Intermediate
Mathematics League
of
Eastern Massachusetts

Doodle Art Alley ©

CLUSTER COORDINATORS - A reminder to all students of some of the rules and of appropriate behavior during this meet:
- **No calculators (or only scientific calculators allowed for meets #4, #5)**
- **Everyone take a moment to turn off any electronic devices that you want to have with you during the rounds. No electronic devices may be on during the rounds. Use of these devices during the rounds will result in a disqualification.**
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1) **When I was four years old, my cousin was three times my age. I am now 30 years old. How many years old is my cousin?**

2) **You and four friends - Ron, Draco, Hermione, and Harry - went out to dinner. The total bill, including tax and tip, was $135.29. Your share to pay was $25.37. Your four friends split the remainder of the bill equally. How much was Draco's share? Express your answer as a decimal.**

3) **How many positive odd divisors, or factors, does the number 5,184 have?**

Answers

1) _____

2) _____

3) _____

Solutions to Category 1
Mystery
Meet #2 - November, 2024

1) **My cousin, being three times as old as I was (4),**
   is now (3)(4), or 12 years old, or 8 years older than
   I was. My cousin will always be 8 years older than
   me. If I am now 30, then my cousin is 30 + 8, or 38
   years old.

2) **Subtract $25.37 from the total bill of $135.29 to**
   compute the cost that the four friends will split.
   135.29 - 25.37 = 109.92. Then 109.92 / 4 = 27.48,
   the amount that each of the four friends must pay.
   So, Draco's share is $27.48.

3) **First find the prime factorization of 5,184:**
   5,184 = 2 x 2,592
   = 2 x 2 x 1,296
   = 2 x 2 x 2 x 648
   = 2 x 2 x 2 x 2 x 324
   = 2 x 2 x 2 x 2 x 2 x 162
   = 2 x 2 x 2 x 2 x 2 x 2 x81
   = 2 x 2 x 2 x 2 x 2 x 2 x 3 x 27
   = 2 x 2 x 2 x 2 x 2 x 2 x 3 x 3 x 9
   = 2 x 2 x 2 x 2 x 2 x 2 x 3 x 3 x 3 x 3
   The question calls for the number of ODD factors of 2,592.
   We must include the number 1 as an odd factor, along with 3, 9, 27,
   and 81. That's a total of FIVE odd factors.

Answers	
1)	38
2)	27.48
3)	5

1) **A square has a perimeter of 80 centimeters.**
   How many square centimeters are in its area?

2) **The perimeter of a rectangle is 36 inches. Its length and width are whole numbers.**

   If A = the maximum (largest) possible area
   and B = the minimum (smallest) possible area,
   then what is the value of A - B ?

3) **Triangle XYZ has a perimeter of 10 inches. The lengths of its sides are all whole numbers. If n is the length of side YZ, then what is the difference between the largest and smallest possible values of n ?**

Answers

1) _____ sq. cm

2) _____

3) _____

Solutions to Category 2
Geometry
Meet #2 - November, 2024

1) **If the perimeter of the square is 80 cm, then one**
    side has a length of 80 / 4, or 20 cm. All sides of
    a square are of equal length, so the area is 20 x 20,
    or 400 square centimeters.

2) **The maximum area occurs when the rectangle is a**
    square, thus making the rectangle of maximum area
    36 / 4, or a 9 by 9 rectangle (square) with area of
    9 x 9, or 81 square inches. So, A = 81.
    The minimum area occurs when the shortest side is 1 inch
    making the longest side 17 inches. Checking that the perimeter
    of this rectangle is 36: 1 + 17 + 1 + 17 = 36. Check.
    The area is 1 x 17, or 17 square inches. So, B = 17.
    Finally, A - B = 81 - 17 = 64.
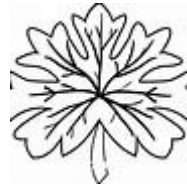
3) **This problem utilizes the notion that the sum of any two sides of a**
    triangle must be larger than the third side. Since the perimeter is a
    fixed value of 10, this chart shows the possibilities for the lengths of
    the sides (and the ones that fail):

side 1	side 2	side 3	possible?
1	1	8	no
1	2	7	no
1	3	6	no
1	4	5	no
2	2	6	no
2	3	5	no
2	4	4	yes
3	3	4	yes

The first six options fail because the sum of the first two sides does
not exceed the third side. The bottom two succeed because the sum
of any two of the sides does exceed the third side.
So, the smallest possible value of n is 2 and the largest possible
value is 4, and their difference is 2.

1) The GCF of a set of numbers is their greatest common factor.
   The LCM of a set of numbers is their lowest common multiple.

   A = the GCF of 8 and 12.
   B = the LCM of 8 and 12.
   C = the GCF of 6 and 24.
   D = the GCF of 9 and 20.

   What is the value of the sum of A + B + C + D ?

2) The prime factorization of 27,300 is 27,300 = 3 x 5 x A x 7 x 5 x B x 2.
   What is the value of A + B ?

