1. Six students in a small class took an exam on the scheduled date. The average of their grades was 75. The seventh student in the class was ill that day and took the exam late. When her score was included, the class average rose to 78. What was the seventh student's score?

   (A) 90 (B) 92 (C) 94 (D) 96 (E) 98

2. The sides of a right triangle are all integers. Two of them are odd numbers that differ by 50. What is the smallest possible value for the third side?

   (A) 48 (B) 54 (C) 60 (D) 66 (E) 72

3. Sean has a drawer containing a dozen black socks, a dozen brown socks, a dozen blue socks, and a dozen green socks, all loose and *not* properly paired. Sean takes socks out of the drawer at random without replacement, in the dark. How many socks must Sean take out to ensure that he has pairs of socks of each of the four colors among those he has chosen?

   (A) 9 (B) 13 (C) 17 (D) 25 (E) 38

4. Let $a=\left(2+\sqrt{3}\right)^2+\left(2-\sqrt{3}\right)^2$, and consider a triangle with sides $a-1,\ a,\ a+1$. What is the area of the triangle?

   (A) 42 (B) 84 (C) $\sqrt{14112}$ (D) $\sqrt{3528}$ (E) none of these

5. For which positive integers n is $\dfrac{\sum_{k=1}^{n} k^2}{\sum_{k=1}^{n} k}$ an integer?

   (A) odd n only (B) even n only (C) $n=1+6k$, integer $k \geq 0$
   (D) $n=1+3k$, integer $k \geq 0$ (E) $n=1$ only

6. A sphere of radius r and a cube of edge length s have the same total surface area. What is s in terms of r?

   (A) $s=r$ (B) $s=r\cdot\sqrt[3]{\dfrac{4\pi}{3}}$ (C) $s=\dfrac{2\pi}{3}r$ (D) $s=r\cdot\sqrt{\dfrac{2\pi}{3}}$ (E) $s=r\sqrt{\pi}$

7. A circle is inscribed in a $60°$ sector of a circle of radius 1 as shown in the diagram to the right. What is the radius of the small circle?

(A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) $\frac{\sqrt{2}}{2}$

(D) $\frac{\sqrt{3}}{2}$ (E) none of these

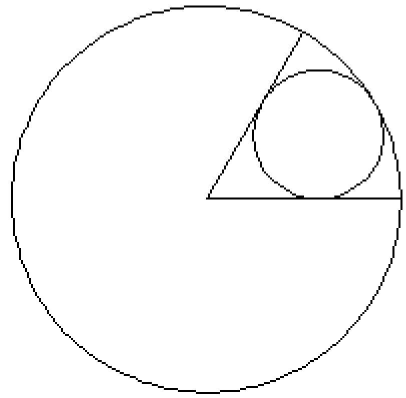

Figure for #7

8. Two "loaded" dice each have the property that a 2 or 4 is three times as likely to appear as a 1, 3, 5, or 6 on each roll. What is the probability that a 7 will be the total sum when the two dice are rolled?

(A) $\frac{1}{8}$ (B) $\frac{1}{7}$ (C) $\frac{1}{6}$ (D) $\frac{7}{50}$ (E) $\frac{7}{25}$

9. Triangle ABC has a right angle at C and an angle of $15°$ at A. Let $BC = x$. Point D is the foot of the perpendicular from C to the opposite side, and point E is the foot of the perpendicular from D to $\overline{AC}$. In terms of x, what is the length EC?

(A) $\frac{x\sqrt{3}}{2}$ (B) $\frac{x}{2}$ (C) $\frac{x}{4}$ (D) $x \cdot \sqrt{\frac{2-\sqrt{3}}{4}}$ (E) $\frac{x}{8}$

10. The terms a_1, a_2, a_3 form an arithmetic sequence whose sum is 18. The terms $a_1 + 1$, a_2, $a_3 + 2$, in that order, form a geometric sequence. Find the sum of all possible values for a_1.

