IMLEM Meet #5
April, 2025

Intermediate Mathematics League of Eastern Massachusetts

Calculator Meet

• CALCULATORS: only *scientific calculators* allowed for meets #4 & #5 • Everyone take a moment to turn off any electronic devices that you want to have with you during the rounds. No electronic devices may be on during the rounds. Use of these devices during the rounds will result in disqualification.

Category 1
Mystery
Meet #5 - April, 2025

Calculator Meet

1) Together, a pen and a pencil cost a total of $1.10. The pen costs a dollar more than the pencil. How many cents does the pencil cost? Your answer should be a whole number. Be careful! This one is a bit tricky!

2) What whole number value of N makes the following equation true? You do not need specialized algebra skills involving radicals in order to solve this problem.

$$\frac{\sqrt{2}+\sqrt{N}}{\sqrt{2}} = 4$$

3) There are 100 people in a room. Ninety-nine are left-handed. How many left-handed people must leave the room so that 98% of the people remaining in the room are left-handed. As with problem #1, this one is a bit tricky!

Answers

1) _____

2) _____

3) _____

Solutions to Category 1
Mystery
Meet #5 - April, 2025

Answers	
1)	5
2)	18
3)	50

1) A classic from the very first year of IMLEM! A bit of guessing and checking should lead to the conclusion that a pen costs $1.05 while the pencil pencil costs 5 cents.

2) A student who has not yet studied the algebra of radicals could begin by approximating the value of X and using the calculator to make better guesses. An algebra student may approach the solution like this:

$$\frac{\sqrt{2}+\sqrt{N}}{\sqrt{2}} = 4$$

$$\sqrt{2}+\sqrt{N} = 4\sqrt{2}$$

$$\sqrt{N} = \sqrt{2}(4-1)$$

$$\sqrt{N} = 3\sqrt{2}$$

$$\sqrt{N} = \sqrt{9}\sqrt{2}$$

$$\sqrt{N} = \sqrt{18}$$

$$N = 18$$

3) Once again, utilizing some number sense can lead a student to some reasoned guessing and checking. Of the 99 left-handed people, 50 would have to leave so that of the 49 remaining, still only one is right-handed. The ratio of left-handed to the total is 49 / 50, or 98%.

Category 2
Geometry
Meet #5 - April, 2025

Calculator Meet

1) A rectangular box measures 5 cm wide by 8 cm long by 3 cm high. By how many square centimeters does its total surface area exceed the number of cubic centimeters in its volume?

2) Three faces of a rectangular box have areas of 40, 45, and 72 square inches, respectively. How many cubic inches are in its volume? All dimensions are whole numbers of inches.

3) It takes 10 seconds to inflate a balloon that is 8 inches in diameter. How many seconds should it take to inflate a balloon that is 24 inches in diameter, if inflated at the same rate? Assume that both balloons are in the shape of a sphere. The formula for the volume of a sphere is given below, where r is the radius of the sphere. Your answer will be a whole number.

$$Volume = \frac{4}{3}\pi r^3$$

Answers

1) _____

2) _____

3) _____

Solutions to Category 2
Geometry
Meet #5 - April, 2025

1) Surface area = 2LW + 2LH + 2WH
= 2(8)(5) + 2(8)(3) + 2(5)(3)
= 80 + 48 + 30
= 158 square centimeters
Volume = LWH
= (8)(5)(3)
= 120 cubic centimeters

The number of square centimeters of the surface area exceeds the number of cubic centimeters of the volume by 158 - 120, or 38.

2) Factoring each of the numbers should yield a unique combination that would comprise the three dimensions of the box.
40 = (5)(8)
45 = (5)(9)
72 = (8)(9)
The box is 5 by 8 by 9 inches and has a volume of (5)(8)(9) = 360 cubic inches.

3) A student could compute the two volumes, divide the greater volume by the smaller, and then multiply by 10 seconds.
A higher order concept in computing the ratio of the volumes of two similar figures is to cube the ratio of any two corresponding linear dimensions.
In this case, the larger balloon is (24/8)^3, or 27 times the volume of the smaller balloon. So, It should take (27)(10), or 270 seconds to inflate it.

