2002 MAML Olympiad Level 1 Questions

1. What is the largest prime factor of $\left(2^4\right)^2 - 1$?

   (A) 5 (B) 7 (C) 11 (D) 17 (E) 19

2. Simplify: $1+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2}}}}$

   (A) $\frac{41}{29}$ (B) $\frac{5}{2}$ (C) $\frac{631}{421}$ (D) $\frac{12}{5}$ (E) $\frac{49}{21}$

3. Suppose $10 is placed in a box today. Every three years from today, $15 will be added to the box; every five years from today, $17 will be removed from the box. How many dollars will be in the box 86 years from today?

   (A) 87 (B) 124 (C) 127 (D) 131 (E) 141

4. Let $S = \{37, 82, 48, A, 94, 30, 2A, 120, 3A\}$. If the arithmetic mean of S is 67, between which two numbers does the median of S lie?

   (A) 44 and 52 (B) 53 and 60 (C) 61 and 68 (D) 69and 76 (E) 77 and 84

5. In how many combinations can 5 dollars be paid in dimes and/or quarters?

   (A) 10 (B) 11 (C) 12 (D) 24 (D) 25

6. During the blizzard of '01, there was one inch of snow on the ground in Worcester at the end of the first hour and 36 inches of snow on the ground at the end of the 48^{th} hour at the end of the storm. Assuming that snow fell at a constant rate throughout the snowstorm, which of the following is closest to the number of inches of snow on the ground at the end of the 36^{th} hour?

   (A) 26.3 (B) 26.7 (C) 26.9 (D) 27.1 (E) 27.3

7. Zoltag the Mind Reader instructs an audience member to write down a four-digit number, add the digits and then subtract that sum from the original number. Finally, he asks the audience member to read out the digits of this new number in any order omitting one digit. If the digits read out are 1, 3, and 4, the Zoltag announces that the missing number is:

   (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

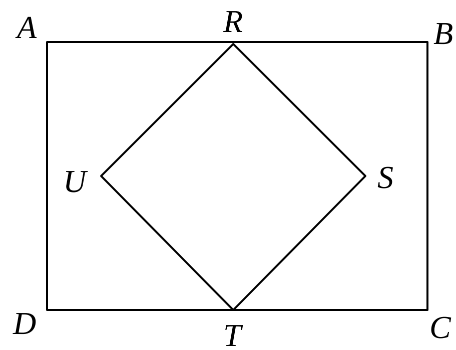
8. If the quadratic equation $2x^2 + ax + 3 = 0$, where a is an integer, has no real solutions, then a can take on how many different values?

(A) 6 (B) 7 (C) 8 (D) 9 (E) 10

9. Given that the six-digit base 10 number, $346xy2$ is divisible by 36, how many distinct order pairs (x, y) are possible?

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

10. Rectangle ABCD has an area of 64 square units and its width is half its length. If $\overline{RT}$ is parallel to $\overline{BC}$, find the area of square RSTU.



(A) 16 (B) $16\sqrt{2}$ (C) 24
(D) 32 (E) $32\sqrt{2}$

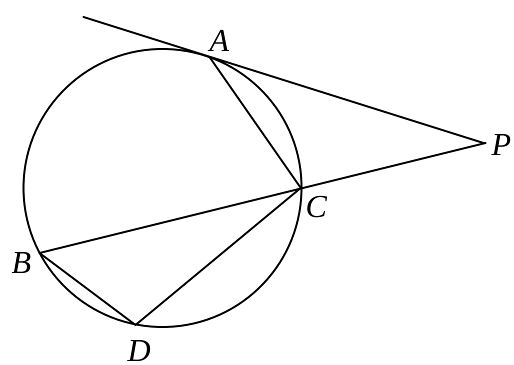
11. A magic square is an $n \times n$ array filled with the integers $1, 2, ..., n^2$ with the property that each row and each column of the array has the same sum. What is the sum of each row and each column in a 10×10 magic square?