(A) 1 (B) 2 (C) 3 (D) 11 (E) 13

11. Consider a cube of side 1. The centers of each pair of faces of the cube sharing a common edge are connected to form a regular octahedron (a regular polyhedron with 8 equilateral triangular faces). What is the volume of this octahedron?

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{1}{6}$ (D) $\frac{1}{8}$ (E) $\frac{1}{10}$

12. What is the sum of the slopes of the lines tangent to both circles: $x^2 + y^2 = 1$ and $(x-6)^2 + y^2 = 4$?

(A) $\sqrt{3}+\sqrt{35}$ (B) 0 (C) $2\sqrt{3}+2\sqrt{35}$ (D) $\sqrt{105}$ (E) None of these

13. Points S and R are endpoints of a diameter of the circle with radius 1, as shown in the figure to the right. $\overline{PR}$ is tangent to the circle at R, and $\overline{PS}$ is a secant line intersecting the circle at Q and S. If $PR = 3$, what is the area of ΔQRS?

(A) $\frac{24}{13}$ (B) $\frac{12}{13}$ (C) $\frac{12\sqrt{13}}{13}$

(D) $\frac{2\sqrt{39}}{13}$ (E) none of these

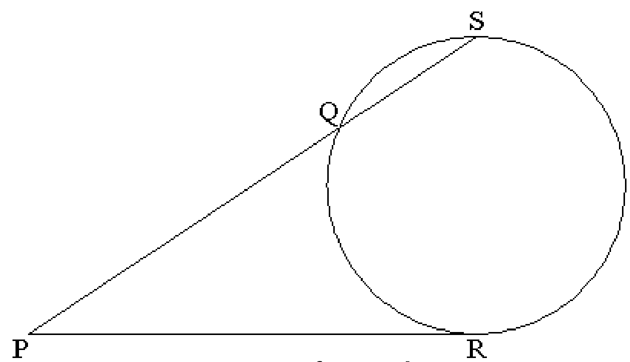

Figure for #13`

14. In the Vigornii alphabet there are 8 consonants and 4 vowels. How many different arrangements of 5 letters can be made if exactly two vowels must be used and no repetition of letters is allowed?

(A) 6 (B) 120 (C) 1400 (D) 33,600 (E) 40,320

15. The polar coordinate equation $r = \sin(\theta)\cos(\theta)$ defines a curve in the plane. What is the largest distance between two distinct points on this curve?

(A) $\sqrt{2}$ (B) $2\sqrt{2}$ (C) 1 (D) $\frac{1}{2}$ (E) 2

16. Find the solution set: $\sqrt{x^2 - 4x + 4} < 3$ and $\frac{1}{4} \le \frac{1}{3-x} \le \frac{1}{2}$.

(A) $0 < x < 1$ (B) $-1 \le x < 1$ (C) $-1 < x \le 1$ (D) $x > -1$ (E) $0 < x \le 1$

17. Three circles of radius r in the same plane are externally tangent in pairs. Consider the triangle whose vertices are the centers of the circles. What percentage of the area of this triangle is not contained in any of the circles (round to the nearest whole number)?

(A) 9 (B) 10 (C) 11 (D) 12 (E) 13

18. Find the sum of $\frac{1}{i}+\frac{3}{i^3}+\frac{5}{i^5}+\frac{7}{i^7}+\frac{9}{i^9}+...+\frac{53}{i^{53}}$, where $i=\sqrt{-1}$.

(A) i (B) $-i$ (C) $26i$ (D) $28i$ (E) $-27i$

19. Let R_1 be the rotation by $\frac{\pi}{3}$ radians counterclockwise about the origin, let T be the translation by the vector $\langle -1,0\rangle$, and let R_2 be the rotation by $\frac{7\pi}{6}$ radians counterclockwise about the origin. Let P be the point $(1,0)$. What are the coordinates of $R_2(T(R_1(P)))$?