Answers

1) 38

2) 360

3) 270

Category 3
Number Theory
Meet #5 - April, 2025

Calculator Meet

1) The intersection of sets A and B consists of the elements that are common to sets A and B. Put another way, if a number is in set A and that same number is in set B, then that number is in the intersection of sets A and B.
Set X = { the multiples of 3 that are between 10 and 29 }
Set Y = { the multiples of 4 that are between 10 and 29 }
What is the sum of all the numbers that are in the intersection of Set X and Set Y ?

2) A survey taken at the Whole Earth Vitamin Bar revealed that 472 people liked pomegranate juice while 387 liked cranberry juice. These were the only two choices surveyed. Of the 613 people surveyed, how many said they liked both cranberry and pomegranate juice?

3) Of the 271 7th graders at the Canton Galvin Middle School, 171 ride the school bus, 124 wear flip flops, and 38 ride the school bus and wear flip flops. How many students do not ride the school bus or wear flip flops?

Answers

1) _____

2) _____

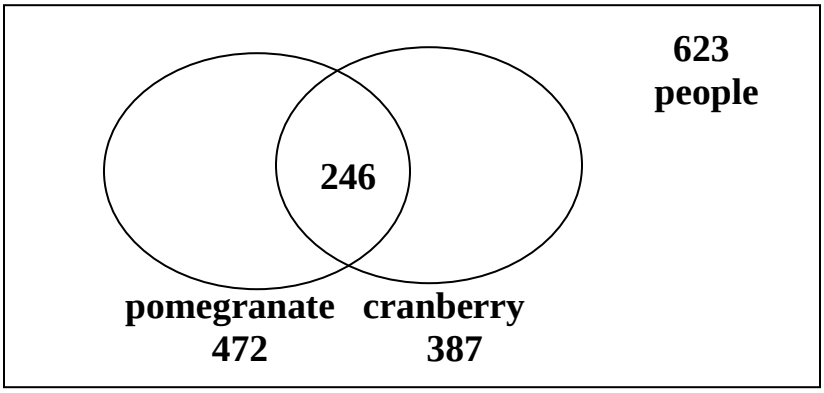
3) _____

Solutions to Category 3
Number Theory
Meet #5 - April, 2025

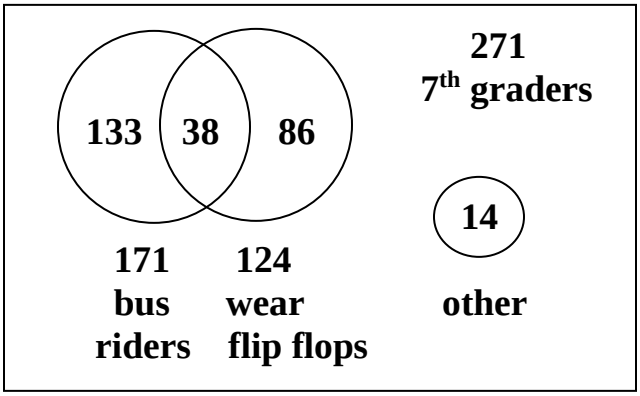
Answers	
1)	36
2)	246
3)	14

1) Set X = { 12, 15, 18, 21, 24, 27 }
Set Y = { 12, 16, 20, 24, 28 }
The intersection is { 12, 24 }
The sum of the numbers in the intersection is 12 + 24, or 36.

2) Since there are fewer juice consumers in total that the number of cranberry and pomegranate juicers combined, there is an overlap (intersection) of those who like both: 472 + 387 – 613 = the number who liked both = 246.



3) Add the first two sets and then subtract their intersection: 171 + 124 – 38 = 257. This is the number of students who ride the school bus or wear flip flops or both. The question asks for the number of the remaining students who neither wear flip flops nor ride the school bus. Subtract the number we just computed from the total number of 7th graders: 271 – 257 = 14.



Category 4
Arithmetic
Meet #5 - April, 2025

Calculator Meet

1) A bag of multi-colored potatoes contains 7 reds, 8 yellows, and 5 purples, all about the same size and texture. Luther chooses one at random. The probability that Luther chose a yellow potato, is expressed as a fraction in lowest terms. What is the value of the sum of its numerator and denominator?

