the scores of the 20 students who averaged 66% is $20 \cdot 66 = 1320$. The sum of the scores of the 30 students who averaged 56% is $30 \cdot 56 = 1680$. The sum of all 50 scores is therefore $1320+1680 = 3000$. Hence, the average of all 50 scores is $\frac{3000}{50} = 60\%$.
3. $\frac{1}{4}$ are nickels and $\frac{5}{8}$ are pennies $\Rightarrow \frac{7}{8}$ are nickels and pennies. Therefore, $\frac{1}{8}$ are quarters. Since there are 6 quarters, there are 48 coins total.

Round 4

1. "Cross-hatch" the Venn diagram. The solution is:
2. There are 20 teachers. The number of teachers who ate something other than just fries is $3 + 3 + 2 + 4 + 1 + 5 = 18$. Hence, only 2 teachers ate only fries.



3. $C' = \{\text{multiples of 3}\}$. Hence, $C' \cap A = \{\text{positive, even multiples of 3}\}$. Then, $(C' \cap A) \cap B = \{0, 6\}$

Team Round

1. $\frac{1}{3}x = \frac{3}{4} \cdot \frac{1}{x} \Rightarrow x = \frac{3}{2}$
2. $x = 3\sqrt{x} - 2 \Rightarrow (x+2)^2 = 9x \Rightarrow x^2 - 5x + 4 = 0 \Rightarrow (x-1)(x-4) = 0 \Rightarrow x = 1, 4$.
3. $\frac{1}{2}x + \frac{1}{3}x + \frac{1}{4}x = 156 \Rightarrow 6x + 4x + 3x = 156 \cdot 12 \Rightarrow 13x = 156 \cdot 12 \Rightarrow x = 144$. The teacher with the fewest number will have $144 \cdot \frac{1}{4} = 36$.
4. $5 = \frac{5}{2}x - \frac{5}{2} \Rightarrow x = 3$.
5. Carefully count them: 166, 266, ..., 566, 766, 866, 966 yields 8. Next, 606, 616, ..., 656, 676, 686, 696 yields 9. Finally, 606, 660, 661, ..., 665, 667, 668, 669 yield 9. Therefore there are $8 + 9 + 9 = 26$ total numbers.
6. S must contain 1, 2, 3 and 6.
7. Let l and w be the length and the width. Then, $l + w = 37$ and $lw = 336$. Solving simultaneously gives $(37 - l)l = 336 \Rightarrow l^2 - 37l + 336 = 0 \Rightarrow (l - 21)(l - 16) = 0$. Therefore the length is 21.
8. The numbers are 1 and all of the primes from 1 to 100, except 2, 3, 5, and 7. There are 25 primes less than 100, hence there are $25 + 1 - 4 = 22$ numbers.