

WOCOMAL

Freshman Meet #3

March 5, 2003

**Hudson High School
Hudson, Massachusetts**

WoCoMaL

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Freshman Meet#3

ROUND#1: Algebraic Word Problems

1. What is the maximum distance a person can walk at 4 kilometers/hour and be able to return from on bicycle at 12 kilometers/hour and be gone no more than a total time of five hours ?
2. How much water should be added to 15 ounces of a solution that is 56 % water to produce a solution that is 75 % water ? Answer in ounces.

3. In a man's will, he leaves everything to his three sons. He specifies that the oldest and next oldest receive amounts in the ratio 4 to 3 and that the oldest and youngest receive amounts in the ratio of 2 to 1. If, however, one son does not survive him, then the other two split his share equally, and if two do not survive him, their shares go to WoCoMaL. The man dies and leaves an estate worth \$ 360,000. What is the largest possible inheritance for one of his sons ?

- Answer here:
1. (1 pt.) _____ kilometers
 2. (2 pts.) _____ ounces
 3. (3 pts.) \$ _____

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ROUND#2: Operations on Polynomials

1. Find and simplify this product. Answer in decreasing powers of x .

$$(x + 1)(x + 2)(x^2 - x + 1)(x - 2)$$

2. Calculate the remainder R if $x^5 + x^4 - x^3 + 2x = Q \cdot (x^2 + 1) + R$

where $0 \leq \text{degree of } R < 2$.

3. If $12x^2 + 5x - 2 = 12(x - a)(x - b)$, then compute and simplify $ab + a + b$.

Answer here: 1. (1 pt.) _____

2. (2 pts.) _____

3. (3 pts.) _____

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ROUND#3: Number Theory

<< No Calculators >>

1. When Alexander Graham Bell telephoned Thomas Watson, it took a number of minutes for the call to go through. The number of minutes is a two-digit prime number between 15 and 40. The sum of the digits is also a prime number and a factor of 40. How many minutes did it take for the call to go through ?
2. Add 123_{base4} , 123_{base5} , and 123_{base6} .
Then express the answer as a base 7 number.
3. Find the digit A if the five-digit number $12A3B$, with $A \neq B$, is divisible by both 4 and 9.

Answer here: 1. (1 pt.) _____minutes

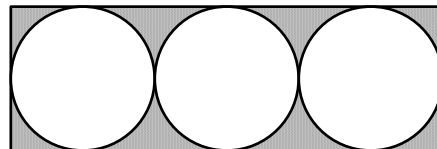
2. (2 pts.) _____ $_{base7}$

3. (3 pts.) _____

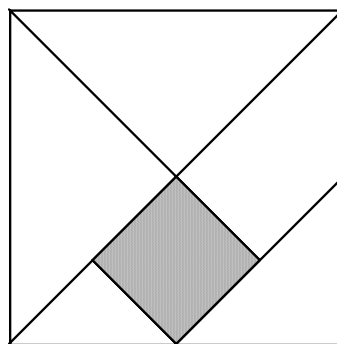
Shrewsbury High, Bancroft School, Notre Dame Academy

ROUND#4: Perimeter and Area, Volume

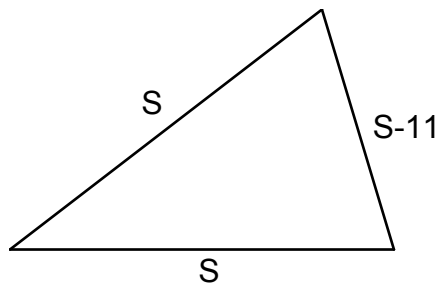
1. Each tangent circle has a radius of R . The shaded rectangular region has an area of $K \cdot R^2$. What is the exact value of K ?



2. In this figure, every point that looks like a midpoint is a midpoint. Calculate the ratio of the area of the shaded square to the area of the largest outer square.



