IMLEM Meet #3
January, 2025

Intermediate Mathematics League of Eastern Massachusetts

CLUSTER COORDINATORS - A reminder to all students of some of the rules and of appropriate behavior during this meet:

- **No calculators (or only scientific calculators allowed for meets #4, #5)**
- **Everyone take a moment to turn off any electronic devices that you want to have with you during the rounds. No electronic devices may be on during the rounds. Use of these devices during the rounds will result in a disqualification.**

Category 1
Mystery
Meet #3 - January, 2025

1) If a two-digit number, 6A, is multiplied by another two-digit number, A2, the result is the four-digit number 5576. What is the value of A ?

2) The sum of two whole numbers, J and K, is 40. Their difference is 8. What is the value of the larger of J and K ?

3) The whole number N has the following properties:

- **its digits are only twos and threes**
- **it has at least one 2 and one 3**
- **it is divisible by both 2 and 3**

What is the smallest possible value of N ?

<u>Answers</u>

1) ______

2) ______

3) ______

Solutions to Category 1
Mystery
Meet #3 - January, 2025

Answers
1) 8
2) 24
3) 2232

1) The product of the units digits, A and 2, results in a units digit of 6. The possibilities for A are 3 or 8. Try both possibilities: (63)(32) = 2016 while (68)(82) = 5576. So, the value of A is 8.

2) This problem can be solved by using number sense, guess and check, or algebra. Here is an algebraic solution: Let J = one number and let K = the other number. Then

$J + K = 40$

$J - K = 8 \quad \text{so} \quad J = 8 + K$

Substituting 8 + K for J in the first equation:

$8 + K + K = 40$

$8 + 2K = 40$

$2K = 32$

$K = 16$

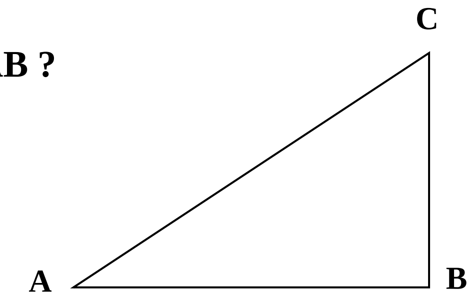
$J = 8 + K = 8m + 16 = 24.$

So, the larger of J and K is the number 24.

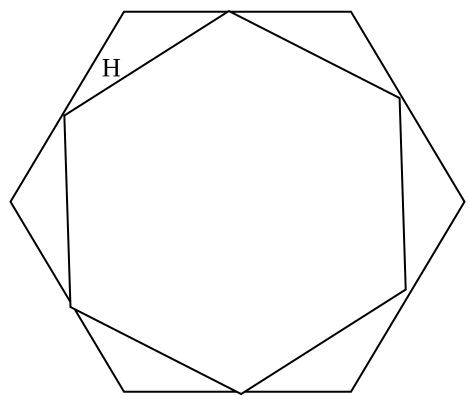
3) The number in question must be even (divisible by 2), so its units digit must be 2. To be also divisible by 3, the sum of the digits must be a multiple of 3. 32 isn't divisible by 3. Try 332 or 322 or 232. The digit sums are not divisible by 3. Go to four digits. Try 2322 or 3222 or 3322 or 3232 or 3332 or 2232. The digit sums are, respectively, 9, 9, 10, 10, 11, and 9. Only 2322, 3222, and 2232 are divisible by 3. The smallest of them is 2232.

Category 2
Geometry
Meet #3 - January, 2025

1) In right triangle ABC, AC = 15 feet and BC = 9 feet. How many feet long is side AB ?



2) A regular hexagon is inscribed inside a larger regular hexagon, intersecting it at the midpoints of its six equal sides. How many degrees are in the acute angle labelled H ? (Note: one interior angle of a regular polygon with N sides measures [(180) (N – 2)] / N.



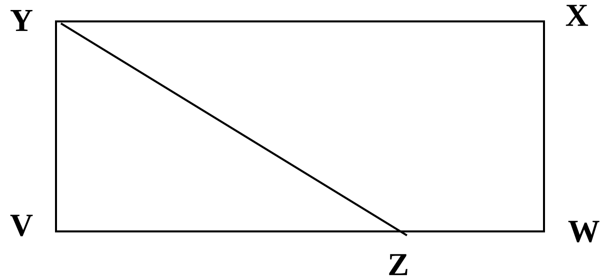
3) VWXY is a rectangle. VZ = 12 inches. XY = 23 inches. The area of triangle VYZ is 30 square inches. How many inches are in the perimeter of trapezoid XYZW ?

Answers

1) _______

2) _______

3) _______



Solutions to Category 2
Geometry
Meet #3 - January, 2025

Answers	
1)	12
2)	30
3)	52

1) Use the Pythagorean Theorem:
A^2 + B^2 = C^2
9^2 + B^2 = 15^2
81 + B^2 = 225
B^2 = 144
B = 12
So, AB = 12 feet.

