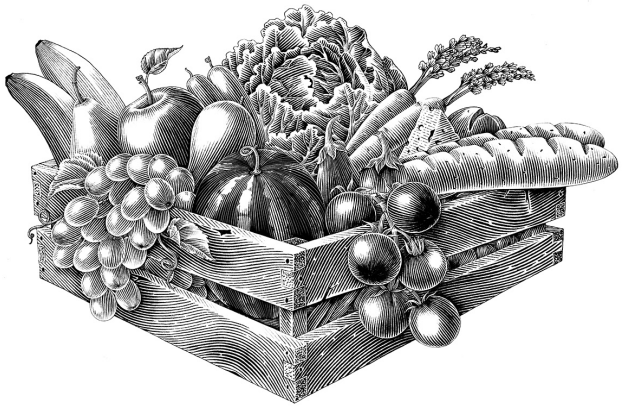
IMLEM Meet #1
October, 2024

Intermediate Mathematics League of Eastern Massachusetts

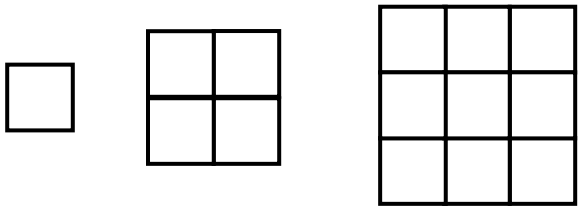


CLUSTER COORDINATORS - A reminder to all students of some of the rules and of appropriate behavior during this meet:

- **Many of you are guests in someone else's school – please be respectful of the classrooms and spaces you are using. Any "out of control" behavior in the halls or during a round is not acceptable. If an adult deems your behavior disrespectful or inappropriate, your score may not be counted.**
- **No calculators (or only scientific calculators allowed for meets #4, #5)**
- **Everyone take a moment to turn off any electronic devices that you want to have with you during the rounds. No electronic devices may be on during the rounds. Use of these devices during the rounds will result in a disqualification.**

Category 1
Mystery
Meet #1 - October, 2024

1) The first image is one small square. The second image consists of four small squares. The third image? Nine small squares. If the pattern continues, then how many small squares are in the ninth image?



2) If the pattern of letters below continues, then what is the value of the 70th letter if A = 1, B = 2, C = 3, and so on?

T E W K S B U R Y T E W K S B U R Y T E W . . .

3) If you reverse the digits of a two-digit positive integer and subtract the resulting integer from the original integer, the difference is 36. What is the positive difference between the two digits?

ANSWERS

1) _____

2) _____

3) _____

Solutions to Category 1
Mystery
Meet #1 - October, 2024

Answers	
1)	81
2)	21
3)	4

1) The images are squares consisting of smaller squares. 1x1, 2x2, 3x3, and so on. The ninth image is a 9x9, consisting of 81 small squares.

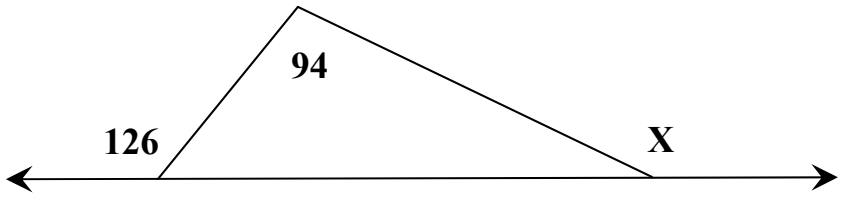
2) TEWKSBURY has nine letters. The 70th letter is seven letters beyond the seventh complete word, or beyond the 63rd letter. The seventh letter is U. The value of U, being the 21st letter, is 21.

3) The student may realize that, for there to be a difference of 36 between the two 2-digit integers, the larger one must be greater than 10 + 36, or greater than 46. Also, for the difference to be even, then the two digit must be either both odd or both even. Guessing and checking will yield the two numbers as 15 and 51. So, the positive difference between the two digits is 5 - 1, or 4.

Alternate (algebraic) solution:
Let X = the tens digit and Y = the units digit.
Then $10X + Y - (10Y + X) = 36$
$10X + Y - 10Y - X = 36$
$10X - X + Y - 10Y = 36$
$9X - 9Y = 36$
$9(X - Y) = 36$
$X - Y = 4$
So, the positive difference between the two digits is 4.

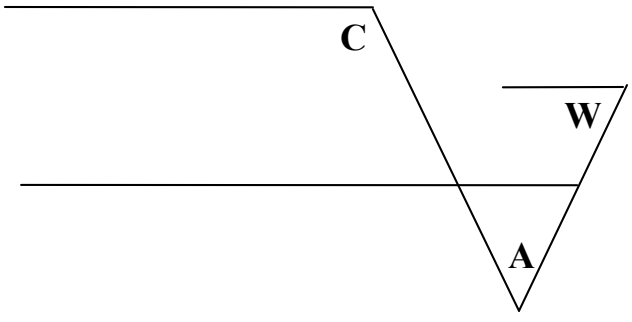
Category 2
Geometry
Meet #1 - October, 2024

1) The base of a triangle is extended in both directions, as shown, with one exterior angle given as measuring 126 degrees and one interior angle measuring 94 degrees. How many degrees are in the exterior angle X ?