2) There is barely enough space on a book shelf to place either four books or five skinnier books. How many more ways could the five books be arranged than the four books?

3) The letters of the word MISSISSIPPI are arranged to form other "words," with a "word" defined as any 9-letter arrangement of the letters in the word MISSISSIPPI. How many different words are possible, including the original word?

ANSWERS

1) _____

2) _____

3) _____

Solutions to Category 4
Arithmetic
Meet #5 - April, 2025

Answers	
1)	7
2)	96
3)	34,650

1) The probability of choosing a yellow potato is (# of yellow potatoes) / (total # of potatoes) = 8 / 20 = 2 / 5 in lowest terms. The sum of the numerator and denominator of the simplified fraction is 2 + 5, or 7.

2) Five books: 5! = (5)(4)(3)(2)(1) = 120 ways
Four books: 4! = (4)(3)(2)(1) = 24 ways
The difference is 120 – 24, or 96 more ways that five books can be arranged than four books.

3) If all eleven letters were different, there would be 11! possible arrangements of those letters. But since some of those letters repeat, we must divide by the number of repetitions as follows, accounting for the four Is, the four Ss, and two Ps:

$$\frac{11!}{(4!)(4!)(2!)} = \frac{(11)(10)(9)(8)(7)(6)(5)(4)(3)(2)(1)}{((4)(3)(2)(1))((4)(3)(2)(1))((2)(1))}$$

$$= \frac{(11)(10)(9)(7)(5)}{1}$$

$$= 34,650$$

Therefore, there are 34,650 ways that the letters in the MISSISSIPPI can be arranged to form different "words."

Category 5
Algebra
Meet #5 - April, 2024

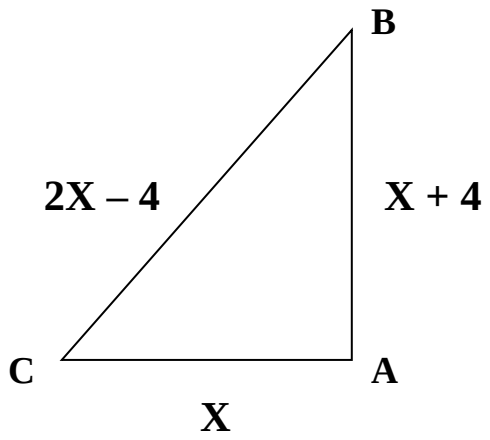
Calculator Meet

1) A quadratic equation is a second-degree equation of the form $ax^2+bx+c=0$ **where a is not zero, while b and c are real numbers. When the values of a, b, and c are known, the equation can be solved in a variety of ways. One such way is to employ the quadratic formula, as posted below.**
If $x^2+2x-48=0$ **then what is the sum of the two possible values of x that make the equation true?**

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

2) If $27+2x^2=15x$ **then what is the product of the two possible values of x that make the equation true? Express your answer as a decimal.**

3) Triangle ABC below is a right triangle with a right angle at vertex A. The two legs are X and X + 4, while the hypotenuse is 2X – 4. How many square units are there in the area of triangle ABC?



ANSWERS

1) _____

2) _____

3) _____

Solutions to Category 5
Algebra
Meet #5 - April, 2025

Answers

1) - 2

2) 13.5

3) 96

1) Although a student may opt to employ the quadratic formula or completing the square, the factoring method provides the quickest solution.

$$x^2 + 2x - 48 = 0$$
$$(x + 8)(x - 6) = 0$$
Either $x + 8 = 0$ or $x - 6 = 0$
Then $x = -8$ or $x = 6$

Finally, the sum of the two possible values of x is $(-8) + (6) = -2$.

2) First, convert the equation to standard form. Again, factoring yields the quickest solution.

$$27 + 2x^2 = 15x$$
$$2x^2 - 15x + 27 = 0$$
$$(2x - 9)(x - 3) = 0$$
Either $2x - 9 = 0$ or $x - 3 = 0$
Then $x = 9/2$ or $x = 3$

Finally, the product of the two possible values of x is $(9/2)(3) = 27/2$. The answer must be a decimal, so $27/2 = 13.5$.