3) Jacs has a rope that is 120 feet long. Gabby's rope is 96 feet long. They
   work together to figure out how to cut the two ropes in such a way that all
   the pieces are of equal length, but as long as possible. How many pieces, in
   all, will they get?

Answers

1) _____

2) _____

3) _____

Solutions to Category 3
Number Theory
Meet #2 - November, 2024

1) A = GCF (8, 12) = 4
   B = LCM (8, 12) = 24
   C = GCF (6, 24) = 6
   D = GCF (9, 20) = 1
   The sum of A + B + C + D = 4 + 24 + 6 + 1 = 35.

2) 27,300 = 100 x 273
   $\quad\quad\quad$ = 10 x 10 x 3 x 91
   $\quad\quad\quad$ = 2 x 5 x 2 x 5 x 3 x 7 x 13
   The missing factors in the question, namely A and B, thus have the values
   2 and 13, in no particular order.
   Finally, A + B = 2 + 13 = 15.

3) Find the GCF, or greatest common factor, of 120 and 96.
   120 = 2 x 2 x 2 x 3 x 5
   $\,$ 96 = 2 x 2 x 2 x 2 x 2 x 3
   The GCF is 2^3 x 3 = 8 x 3 = 24.
   Therefore, the longest possible piece is 24 feet long. Jacs' rope can be cut
   into 5 such lengths while Gabby's rope can be cut into 4 such lengths. The
   total number of pieces is 5 + 4, or 9.

1) **What is the product of the hundreds and hundredths digits of the decimal numeral 12345.6789 ?**

2) **What is 80% of $\frac{3}{4}$ of 720 ?**

3) **If the repeating decimal 0.3666666 . . . is converted to a fraction and then reduced to lowest terms, then what is the sum of the numerator and denominator ?**

Answers

1) _____

2) _____

3) _____

Solutions to Category 4
Arithmetic
Meet #2 - November, 2024

1) **The hundreds digit is 3.**
   The hundredths digit is 7.
   Their product is (3)(7), or 21.

2) **This can be computed in many ways. Here is one:**
   1/4 of 720 is 720 / 4 = 180.
   So, 3/4 of 720 = (3)(180), or 540.
   Then 10% of 540 is 54.
   So, 80% of 540 = (8)(54), or 432.
   Alternatively, (0.8)(0.75)(720) = 432,

3) **Let   10X = 3.66666666 . . .**
   and   X = 0.36666666 . . .

   Then   9X = 3.3 and X = 3.3 / 9 = 33 / 90
   **   = 11 / 30**
   Finally, the sum of the numerator and denominator is 11 + 30, or 41.

Answers	
1)	21
2)	432
3)	41

1) The sum of three consecutive even integers is 114.
    What is the largest of those three even integers?

2) If   2A + 10B = 88
    and   6C - 15D = 45
    then what is the value of 3A + 15B + 8C - 20D ?

3) The formulas that relate the temperatures, in degrees, of the Celsius (C)
    and Fahrenheit (F) scales are

$$F = (9/5)C + 32 \quad \text{and} \quad C = (5/9)(F - 32)$$

    At how many degrees are the Fahrenheit and Celsius temperatures the same?

Answers

1) _____

2) _____

3) _____

Solutions to Category 5
Meet #2 - November, 2024

1) **Let X, X + 2, and X + 4 represent three**
   Consecutive even integers.
   Then X + X + 2 + X + 4 = 114
   $$3X + 6 = 114$$
   $$3X = 108$$
   $$X = 36$$

2) **Although a strategy of guessing and checking**
   Could yield a successful outcome, a student's
   Knowledge of the distributive property could a
   Accelerate the process toward an answer.
   If 2A + 10B = 88, then A + 5B = 44 and 3A + 15B = 132.
   If 6C – 15D = 45, then 2C – 5D = 15 and 8C – 20D = 60.
   So, 3A + 15B + 8C – 20D = 132 + 60 = 192.

3) **Set C equal to F in one of the two formulas, then solve for F:**

   $$C = 5/9 \ (F – 32)$$
   $$F = 5/9 \ (F – 32) \qquad \text{substituting F for C}$$
   $$F = 5/9F \text{ - } (5/9)(32) \quad \text{distribution}$$
   $$F = 5/9F – 160/9$$
   $$9/5F = 9/5(5/9F – 160/9)$$
   $$9/5F = F – 32$$
   $$4/5F = \text{ - } 32$$
   $$F = (\text{ - } 32)(5/4)$$
   $$F = \text{ - } 40$$

   So, at - 40 degrees, both the Fahrenheit and Celsius temperatures are
   the same.

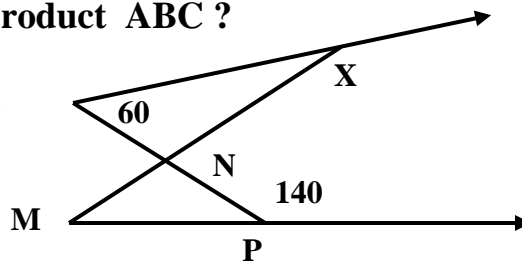| Answers |
| --- |
| 1) 40 |
| 2) 192 |
| 3) - 40 |

Category 6
Team Round
Meet #1 - November, 2024

Each of the following nine
problems is worth <u>four</u> points.