3. If the perimeter of this isosceles triangle is 64, compute its area.



- Answer here:
1. (1 pt.) _____
 2. (2 pts.) _____
 3. (3 pts.) _____

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Team Round:

1. A recipe for bug juice uses only perfect insects (one head and six legs) and perfect spiders (one head and eight legs) as the main ingredients in the blend. If the recipe requires a 5 to 2 ratio of spiders' legs to insects' legs and a total of 345 heads are used, how many of the bugs were spiders ?
2. The area of a rectangle is $x^2 + 4x - 8$ sq. cm. The side of a square of equal area is $x + 1$ cm. Compute the decimal which is the exact area of the square in cm.².
3. A number is Beprisque if it is the only natural number between a prime natural number and a perfect square natural number. [Example: 10 is Beprisque because it is between 11 and 9, but 12 is not Beprisque.] How many Beprisque numbers are there less than 100 ?
4. The new square tiles that I put on my kitchen floor each have a perimeter of 18 inches. No tiles had to be cut, and I used a total of 384 of them to cover the perfectly rectangular floor. If the floor's perimeter is as small as possible, compute the floor's area in square feet. [1 foot = 12 inches]
5. A worker in an apple orchard was filling two boxes of apples. He found that box A had 20 more apples than box B, so he took two-fifths of the apples from box A and transferred them to box B. After checking both boxes again, he took 10 of the apples from box B and put them in box A. He now found that both boxes contained the same number of apples. Totally, how many apples were in the two boxes ?
6. Hat A contains 2 red and 4 blue marbles. Hat B contains 5 red and 3 blue marbles. A fair coin is tossed to determine which hat to choose, and one marble is drawn from it. What is the probability the marble is red ?
7. In a class of boys and girls, the 40 boys averaged 74.0 on a test, while the entire class averaged 78.0. At the very least, how many girls are in this class ? [Assume grades from 0 to 100.]
8. Snit River starts at Mt. Goben and flows 40 miles downstream to Mt. Lesen. Starting from halfway, Rog paddled downstream to Mt. Lesen. His trip from there all the way back to Mt. Goben took five times as long as the part downstream. If the current flows at 3 miles per hour, how long did the trip upstream take ?

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Team Round

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3 Points Each

Answers must be **exact** or rounded to **three decimal** places, except where stated otherwise.

Answers here ↓ :

1. _____ spiders

2. _____ cm.²

3. _____

4. _____ sq. ft.

5. _____ apples

6. _____

7. _____ girls

8. _____ hours

School: _____

Team#: _____

Players' Names ↓ :

#1: _____

#2: _____

#3: _____

#4: _____

#5: _____

WOCOMAL Answers Freshman Meet #3 March 5, 2003

R#1: 1. 15

2. $11.4 = 11\frac{2}{5}$

3. \$ 220,000

R#2: 1. $x^5 - 4x^3 + x^2 - 4$
[No other form is allowed.]

2. $4x + 1$

3. $-\frac{7}{12} = -0.58\bar{3} \approx -0.583$

Team: 1. 225

2. $30.25 = 30\frac{1}{4} = \frac{121}{4}$

3. Eight

4. 54

R#3: 1. 23

2. 224

3. 1

5. 80

6. $\frac{23}{48} = 0.479\bar{16} \approx 0.479$

7. 8

R#4: 1. $12 - 3\pi$ or $3(4 - \pi)$

2. $\frac{1}{8} = 0.125 = 12.5\% = 12\frac{1}{2}\%$

3. 168

8. 10 hours

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F3 - Solutions

Mar. 5, 2003

Round#1 1. The max. distance D occurs when all 5 hours are used moving.

$$\text{Equation is } \frac{D}{4} + \frac{D}{12} = 5. \text{ So, } D = 15 \text{ km.}$$

2. $100\%(W) + 56\%(15) = 75\%(W + 15)$ yields $W = 11.4$ oz.

3. If the father is survived by all three sons, they share the estate in the ratio 4 to 3 to 2, a total of nine parts. If the 2nd son dies before the father, the two remaining share in the ratio 5.5 to 3.5. The 5.5 out of nine parts is the most any son can ever expect.

$$\frac{5.5}{9}(\$360,000) = \$220,000.$$

Round#2 1. It's easier if one regroups as $[(x+2)(x-2)][(x+1)(x^2-x+1)] = (x^2-4)(x^3+1)$.

2. It is easiest to *long divide*.

3. Rewrite $12x^2 + 5x - 2 = 12(x-a)(x-b)$ as $\frac{12x^2 + 5x - 2}{12} = x^2 - (a+b)x + ab$
and notice that $a+b = -\frac{5}{12}$ and $ab = -\frac{2}{12}$, for a sum of $-\frac{7}{12}$.