3) Employ the Pythagorean Theorem:

$$x^2 + (x + 4)^2 = (2x - 4)^2$$
$$x^2 + x^2 + 8x + 16 = 4x^2 - 16x + 16$$
$$0 = 2x^2 - 24x$$
$$0 = 2x(x - 12)$$
Either $2x = 0$ or $x - 12 = 0$
Then $x = 0$ or $x = 12$

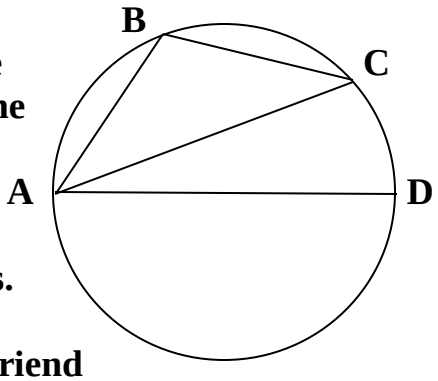
Since x is the length of a side of a triangle, we discard $x = 0$ as a solution, so that $x = 12$, $x + 4 = 16$, and $2x - 4 = 20$.

To compute the area of the right triangle, use Area $= (1/2)$(base)(altitude)
$= (1/2)(12)(16)$
$= 96$ square units.

Category 6
Team Round
Meet #5 - April, 2025

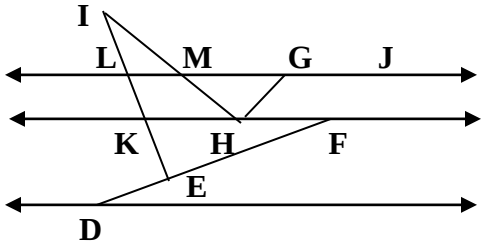
Each of the following nine problems is worth <u>four</u> points.

1) In this circle, AD is the diameter, segments AB and BC are congruent, and inscribed angle ACB = 37 degrees. How many degrees are in the measure of angle CAD ?



2) Maxine walked at 4 km per hour and was late in meeting a friend for lunch by 10 minutes. different day, Maxine walked at 6 km per hour and was 10 minutes early in meeting the same friend for lunch at the same place, having followed the same route. How many kilometers is the distance to the restaurant from Maxine's starting point?

3) In the picture to the right, the horizontal lines are parallel. Several non-horizontal lines intersect with the horizontal lines to form many angles. GH is perpendicular to IH. IE is perpendicular to DF. Angle EFH measures 20 degrees. Angle HGJ measures 150 degrees. How many degrees are in the measure of angle EIH ?



<u>ANSWERS</u>

1) _____

2) _____

3) _____

4) _____

5) _____

6) _____

7) _____

8) _____

9) _____

4) A time of 7 minutes 30 seconds is expressed as a simplified fraction of an hour. What is the sum of the numerator and denominator of that fraction?

5) What is the value of W if $7^{7+7W} = 1$ **?**

6) The product of three consecutive odd integers is 357,627. What is the sum of the smallest and largest of these three odd integers?

7) A palindrome is a number that reads the same as when its digits are reversed. For example, the number 4725274 is a palindrome. What is the largest prime number less than 400 that is a palindrome?

PROBLEMS #8 AND #9 ARE ON THE NEXT PAGE.

Category #6 – Team Round – continued . . .

8) More than 20 students, but fewer than 40 students, paid a total of $527 for concert tickets. If each ticket costs the same whole number of dollars, then how many students are in the group?

9) The length of each side of a scalene triangle is a prime number of units. The perimeter is also a prime number of units. What is the number of units in the smallest possible perimeter that this triangle could have?

**** Note to students and coaches from the contest writer:**

Hello, on behalf of the league officers and all who have made it possible for providing students with a high quality, challenging and rewarding competition experience. My name is Evagrio Mosca. This has been my 50th year with IMLEM - the Intermediate Math League of Eastern Massachusetts. Initially, I served as the coach of one of the participating schools in 1975. Since then, I have served in various leadership capacities, including president, vice president, website designer, and webmaster. It has been an honor and privilege to have worked with students and coaches so dedicated to helping to make this one of the premier and longest continuously running mathematics leagues in the nation.