1) The 5-digit number 2018U is divisible by 9. What is the remainder when
   That 5-digit number is divided by 8 ?

2) The harmonic mean of a set of non-zero numbers is the reciprocal of the
   average of the reciprocals of the numbers in that set. If the harmonic
   mean of 1, 2, and 4 is a fraction in lowest terms, then what is the
   product of the numerator and denominator of that fraction? Note: The
   harmonic mean can be used when an average rate is desired.

3) Let F be the largest 5-digit number whose digits have a product of 120.
   What is the sum of the digits of F ?

4) A, B, and C are prime numbers. There are two possible sets of
   of numbers { A, B, C }, such that A + B + C = 20. What
   is the largest possible value of the product ABC ?

5) How many degrees are in the angle
   marked X if MN = NP?

ANSWERS

1) _____

2) _____

3) _____

4) _____

5) _____

6) _____

7) _____

8) _____

9) _____

6) When I was three years old, my uncle was
   eight times my age. I am now 13 years old.
   How many years old is my uncle?

7) A food truck sells sandwiches for $6.50 each
   and drinks for $2 each. The food truck's
   revenue for selling a total of   209 sandwiches
   and drinks was $836.50. How many drinks
   were sold?

8) A farmer can husk at least 12 dozen ears of
   corn per hour and, at most 18 dozen ears of
   corn per hour. How many more hours would
   it take if the farmer worked at minimum speed
   than at maximum speed in order to husk 72
   dozen ears of corn?

9) If A + B + C = 52   and   A + C = 37, then
   What is the value of B ?

Solutions to Category 6
Team Round
Meet #2 - November, 2024

1) If U = 7, then 2018U is divisible by **9**
   because the sum of its digits, 18, is
   divisible by 9. When 20187 is divided
   by 8, the quotient is 2,523 with
   remainder **3.**

2) The avg of reciprocals is (1/1 + 1/2 + 1/4) / 3
   (4/4 + 2/4 + 1/4) / 3, or 7/12. The reciprocal
   of 7/12 is 12/7. The product of the
   numerator and denominator is (12)(7) = **84.**

3) Use the prime factorization of 120 to view
   its prime factors: 120 = 2 x 2 x 2 x 3 x 5
   Then include 1 as a factor when considering
   possible 5-digit combinations. The largest
   possible 5-digit number occurs when the
   2 x 2 x 2 produces the largest ten-thousands
   digit of 8. So, 85311 is the largest. The
   sum of the digits is 8 + 5 + 3 + 1 + 1 = **18.**

4) One possibility is   2 + 7 + 11 = 20.
   Another is 2 + 5 + 13 = 20.
   2 x 7 x 11 = 154   while   2 x 5 x 13 = 130.
   Therefore, the largest possible value of
   A x B x C = **154.**

ANSWERS	
1)	3
2)	84
3)	18
4)	154
5)	160
6)	34
7)	116
8)	2
9)	15

5) Since MN = NP, the base angles are congruent. The angle of
   triangle MNP that is adjacent to the 140-degree angle measures
   40 degrees, so the other base angle also measures 40 degrees. The
   apex angle of the triangle measures 180 – (40 + 40), or 100 degrees.
   Its vertical angle is the same measure of 100 degrees. The remaining
   angle of the upper triangle measures 180 – (100 + 60), or 20 degrees.
   Its adjacent angle, X, is supplementary, or 180 – 20, or **160** degrees.

6) If I was 3, then my uncle was (3)(8), or 24 years old. My uncle was
   21 years older and would always be 21 years older than I. If I am 13,
   then my uncle is 21 + 13, or **34** years old.

   NEXT PAGE FOR SOLUTIONS TO #7-9

7) S = # of sandwiches   D = # of drinks

$S + D = 209$, so $S = 209 - D$

$6.5\,S + 2D = 836.5$ Then substitute $209 - D$ for S:

$$6.5(209 - D) + 2D = 836.5$$
$$1358.5 - 6.5D + 2D = 836.5$$
$$1358.5 - 4.5D = 836.5$$
$$1358.5 - 836.5 = 4.5D$$
$$522 = 4.5D$$
$$116 = D$$

So, 116 drinks were sold.

Suggestion: The equation involving decimals can be easily transformed to an equation with integers by multiplying both sides by 10, resulting in $65S + 20D = 8365$.

8) At minimum speed, the farmer would require 6 times as much time to husk 72 dozen ears, or 6 hours. $72 / 12 = 6$. At maximum speed, the farmer would require 4 times as much time to husk 72 dozen ears, or 4 hours. $72 / 18 = 4$.

The difference in time is $6 - 4$, or 2 hours.

9) Subtracting the two equations yields $B = 15$.