(A) $\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right)$ (B) $\left(\frac{\sqrt{3}}{2},-\frac{1}{2}\right)$ (C) $\left(-\frac{\sqrt{2}}{2},-\frac{\sqrt{2}}{2}\right)$ (D) $\left(-\frac{\sqrt{3}}{2},-\frac{1}{2}\right)$
(E) $\left(\frac{\sqrt{2}}{2},-\frac{\sqrt{2}}{2}\right)$

20. Which of the following formulas define functions always equal to $\cos^2(2x)$?

I. $\frac{1}{1+\tan^2(2x)}$
II. $\sin^2 x\cot^2 x$
III. $\frac{1}{2}(1+\cos(4x))$
IV. $\frac{\csc^2(2x)-1}{\csc^2(2x)}$

(A) None (B) III only (C) I and III only
(D) II and III only (E) I, II, III and IV

21. Let $2a>-1$. What is the area of the region of the plane defined by
$$\{(x,y): x\ge 0,\ 2y-x\ge 0,\ ax+y-3\le 0\}?$$

(A) $\frac{18}{2a+1}$ (B) $\frac{9}{2a+1}$ (C) $\frac{18}{3a+2}$ (D) $\frac{9}{a+1}$ (E) the area is infinite

22. How many pairs of decimal digits are there with the property that if N is an integer ending in those two digits, then N^2 ends in the same two digits (repitition of the digits in a pair is allowed)?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

23. Let $\left\{ f_1(x)=1-x,\ f_2(x)=\frac{1}{1-x},\ f_3(x)=\frac{x-1}{x},\ f_4(x)=\frac{x}{x-1},\ f_5(x)=\frac{1}{x} \right\}$. For $x \neq 0,1$, which n in $\{1,2,3,4,5\}$ is $f_3(f_4(x)) = f_2(f_n(x))$?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

24. If $\tan x = a$, then $\cot\left(x-\frac{\pi}{4}\right)=$?

(A) $\frac{a-1}{a+1}$ (B) $\frac{a^2-1}{a^2+1}$ (C) $\frac{a+1}{a-1}$ (D) $\frac{a^2+1}{a^2-1}$ (E) none of these

25. Set A has a number of elements *strictly* between the number of elements in set B and twice the number of elements in set B. Set B has 16 more subsets than set C (as usual, we count the empty set as a subset of every set). What is the largest number of subsets A can have?

(A) 128 (B) 256 (C) 512 (D) 1024 (E) 2048

26. If $\frac{\log_2\left(\frac{b^3}{8}\right)}{\log_3\left(\frac{27}{a^2}\right)}=1$ and $\log_3\left(\frac{9}{a}\right)=\log_2\left(\frac{b}{4}\right)$, what is $\frac{a}{b}$?

(A) $\frac{729}{4}$ (B) -3 (C) 2916 (D) 972 (E) $-\frac{1}{3}$

27. How many different polynomials of the form $x^4+ax^3+bx^2+cx+d$ with rational coefficients are factors of the polynomial $x^{12}-1$?

(A) 3 (B) 5 (C) 7 (D) 11 (E) 15

28. What is the remainder when 45^{2001} is divided by 41?

(A) 1 (B) 4 (C) 6 (D) 8 (E) 32

29. Which of the following equals $\sin\left(\frac{a}{2}\right)\cdot\left(\frac{1}{2}+\sum_{k=1}^{n}\cos(ka)\right)$?

(A) $\frac{1}{2}\sin\left(\left(n+\frac{1}{2}\right)a\right)$ (B) $\frac{1}{2}\sin\left(\left(n-\frac{1}{2}\right)a\right)$ (C) $\sin\left(\left(n+1\right)a\right)$

(D) $\cos^{n}(a)$ (E) none of these

30. A container has the shape of a right circular cone with radius R units and height H units. It is originally in an inverted position (vertex down), and partially filled with water. If the depth of the water is $H-2$, what will the *upper radius* of the water be when the cone is returned to an upright position?

(A) $R\cdot\frac{H-2}{H}$ (B) $R\cdot\sqrt{1-\left(\frac{H-2}{H}\right)^2}$ (C) $R\cdot\sqrt[3]{1-\left(\frac{H-2}{H}\right)^3}$

(D) $\frac{RH}{2}$ (E) $\frac{R\cdot\sqrt[3]{6}}{H}$