It is with sincere gratitude that I thank all those who make this experience possible:

- **** coaches, for their teaching and inspiration**
- **** support staff, for helping to run the meets**
- **** cafeteria personnel, for supplying the refreshments at meets**
- **** teachers and administrators, for their support and encouragement**
- **** school systems, for financial support**
- **** students, for participation and commitment to their teams.**

I am also deeply grateful to the league officers who dedicate their time and professionalism. The following officers have devoted the past decade or more to this endeavour:

- **** Kelly Hagan, president**
- **** Pat Kavanaugh, treasurer**
- **** Richard Yanco, tabulator and webmaster**
- **** Alice Mandel, test checker**
- **** Sergei Bernstein, test checker and former IMLEM champion**

We all hope you have enjoyed the competitions this year and the camaraderie with your friends as well as with students and coaches in other schools. I welcome your input and suggestions. Access the website to send me a note or ask your coach - imlem.org

Solutions to Category 6
Team Round
Meet #5 - April, 2025

ANSWERS	
1)	16
2)	4
3)	10
4)	9
5)	- 1
6)	142
7)	383
8)	31
9)	23

1) Whereas triangle ABC is isosceles, with two sides congruent, the base angles are also congruent so that angle BAC also equals 37 degrees. The key to solving this problem involves knowing that an inscribed angle subtends an arc that measures double the degrees of the inscribed angle. So, minor arcs Ab and BC each measure (2)(37), or 74 degrees. The remaining minor arc of the semi-circle, arc CD, measures 180 - (74 + 74), or 180 – 148, or 32 degrees. Finally, angle CAD measures (1/2)(32), or 16 degrees.

2) Rate x Time = Distance. Also, 10 minutes = 1/6 hr.
(4 km/hr)(x + 1/6 hr) = (6 km/hr)((x – 1/6 hr)
4x + (4)(1/6) = 6x - (6)(1/6)
4x + 2/3 = 6x - 1
5/3 = 2x
5/6 = x
So, the distance from the starting point to the restaurant can be computed either of two ways:
(4)(5/6 + 1/6) = (4)(1) = 4 kilometers, or
(6)(5/6 – 1/6) = (6)(4/6) = 4 kiolometers.

3) Angle EKF = 180 – (90 + 20) = 180 – 110 = 70 degrees.
Angle LKH = 180 – 70 = 110 degrees.
Angle ILG = angle LKH = 110 degrees.
Angle HGL = 180 – 150 = 30 degrees.
Angle GMH = 180 – (30 + 90) = 180 – 120 = 60 degrees.
Angle IML = angle GMH = 60 degrees.
Finally, angle EIH = 180 – (110 + 60) = 180 – 170 = 10 degrees.

4) 7 minutes 30 seconds, as a fraction of an hour:
= [(7)(60) + 30 seconds] / (3600 seconds)
= 450 / 3600 = 45 / 360 = 1/8 of an hour
The sum of the numerator and denominator is 1 + 8, or 9.

THE SOLUTIONS TO PROBLEMS #5-9 ARE ON THE NEXT PAGE.

5) Since 7^0 = 1, the exponent of 7 + 7W = 0.

$$7W = -7$$

$$W = -1$$

6) The three consecutive odd integers are in the vicinity of the cube root of 357,627, or about 70.98. Then 71 should be the median of the three odd integers. Test that (69)(71)(73) is exactly 357,627 . . . and it is!
Finally, the sum of the smallest and largest of these odd integers is 69 + 73, or 142.

7) 393 is divisible by 3, so is not prime.
383 is prime.

8) 527 = 17 x 31
Since the number of students must be between 20 and 40, then the number of students is 31 and the price per ticket is $17.

9) Each side of a scalene triangle, by definition, must have a different length, In addition, the sum of the lengths of two sides of any triangle must be greater than the length of the third side. Start small to find the smallest perimeter.

2, 3, 5 No. 2 + 3 = 5, so not a triangle.
3, 5, 7 No. The perimeter, 15, is not prime.
3, 5, 11 No. 3 + 5 < 11.
3, 7, 11 No. 3 + 7 < 11.
5, 7, 11 Yes! The perimeter, 23, is prime and 5 + 7 < 11.

So, the smallest possible perimeter is 23 units.