

Worcester County Mathematics League

Freshman Meet 2 – January 7, 2004

Round 2: Number Theory

All answers must be in simplest exact form!

NO CALCULATOR ALLOWED!

1. The sum $110_8 + 110_2$ is equal to what number in base ten? (Note: the subscript indicates the number's base)

2. The greatest common factor of two numbers is 2 and their least common multiple is 2160. If one of the numbers is 80, what is the other number?

3. Find the *sum* of the greatest common factor and least common multiple for the set $\{108, 162, 216\}$.

ANSWERS

(1 pt.) 1. _____

(2 pts.) 2. _____

(3 pts.) 3. _____

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Round 3: Operations on Fractions, Decimals, Percents
and Percentage Word Problems

All answers must be in simplest exact form!

NO CALCULATOR ALLOWED!

1. First, Jim paints two-fifths of a fence. Joan then paints one-half of what is left. What fraction of the fence is left unpainted?

2. If a worker receives a 20% reduction in his wages, find the percent raise required for him to obtain his original salary.

3. Express the difference $0.\overline{5} - 0.\overline{18}$ as a common fraction.

ANSWERS

(1 pt.) 1. _____

(2 pts.) 2. _____ percent

(3 pts.) 3. _____

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Round 4: Techniques of Counting and Probability

For your answers: follow the directions in each problem.

1. A jar contains two red, four blue, and five white marbles. If a marble is drawn at random, what is the probability that the marble is not red? Express your answer as a reduced fraction.

2. If you rolled a fair, standard pair of six-sided dice, what is the probability that the sum showing on the two dice is 9 or more? Express your answer as a reduced fraction.

3. At McKing's fast food restaurant, you can order a hamburger with any of the following "toppings": cheese, onion, pickle, relish, mustard, lettuce, tomato, and mayonnaise. How many different combinations of these toppings can you order, choosing *at least* 1 and *at most* 4 of them?

ANSWERS

(1 pt.) 1. _____

(2 pts.) 2. _____

(3 pts.) 3. _____

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TEAM ROUND

All answers must *either* be in simplest exact form *or* as decimals rounded correctly to at least three decimal places! (3 pts. each)

1. Ann can ride 17 kilometers on her bicycle in the same amount of time that it takes her to walk 9 kilometers. If her riding speed is 4 km/h faster than her walking speed, find her walking speed in kilometers per hour.
2. If $\sqrt{2 + \sqrt{x}} = 3$, find x .
3. The radii of two concentric circles are in the ratio 4:3. If the area of the larger circle is 16π , find the area of the smaller circle. (If necessary for approximating your answer, use $\pi \approx 3.14159$.)
4. Find the value for the digit K that makes the base ten number K43,4K0 divisible by 36.
5. In a certain class, two-thirds of the female students and one-half of the male students speak Spanish. If there are three-fourths as many girls as boys in the class, what fraction of the entire class speaks Spanish?
6. For what value of m are the lines $2x + 3y = 4m$ and $x - 2my = 7$ perpendicular?
7. What is the degree measure of the angle formed between the hour hand and the minute hand when a standard, non-digital clock reads 2:20 p.m.?
8. How many four-digit positive integers, when represented in base-ten, contain at least one three?

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All answers must be in simplest exact form!

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ANSWER SHEET – TEAM ROUND

All answers must *either* be in simplest exact form or as decimals rounded correctly to at least three decimal places! (3 pts. each)

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

7. _____

8. _____

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ANSWERS

Round 1

1. 15, 17, 19

2. 29

3. $\frac{1}{4}$

Round 2

1. 78

2. 54

3. 702

Round 3

1. $\frac{3}{10} = 0.3$

2. 25

3. $\frac{11}{30}$

Round 4

1. $\frac{9}{11}$ only

2. $\frac{5}{18}$ only

3. 162

Team Round

1. $4.5 = 4\frac{1}{2} = \frac{9}{2}$ (km/h not needed)

2. 49

3. $9\pi \approx 28.274$

4. 8

5. $\frac{4}{7} = 0.\overline{571428} \approx 0.571$

6. $\frac{1}{3} = 0.\overline{3} \approx 0.333$

7. 50 (or 50 degrees)

8. 3168

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SOLUTIONS

Round 1

1. Call the integers x , $x + 2$ and $x + 4$. Then, $2x = x + 4 + 11 \Rightarrow x = 15$. So, the integers are 15, 17 and 19.
2. Let x be the age of the son now. So, Mrs. Neutron is $x + 19$ years old now. Also, in fifteen years, Mrs. Neutron will be $x + 34$ years old and her son will be $x + 15$ years old. Using the given info to set up an equation we have
 $x + 34 + 6 = 2(x + 15) \Rightarrow x = 10$. Mrs. Neutron's son is 10 years old, which makes her 29 years old.
3. Set up and solve an equation for the ratio $\frac{A}{B}$: $0.30A + 0.45B = 0.42(A + B) \Rightarrow \frac{A}{B} = \frac{1}{4}$.

