

**FORTY-SEVENTH ANNUAL OLYMPIAD**  
**HIGH SCHOOL PRIZE COMPETITION**  
**IN MATHEMATICS**  
**2010-2011**

**Conducted By**

**The Massachusetts Association  
of  
Mathematics Leagues  
(MAML)**

**Sponsored By  
The Actuaries' Club of Boston**

**FIRST LEVEL EXAMINATION**

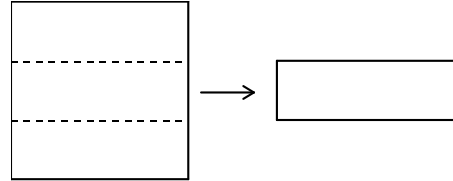
**Thursday, October 21, 2010**



1. A car is traveling at 70 miles per hour. To the nearest tenth, how many seconds does it take to travel one mile?

(A) 51.3      (B) 51.4      (C) 52.3      (D) 52.4      (E) 53.3

2. A square is folded into thirds along the dotted lines producing the rectangle shown. If the perimeter of the rectangle is 24, find the number of units in the perimeter of the original square.

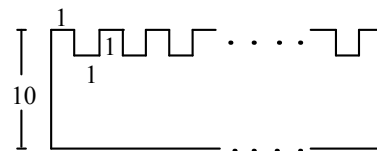


(A) 36      (B) 42      (C) 48      (D) 27      (E) 28

3. In the 2010 World Cup of Soccer the goalkeeper of the Kenyan team stopped 80% of the shots on goal prior to the game against South Africa. Against South Africa he did not stop any of the 15 shots the South Africans took and his percentage dropped to 50%. How many shots on goal did he stop before the game against South Africa?

(A) 10      (B) 15      (C) 18      (D) 20      (E) 25

4. Shown is a figure with 100 teeth. Each tooth is a 1 by 1 square. Find the number of square units in the area enclosed by the figure.



(A) 1791      (B) 1890      (C) 1891      (D) 1900      (E) 1901

5. If a single digit is removed from the decimal expansion of  $\frac{8}{11}$ , resulting in a new decimal, determine the largest possible result.

(A)  $\frac{73}{100}$       (B)  $\frac{76}{99}$       (C)  $\frac{7}{9}$       (D)  $\frac{77}{100}$       (E)  $\frac{17}{22}$

6. A father and son drove out to California. The father drove 80% of the time and covered 60% of the distance. Assuming that each drove at a constant rate, determine the ratio of the father's speed to the son's speed.

(A)  $\frac{8}{25}$       (B)  $\frac{3}{8}$       (C)  $\frac{12}{25}$       (D)  $\frac{3}{5}$       (E)  $\frac{3}{4}$

7. In the following list of 4 people, if exactly one person is telling the truth and exactly one person did it, then who did it?

Al: I didn't do it.  
 Betty: Carl did it.  
 Carl: Debby did it.  
 Debby: I did it.

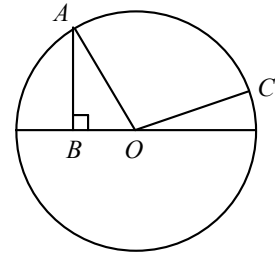
- (A) Al (B) Betty (C) Carl (D) Debby (E) Cannot be determined.

8. Simplify:  $\sqrt{\frac{5}{4} - \sqrt{\frac{3}{2}}} - \sqrt{\frac{5}{4} + \sqrt{\frac{3}{2}}}$ .

- (A)  $-\sqrt{3}$  (B)  $-\sqrt{2}$  (C)  $\sqrt{2}$  (D)  $\sqrt{3}$  (E)  $\frac{-\sqrt{5}}{2}$

9. In circle  $O$ ,  $\overline{AB} \perp \overline{BO}$ ,  $AB + BO + OC = 10$  and

$(AB)(BO) = 5$ . Find  $OC$ .