2) Each interior angle of a regular hexagon measures [(180) (6 – 2)] / 6 or 120 degrees. The triangle with the angle marked H has a vertex angle of 120 degrees. The base angles are equal, as the opposite sides are congruent. Therefore, H = (180 – 120) / 2, or 30 degrees.

3) Given that the area of right triangle VYZ is 30 square inches, the length of side AB can be found: Area = (1/2)(base)(altitude)
30 = (1/2)(12)(altitude)
30 = (6)(altitude)
5 = altitude
So, VY = XW = 5 inches as the opposite sides of a rectangle are congruent.
So, also, VW = XY = 23 inches, and VW – VZ = 23 – 12 = 11 inches.
To calculate the value of YZ, use the Pythagorean Theorem:
5^2 + 12^2 = (YZ)^2
25 + 144 = (YZ)^2
169 = (YZ)^2
13 = YZ
Finally, the perimeter of trapezoid XYZW is the sum of its four sides:
5 + 11 + 23 + 13 = 52 inches.

Category 3
Number Theory
Meet #3 - January, 2025

1) The number 71,300,000 when written in scientific notation is 7.13×10^{W}. What is the value of W ?

2) The number 3201 is written in base 5. What is its value in base 10 ?

3) The number 1010010110 is written in base 2. What is its value in base 6 ?

Answers

1) ________

2) ________

3) ________

Solutions to Category 3
Number Theory
Meet #3 - January, 2025

Answers	
1)	7
2)	426
3)	3022

1) Count the number of decimal places between the decimal point and the end of the number. The exponent is, therefore, 7.

2) The base 5 number 3201, when converted to base 10: 1(1) + 0(5) + 2(25) + 3(125) = 1 + 0 + 50 + 375 = 426.

3) First convert the base 2 number 1010010110 to base 10:
0(0) + 1(2) + 1(4) + 0(8) + 1(16) + 0(32) + 0(64) + 1(128) + 0(256) + 1(512)
= 0 + 2 + 4 + 0 + 16 + 0 + 0 + 128 + 0 + 512
= 662.
Then convert the base 10 number 662 to base 6. Divide 662 by the highest power of 6 that is less than 662, namely 6^3, or 216.
662 / 216 = 3 with remainder 14. Now divide the remainder of 14 by the highest power of 6 that is less than 14, namely 6^1, or 6.
14 / 6 = 2 with remainder 2.
The base 6 value is 3022.
Check: 3(216) + 0(36) + 2(6) + 2(1) = 648 + 0 + 12 + 2 = 662.

Category 4
Arithmetic
Meet #3 - January, 2025

1) What is the value of this exponential expression?

$$7^0 + 6^1 + 5^2 + 4^3 + 3^4 + 2^5 + 1^6$$

2) What is the value of this square root expression?

$$\sqrt{\sqrt{441} + \sqrt{\sqrt{169} + \sqrt{7 + \sqrt{4}}}}$$

3) How many whole numbers are between $\sqrt[3]{209}$ **and** $\sqrt[4]{9813}$ **?**

ANSWERS

1) ________

2) ________

3) ________

Solutions to Category 4
Arithmetic
Meet #3 - January, 2025

Answers	
1)	**210**
2)	**5**
3)	**4**

1) $7^0 + 6^1 + 5^2 + 4^3 + 3^4 + 2^5 + 1^6$

= 1 + 6 + 25 + 64 + 81 + 32 + 1
= 210

2) $\sqrt{\sqrt{441} + \sqrt{\sqrt{169} + \sqrt{7 + \sqrt{4}}}}$

$= \sqrt{21 + \sqrt{13 + \sqrt{7 + 2}}}$

$= \sqrt{21 + \sqrt{13 + \sqrt{9}}}$

$= \sqrt{21 + \sqrt{13 + 3}}$

$= \sqrt{21 + \sqrt{16}}$

$= \sqrt{21 + 4}$

$= \sqrt{25}$

$=$ **5**

3) The whole number that is immediately larger than the cube root of 209 is 6 and the whole number that is immediately smaller than the fourth root of 9814 is 9. So, the whole numbers between the two radical expressions are 6, 7, 8, and 9 for a total of 4 whole numbers.

Category 5
Algebra
Meet #3 - January, 2025

1) What is the value of this absolute value expression?

$$|17| + |0| + |-6|$$

2) What is the smallest integer value of A that makes the following inequality true?

$$2(3A + 7) < 3(4A - 5) - 9$$

3) In this absolute value inequality, if Y is an integer and if

$|4Y + 2| \leq 10$ **then what is the sum of all possible values of Y ?**

Answers

1) ______

2) ______

3) ______

Solutions to Category 5
Algebra
Meet #3 - January, 2025

Answers	
1)	23
2)	7
3)	- 3

1) $|17| + |0| + |-6|$

$= 17 + 0 + 6 = 23$

2) $2(3A + 7) < 3(4A - 5) - 9$

$6A + 14 < 12A - 15 - 9$

$-6A < -38$

$A > 38/6$

The smallest integer value that makes this inequality true is 7.