(A) 505 (B) 510 (C) 515 (D) 495 (E) 500

12. Given the set of natural numbers $\{1, 2, 3, 4, ...\}$, find the 2002nd number in this set not divisible by 2, 3 or 5.

(A) 7159 (B) 7229 (C) 7507 (D) 8417 (E) 8723

13. In the diagram at the right, if $\overline{PA}$ is tangent to the circle at A, $\overline{PB}$ is a secant line, $m\angle BDC = 125°$, and $m\angle CAP = 40°$, find the measure of $\angle ACP$. (Note: the diagram is not drawn to scale.)



(A) 90° (B) 95° (C) 105°
(D) 107.5° (E) 115°

14. How many zeros are there at the end of the decimal expansion of 150! (! = factorial)?

(A) 36 (B) 37 (C) 38 (D) 39 (E) 40

15. Given the linear function *f*, such that $f\left(2x+4\right)=6x+13$, find $f\left(5-3x\right)+f\left(5x-1\right)$.

(A) $6x+14$ (B) $6x+18$ (C) $4x+14$ (D) $4x+12$ (E) $8x+16$

16. A regular hexagonal pyramid has one of its lateral edges four times the length of one side of the regular hexagon. If the volume of the pyramid is 60 cm^3, then the number of centimeters in the length of one side of the hexagon is

(A) $2\sqrt[3]{5}$ (B) $2\sqrt[6]{5}$ (C) $3\sqrt[3]{5}$ (D) $\sqrt[3]{5}$ (E) $\sqrt[6]{10}$

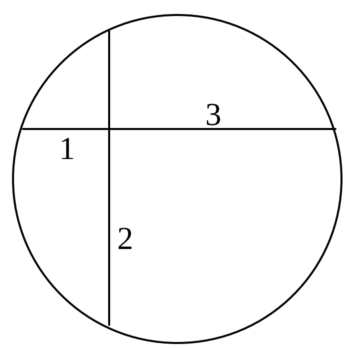
17. In a tennis tournament, the odds that player A will be the champion is 4 to 3, and the odds that player B will be the champion is 1 to 4. What are the odds that either A or B will become the champion?

(A) 5 to 2 (B) 25 to 7 (C) 27 to 8 (D) 4 to 1 (E) 27 to 35

18. If $AB_{13} = BA_{15}$, find the largest possible integer value of A.

(A) 6 (B) 7 (C) 8 (D) 10 (E) 12

19. In the diagram to the right with perpendicular chords, find the radius of the circle with the given lengths.



(A) $\dfrac{\sqrt{5}}{4}$ (B) $\dfrac{9}{4}$ (C) $\dfrac{\sqrt{451}}{4}$

(D) 2 (E) $\dfrac{\sqrt{65}}{4}$

20. If $\left[\sin\left(\dfrac{5p}{2}+x\right)\right]\left[\cos\left(\dfrac{7p}{2}-x\right)\right]=\left(\cos\dfrac{4p}{3}\right)\left(\sin\dfrac{7p}{4}\right)$, find the sum of all solutions for all x where $0\le x<2p$.

(A) p (B) $\dfrac{5p}{4}$ (C) $\dfrac{5p}{2}$ (D) $3p$ (E) $5p$

21. The circle which passes through the three points (0, 0), (1, 1), and (2, 0) has diameter

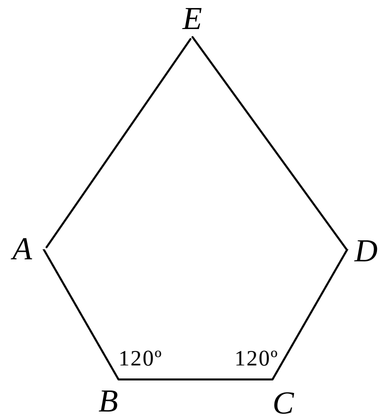
(A) $3\sqrt{2}$ (B) $2\sqrt{2}$ (C) 1 (D) 2 (E) 3

22. A box contains 6 red, 5 blue and 4 white marbles. Four marbles are chosen at random without replacement. What is the probability that there is at least one marble of each color among the four chosen?