- (A) 3 (B) 3.5 (C) 4 (D) 4.5 (E) 5
10. Determine the number of square units in the area of the region bounded by the graphs of  $|x| + y = 4$  and  $x + 5y = -4$ .
- (A) 16 (B) 20 (C) 24 (D) 28 (E) 32
11. Given  $x^2 + bx + c = 0$ , let  $d$  equal the positive difference in the roots. If  $b$ ,  $c$ , and  $d$  form an increasing sequence of consecutive integers, find the sum  $b + c + d$ .
- (A) 0 (B) -2 (C) 2 (D) -1 (E) 1

12. Find all values of  $k$  such that all solutions  $(x, y)$  to the system  $x + 4y = 2k^2$  and  $x + y = k$  are such that  $x, y > 0$ .

(A)  $0 < k < 2$     (B)  $0 < k < \frac{1}{2}$     (C)  $k < 0$  or  $k > 2$     (D)  $k < \frac{1}{2}$  or  $k > 2$     (E)  $\frac{1}{2} < k < 2$

13. For  $N > 1$ , simplify the product  $\frac{1}{\log_2 N} \cdot \frac{1}{\log_N 8} \cdot \frac{1}{\log_{32} N} \cdot \frac{1}{\log_N 128}$ .

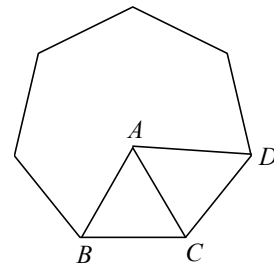
(A)  $\frac{3}{7}$     (B)  $\frac{3}{5}$     (C)  $\frac{3}{7 \ln 2}$     (D)  $\frac{3}{5 \ln 2}$     (E)  $\frac{5}{21}$

14. Let  $i = \sqrt{-1}$ . For real values of  $x$  and  $y$ , if  $(x + yi)(1 + 2i) = 4 + 2i$ , determine the value of the sum  $x + y$ .

(A)  $\frac{2}{5}$     (B)  $-\frac{2}{3}$     (C)  $\frac{14}{5}$     (D)  $-\frac{6}{5}$     (E) 2

15. Equilateral triangle  $ABC$  shares a side with a regular 7-gon.

Find the degree measure of  $\angle ADC$ .



(A)  $\frac{240}{7}$     (B)  $\frac{360}{7}$     (C)  $\frac{390}{7}$     (D)  $\frac{450}{7}$     (E)  $\frac{480}{7}$

16. Find the number of distinct ways to rearrange the characters in KEEN225 such that there is a numeral at the end of the string.

(A) 300    (B) 480    (C) 540    (D) 630    (E) 1260

17. Given the equation  $x^3 - kx + k - 1 = 0$ , find the real value for  $k$  such that the equation has exactly two real solutions.

(A)  $\frac{3}{4}$     (B)  $\frac{4}{5}$     (C)  $\frac{4}{3}$     (D)  $\frac{5}{4}$     (E)  $\frac{3}{2}$

18. Let  $S$  be the set of all five-digit palindromes that are divisible by 11. Find the sum of the digits of the three smallest numbers in  $S$ .

- (A) 18            (B) 19            (C) 20            (D) 23            (E) 24

19. If  $\tan x + \tan y = a$  and  $\cot x + \cot y = b$ , find  $\tan(x + y)$ .

- (A)  $\frac{a+b}{ab}$       (B)  $\frac{a-b}{ab}$       (C)  $\frac{ab}{a+b}$       (D)  $\frac{ab}{a-b}$       (E)  $\frac{ab}{b-a}$

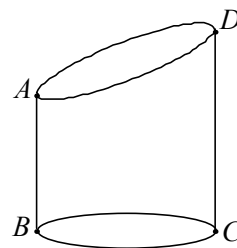
20. For  $k \geq 0$  and  $x$  in radians with  $0 \leq x \leq 2\pi$ , find the largest possible sum of all  $x$  such that  $\sin x + \cos x = k(\sin x - \cos x)$ .