Round 2

1. Convert to base ten first and then add. $110_8 = 72$ and $110_2 = 6 \Rightarrow 110_8 + 110_2 = 78$.
2. For any two positive integers, their product is equal to the product of their GCF and LCM. So, $80x = 2 \cdot 2160 \Rightarrow x = 54$.
3. Prime factor each of the three numbers to determine the GCF and LCM for the set.
 $108 = 2^2 \cdot 3^3$; $162 = 2 \cdot 3^4$; and $216 = 2^3 \cdot 3^3$. So, the GCF = $2 \cdot 3^3 = 54$ and the LCM = $2^3 \cdot 3^4 = 648$. Their sum is 702.

Round 3

1. Three-fifths of the fence is unpainted after Jim paints. Joan paints half of this, which leaves $\frac{1}{2} \cdot \frac{3}{5} = \frac{3}{10}$ unpainted.
2. Call the worker's original salary x . So after the 20% reduction the worker's new salary is $0.8x$. Let p be the increase needed to bring the worker's salary back to x . So, $p \cdot 0.8x = x \Rightarrow p = 1.25$. The result is a 25% increase.

$$3. 0.\bar{5} = \frac{5}{9} \text{ and } 0.1\bar{8} = \frac{17}{90} \Rightarrow 0.\bar{5} - 0.1\bar{8} = \frac{5}{9} - \frac{17}{90} = \frac{33}{90} = \frac{11}{30}.$$

Round 4

1. There are 11 marbles in the jar, and 9 of them are not red. So, the probability is $\frac{9}{11}$.
2. There is 1 way to roll a "12" {(6,6)}; 2 ways to roll an "11" {(5,6) and (6,5)}; 3 ways to roll a "10" {(5,5); (6,4); and (4,6)}; and there are 4 ways to roll a "9" {(5,4); (4,5); (6,3) and (3,6)}. There are 36 unique rolls of two dice, so the probability of rolling a 9 or higher is $\frac{1+2+3+4}{36} = \frac{10}{36} = \frac{5}{18}$.
3. There are 8 toppings. There are ${}_8C_1 = 8$ ways to choose one topping; ${}_8C_2 = 28$ ways to choose two toppings; ${}_8C_3 = 56$ ways to choose exactly 3 toppings and ${}_8C_4 = 70$ ways to choose 4 toppings. In total, there are $8 + 28 + 56 + 70 = 162$ ways to choose at least one and at most four toppings.

Team Round

1. Set up and solve a proportion. Let x be her walking speed. Then,

$$\frac{17}{x+4} = \frac{9}{x} \Rightarrow x = 4.5.$$
2. Square both sides, subtract 2 from both sides and then square again:

$$\sqrt{2+\sqrt{x}} = 3 \Rightarrow 2+\sqrt{x} = 9 \Rightarrow \sqrt{x} = 7 \Rightarrow x = 49$$
3. Let K be the area of the smaller circle. Then, $\frac{K}{16\pi} = \left(\frac{3}{4}\right)^2 = \frac{9}{16} \Rightarrow K = 9\pi$.
4. In order for the given number to be divisible by 4, the last two digits must be divisible by 4. So, K must be even (K cannot be 0 else the leading digit would be 0). Also, in order to be divisible by 9, the sum of the numbers digits must sum to a multiple of 9. The sum of the digits is $11+2K$. The only even value of K that will make $11 + 2K$ divisible by 9 is 8.

5. Let m be the number of males in the class. Then there are $\frac{3}{4}m$ females,
 $\frac{2}{3} \cdot \frac{3}{4}m + \frac{1}{2}m = m$ students who speak Spanish; and there are $m + \frac{3}{4}m = \frac{7}{4}m$ students
 in the class. The fraction of students that speak Spanish is therefore $\frac{m}{\frac{7}{4}m} = \frac{4}{7}$.
6. The slope of $2x + 3y = 4m$ is $-\frac{2}{3}$. The slope of $x - 2my = 7$ is $\frac{1}{2m}$. The lines will
 be perpendicular when the slopes are “negative reciprocals” of each other. Hence,
 $-\frac{2}{3} = -2m \Rightarrow m = \frac{1}{3}$.
7. The “angle” between two consecutive numbers on a standard clock is 30° . At 2:20,
 the minute hand is pointing toward the 4 and the hour hand is $\frac{1}{3}$ of the way between
 the 2 and the 3 (closer to 2). So the angle is $30^\circ + \frac{2}{3} \cdot 30^\circ = 50^\circ$.
8. There are 9000 four-digit numbers. There are $8 \cdot 9 \cdot 9 \cdot 9 = 5832$ numbers that do not
 contain the digit 3. Hence there are $9000 - 5832 = 3168$ four-digit numbers that
 contain at least one three.