3) $4Y + 2 \leq 10$ and $4Y + 2 \geq -10.$

$4Y \leq 8$ and $4Y \geq -12$

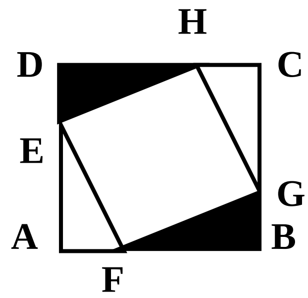
$Y \leq 2$ and $Y \geq -3$

The integer values of Y that satisfy this compound condition are
-3, - 2, - 1, 0, 1, and 2,
The sum of these values is (-3) + (-2) + (-1) + 0 + 1 + 2 = - 3

Category 6
Team Round
Meet #3 - January, 2025

Each of the following nine problems is worth four points.

1) **Square ABCD has a perimeter of 112 inches. Square EFGH has a perimeter of 80 inches. How many total square inches are in the shaded area?**



2) **How many prime numbers less than 100 have the digit 7 in the units (ones) place?**

3) **What 2-digit number is equal to three times the sum of its digits?**

4) **The numbers 0, 1, 2, and 3 can be arranged to create a 4-digit number that is divisible by the four smallest prime numbers. What is that 4-digit number?**

5) **One thousand people are standing in a circle, equally spaced, and are numbered consecutively from 1 to 1000. A diameter is drawn that connects person #47 to person #T. What is the value of T ?**

ANSWERS

1) ________

2) ________

3) ________

4) ________

5) ________

6) ________

7) ________

8) ________

9) ________

6) **Jas turns N years old in the year N^2. If the year N^2 is in the 20th century, then in what year was Jas born?**

7) **The product $12^4 \times 15^6$ ends in how many zeroes?**

8) **When the fraction $\frac{3}{7}$ is converted to a decimal, then what is the 2025th digit?**

9) **If $7^{7+7X} = 1$, then what is the value of X ?**

Solutions to Category 6
Team Round
Meet #3 - January, 2025

ANSWERS	
1)	192
2)	6
3)	27
4)	2310
5)	547
6)	1892
7)	6
8)	8
9)	- 1

1) If square ABCD has a perimeter of 112 inches, then one side is 112 / 4, or 28 inches and has an area of (28)(28), 784 square inches. If square EFGH has a perimeter of 80 inches, then one side is 80 / 4, or 20 inches and has an area of (20)(20), or 400 square inches. Subtract the two areas 784 – 400 = 384 square inches to get the total area of the four triangles. Half that area is shaded = 192 square inches.

2) The prime numbers less than 100 that have the digit 7 in the units place are 7, 17, 37, 47, 67, and 97. That is six such numbers.

3) The number described is 27, as 27 = 3 (2 + 7) = 3 (9).

4) The smallest four prime numbers are 2, 3, 5, and 7. The units digit must be 0 in order to make the 4-digit number divisible by 2 and 5. The sum of the digits is 6, so that any combination of the digits is divisible by 3. The challenge is to correctly place the 1 so that the 4-digit number is divisible by 7. 1230 is not divisible by 7 and neither are 1320, 2130, 3120, or 3210. However, 2310 is divisible by 7.

5) Simplify the problem and test a few hypotheses. For a circle with just the numbers from 1 through 6, the numbers opposite one another have a difference of 3, or half of 6. For a circle with just the numbers from 1 through 10, the numbers opposite one another have a difference of 5, or half of 10. So, for a circle with just the numbers from 1 through 1000, the numbers opposite one another have a difference of 500, or half of 1000. The number opposite #47 is 47 + 500, or #547.

SOLUTIONS #6-9 ARE ON THE NEXT PAGE.

6) There is only one square numbered year in the 20th century. That is the square of 44, or the year 1936. If Jas is 44 years old in 1936, then Jas was born in 1892, because 1936 - 44 = 1892.

7) 12^4 x 15^6
= (3 x 4)^4 x (3 x 5)^6
= 3^4 x 4^4 x 3^6 x 5^6
= 3^4 x 2^8 x 3^6 x 5^6.
When six factors of 2 are multiplied by six factors of 5, the result is (2 x 5)^6 or 1,000,000 which ends in six zeroes, even when multiplied by the remaining factors in the product.

8) Convert the fraction 3 / 7 to a decimal by dividing 3 by 7, resulting in 0.428571428571 . . . in blocks of six digits of 428571. Divide 2025 by 6 to get how many such blocks are in the decimal representation, then focusing on the remainder: 2025 / 6 = 337 with remainder of 3. 337 blocks of the six digits consumes 2022 digits. Three digits later is the 2025th digit, or 8.

9) Since 7^0 = 1, then then value of 7 + 7X = 0.
7X = - 7, and X + - 1.