Round#3 1. The candidates are 17, 19, 23, 29, 31, 37. Their sums are 8, 10, 5, 11, 4, 10.
Only 5 is prime and a divisor of 40. So, 23 minutes is the answer.

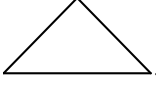
2. $123_{base4} + 123_{base5} + 123_{base6} = 27 + 38 + 51 = 116$, which converts into 224_7 .

3. If $3B$ is divisible by 4, then $B = 2$ or $B = 6$. If $12A3B$ is divisible by 9, then the sum $1+2+A+3+B = 6+A+B$ must also be divisible by 9. If $B = 2$, then $A = 1$.

If $B = 6$, then $A = 6$, which we reject because A and B are equal. So, $A = 1$.

Round#4 1. The rectangle is $2R$ by $6R$ and has area $12R^2$. The circles have total area $3\pi R^2$. So, $K = 12 - 3\pi$.

2. There should be no doubt about which point are midpoints and which are not.

Simply divide the large square up into 16 triangles which look like this: 
Two of the 16 are in the shaded square. So, the answer is $\frac{2}{16} = \frac{1}{8}$.

3. From perimeter equation, $S = 25$. Draw the altitude from the vertex angle, forming two 7-24-25 right triangles. Area = $\frac{1}{2} \times 14 \times 24 = 168$.

Team 1. Suppose S spiders, then $345 - S$ insects. $8S$ is the number of spider legs and $6(345 - S)$ is the number of insect legs. Solve $\frac{8S}{5} = \frac{6(345 - S)}{2}$ to obtain $S = 225$ spiders.

2. Solve $x^2 + 4x - 8 = (x + 1)^2$ for $x = 4.5$. So, answer is $5.5^2 = 30.25$.

3. Make a list of the perfect squares 1 to 81, as the lowest of three, and of 4 to 100, as the highest of three. Then, check out the number 2 units higher, in the first case, or 2 units lower, in the second. If primes, then they capture 2, 3, 8, 10, 24, 48, 80, and 82. So, there are 8 Beprisque numbers.

4. The square tiles are $4\frac{1}{2}$ in. on a side. Since there are 384 of them forming a rectangular floor, the floor cannot be square because 384 is not a square number. A rectangle as close to a square as possible will have the least perimeter. Since $384 = 2^7 \cdot 3^1$, the optimum breakup is $2^4 \times 2^3 \cdot 3^1$ or 16×24 . So, the width is $16 \times 4\frac{1}{2} = 72$ in. = 6 ft., and the length is $24 \times 4\frac{1}{2} = 108$ in. = 9 ft. for an area of $6 \times 9 = 54$ sq.ft.

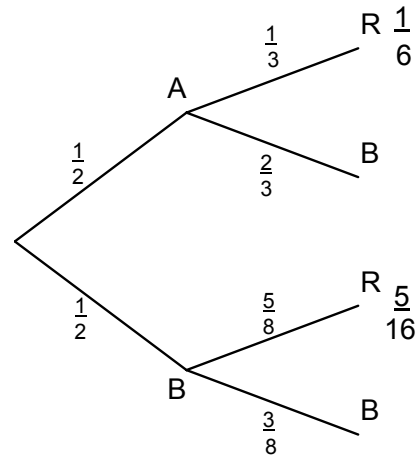
5. Start with A and $A - 20$ apples in the two boxes. After the first transfer, the boxes will contain $\frac{3}{5}A$ and $\frac{7}{5}A - 20$. After the second transfer, the amounts will be $\frac{3}{5}A + 10$ and $\frac{7}{5}A - 30$. Set these equal to find $A = 50$ and the apple total is $2A - 20 = 80$.

6. Use a tree diagram:

And the result is

$$\frac{1}{6} + \frac{5}{16} = \frac{23}{48}$$

It is definitely not $\frac{1}{2}$.



7. Let G be the no. of girls and A be their average score. Then, total points can be tallied in two ways that are equal: $40(74) + G(A) = (40 + G)(78)$. This can be rewritten as $G(A - 78) = 160$. Since $A \leq 100$, $A - 78 \leq 22$. $A - 78 = 20$ and $G = 8$ describe the situation where the girls' average is as high as possible and the number of girls is the least.

8. $\frac{40}{p-3} = 5 \frac{20}{p+3}$ yields $p = 7$. So, the time upstream is $\frac{40}{7-3} = 10$ hours.