(A) $\frac{40}{91}$ (B) $\frac{44}{91}$ (C) $\frac{46}{91}$ (D) $\frac{48}{91}$ (E) $\frac{50}{91}$

23. Given pentagon *ABCDE* with $m\angle B = m\angle C = 120°$, $AB : BC : CD : DE : AE = 2:2:2:3:3$, and the area of the pentagon is 20 square meters, then the number of meters in the perimeter of this pentagon is approximately

(A) 17.3 (B) 17.5 (C) 17.7 (D) 17.9 (E) 18.1



24. Find all real values of x such that $\log_2(x-1)+\log_2(x+2)-\log_2(3x-1)<1$.

(A) $\frac{1}{3}<x<5$ (B) $0<x<5$ (C) $1<x<5$ (D) $\frac{1}{3}<x<2$ (E) $x>1$

25. Find the sum of all solutions to $(x^2+3x+1)^{x^2+2x-8}=1$.

(A) –8 (B) –7 (C) –5 (D) –3 (E) –2

26. Given an arithmetic sequence which has its fourth and seventh terms in the ratio 2:3, and the sum of the first 100 terms equals 7000, find its 2002nd term.

(A) 2432 (B) 2452 (C) 2472 (D) 2582 (E) 2672

27. Given $0 < x < \frac{p}{2}$ and $\sin\left(x + \frac{p}{4}\right) - \sin\left(x - \frac{p}{4}\right) = t$, for $0 < t < 1$. Find x.

(A) $\text{Arcsin}\left(\frac{t}{2}\right)$ (B) $\text{Arccos}(t)$ (C) $\text{Arcsin}\left(\frac{t}{\sqrt{2}}\right)$ (D) $\text{Arccos}\left(\frac{t}{2}\right)$ (E) $\text{Arccos}\left(\frac{t}{\sqrt{2}}\right)$

28. A parallelogram containing a $60°$ angle has perimeter of length p and its longer diagonal is of length d. Find its area in terms of p and d.

(A) $p^2 - d^2$ (B) $\frac{\sqrt{3}}{8}\left(p^2 - 4d^2\right)$ (C) $p^2 - 3d^2$ (D) $\frac{\sqrt{3}}{12}\left(p^2 - 2d^2\right)$ (E) $\frac{\sqrt{3}}{24}\left(p^2 - 4d^2\right)$

29. Given point $P(4,0)$, and points Q and R on line $\{(x,y) : y = 2x - 4\}$ such that the first coordinate of R is twice the first coordinate of Q. If the area of ΔPQR is 6 square units, then there are two possibilities for the coordinates of point Q. Call these points Q and Q'. Find the length of $\overline{QQ'}$.

(A) $4\sqrt{5}$ (B) $6\sqrt{5}$ (C) $8\sqrt{5}$ (D) $9\sqrt{5}$ (E) $10\sqrt{5}$

30. Given the complex number $z = 1 + ai$, where a is a positive real number and z^3 is a real number, find the sum $1 + z + z^2 + \ldots + z^{11}$.

(A) $-1365i\sqrt{3}$ (B) $-1200i$ (C) $800 - 1200i$ (D) $-1200 - 800i$ (E) $-1250i\sqrt{3}$

MAML Level 1
2002 Answer Key

1	D	16	B
2	A	17	C
3	E	18	B
4	C	19	E
5	B	20	E
6	D	21	D
7	A	22	D
8	D	23	A
9	C	24	C
10	A	25	B
11	A	26	E
12	C	27	E
13	B	28	B
14	B	29	B
15	A	30	A