- (A)  $2\pi$             (B)  $\frac{5\pi}{2}$             (C)  $3\pi$             (D)  $4\pi$             (E)  $5\pi$

21. Two non-congruent circles are externally tangent. The product of their radii is an integer  $K$  between 1 and 100 inclusive. For how many values of  $K$  is the length of the external tangent also an integer?

- (A) 10            (B) 9            (C) 8            (D) 7            (E) 6

22. As shown at the right, a right circular cylinder of diameter 4 is sliced diagonally. If  $AB = 6$  and  $DC = 9$ , determine the number of square units in the total surface area of the figure.



- (A)  $34\pi$             (B)  $37\pi$             (C)  $38\pi$             (D)  $39\pi$             (E)  $40\pi$

23. Let  $S$  be a set of  $n$  consecutive positive integers starting with  $a_1$ . Pick two numbers, remove them from the set, and replace the two numbers with their sum. Continue this process until one number is left. If 162 is the number that is left, find the least possible value for  $a_1$ .
- (A) 2            (B) 4            (C) 6            (D) 8            (E) 14
24. Let  $m$  be an integer such that  $1 \leq m \leq 2010$ . Consider parabolas of the form  $y = \frac{x^2}{m} - k$ . Determine the number of integer values of  $k$  such that the focal point of all the parabolas is  $(0, 1)$ .
- (A) 500            (B) 501            (C) 502            (D) 503            (E) 504
25. Consider an arithmetic sequence of integers with the property that  $a_{n+1} = a_n - 6$ . The sum  $a_1 + a_2 + a_3 + a_4 + a_5$  equals  $k^2$  where  $k$  is a positive integer. If  $k > a_5$ , what is the largest possible value of  $a_2$ ?
- (A) 6            (B) 11            (C) 26            (D) 51            (E) 86

1. ***DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR GIVES THE SIGNAL TO BEGIN.***
2. This is a 25-question, multiple-choice test. Each question is followed by answers marked (a), (b), (c), (d), and (e). Only one of these is correct.
3. You are permitted to use scratch paper, graph paper, rulers, compasses, protractors, and erasers.
4. **CALCULATORS ARE NOT PERMITTED.** Students with a calculator near their desk, even if it is not used, will be disqualified.
5. Figures are not necessarily drawn to scale.
6. Mark your answer to each problem on the ParSCORE scoresheet with a #2 pencil. Check your blackened ovals for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded. The scoresheet has numbers up to 200; leave numbers 26 to 200 blank.
7. **SCORING:** You will receive 6 points for each correct answer, 2 points for each problem left unanswered, and 0 points for each incorrect answer. While random guessing will almost certainly lower your score, guessing might be advantageous if you can eliminate more than two answers.
8. Before beginning the contest, your proctor will ask you to record specific information on the answer form. Specifically:
  - Your I.D. NUMBER is your school’s six-digit CEEB code followed by a three-digit student ID number, of the form AAAAAABBB. Leave the last column blank.
  - For your NAME, enter your complete last name, skip one space, then enter as much of your first name as you can. If your entire first name fits, skip a space and enter your middle initial, if it fits. If your first name and middle initial do not fit, you will hand-write them elsewhere.
  - For your CODE, in the first column enter:
    - A if you are in grade 8 or below
    - B if you are in grade 9
    - C if you are in grade 10
    - D if you are in grade 11
    - E if you are in grade 12
    - F if you have graduated from high school (and are completing a postgraduate year)
  - Also for your CODE, in the second column enter Y if you were able to fit your complete name on the scoresheet; otherwise enter N. If you enter N, and score within the top 200 in the state, your handwritten name will be used, so **write legibly**. For example, the code “CY” specifies that you are a sophomore whose name fit completely; “EN” specifies that you are a senior whose name did not fit completely.
    - **For TEST FORM, enter A.**
    - Leave the PHONE NUMBER and SUBJECT CODE blank.
    - Turn your paper clockwise and carefully hand-write your name and today’s date.
    - Underneath each letter or number, blacken the corresponding oval.
9. When your proctor gives the signal, begin working on the problems. You will have 90 MINUTES to complete the contest.