2) If the angle measures of a triangle are in the ratio 3 : 5 : 7, then how many more degrees is the measure of the largest angle than the measure of the smallest angle?

3) Angle C is the supplement of a 61 degree angle. Angle A is the complement of a 52 degree angle. The three horizontal segments are parallel. How many degrees are in the measure of angle W ?



Answers

1) _____

2) _____

3) _____

Solutions to Category 2
Geometry
Meet #1 - October, 2024

Answers
1) 148
2) 48
3) 81

1) The interior angle adjacent to the 126-degree is its supplement, so 180 - 126 = 54 degrees. The remaining interior angle of the triangle is 180 - (54 + 94) = 180 - 148 = 32 degrees. Finally, the angle marked "X" is the supplement of the 32-degree angle, so 180 - 32 - 148 degrees.

2) Let N = the scale factor.
3X + 5X + 7X = 180
15X = 180
X = 180 / 15
X = 12
The scaling factor is 12, so the three angles of the triangle measure
3(12) = 36 degrees
5(12) = 60 degrees
7(12) = 84 degrees
The difference between the largest and smallest angles is 84 - 36, or 48.

3) Angle C = 180 - 61 = 119 degrees.
Angle A = 90 - 52 = 38 degrees.
The upper left corner of the triangle that contains angle A is opposite to, or vertical to the other interior angle of the two parallel lines that contain angle C. Each of those angles is the supplement of angle C, or 61 degrees. The remaining angle of the triangle measures 180 - (61 + 38) = 180 - 99 = 81 degrees.
Finally, angle W and the 81-degree angle are equal, as when two parallel lines are cut by a transversal, the alternate exterior angles are congruent.
So, angle W = 81 degrees.

Category 3
Number Theory
Meet #1 - October, 2024

1) What is the sum of the six prime numbers between 10 and 30 ?

2) A composite number is a positive whole number that has more than two factors, unlike prime numbers that have exactly two factors. There are six composite numbers between 1 and 200 that are divisible by 2, 3, and 5. What is the sum of these six composite numbers?

3) The letter N is the hundreds digit of the five-digit whole number 27N41. There are three possible values of N that make this five-digit number divisible by 3. What is the sum of those three possible values of N ?

Answers

1) _____

2) _____

3) _____

Solutions to Category 3
Number Theory
Meet #1 - October, 2024

1) The sum of the prime numbers between 10 and 30 is 11 + 13 + 17 + 19 + 23 + 29 = 112.

2) Whole numbers that are divisible by 2, 3, and 5 are divisible by 30. The sum of the six multiples of 30 that lie between 1 and 200 is 30 + 60 + 90 + 120 + 150 + 180 = 630.

3) For a positive whole number to be divisible by 3, the sum of its digits must be divisible by 3. The sum of the digits of 27N41 is 2 + 7 + N + 4 + 1 = 14 + N.
If N is 1 or 4 or 7, then 14 + N is divisible by 3 and = 15 or 18 or 21, all divisible by 3.
The sum of these three values of N is 1 + 4 + 7 = 12.

Answers

1) 112

2) 630

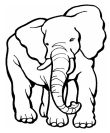
3) 12

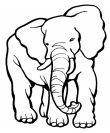
Category 4
Arithmetic
Meet #1 - October, 2024

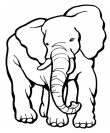
1) Find the value of the following expression, according to the correct order of operations.

7 x 8 + 3 x 5

2) Consider the set  **= { 4, 11, 9, 6, 10, 6, 4, 15, 7, 6, 10 }**

 **= the median of** 

 **= the mode of** 

 **= the mean of** 

What is the value of the sum  **+**  **+**  **?**

3) If the arithmetic mean, or average, of three positive integers is 5, then the product of all three integers is, at most, E. What is the value of E ?

Answers

1) _____

2) _____

3) _____

Solutions to Category 4
Arithmetic
Meet #1 - October, 2024

Answers
1) 71
2) 21
3) 125

1) Multiply before adding.
7 x 8 + 3 x 5
= 56 + 15
= 71

2) Cat = median = middle number of the set , once once the numbers have been arranged in order, as such: 4 4 6 6 6 7 9 10 10 11 15
The middle number, or median, is 7.
Dog = mode - the number that appears most frequently in the set.
The mode is 6.
Bird = mean = average = the sum of the numbers divided by the number of numbers in the set.
The sum is 4 + 4 + 6 + 6 + 6 + 7 + 9 + 10 + 10 + 11 + 15 = 88
The number of numbers is eleven.
The mean is 88 / 11 = 8.
Finally, cat + dog + bird = 7 + 6 + 8 = 21.

3) Experimenting with the products of various trios of numbers yields the following conclusion - that the three numbers must be as close to the value of 5 as possible. In fact, those three numbers are 5, 5, and 5, yielding a product of 5 x 5 x 5, or 125. So, E = 125.

Category 5
Algebra
Meet #1 - October, 2024

1) If ♥ = 7, ♦ = 3, ♣ = - 5, and ♠ = 10, then what is the value of 6♣ + 9♦ + 2♠ - 8♥ ?

2) What value of N makes the following equation a true statement?

$$3N + 5(2N - 7) = 4N + 37$$

3) An identity is an equation that is true for all values of the variable. What value of C makes the following equation an identity in the variable H ?

$$5(H + 2) + 11 = 3(8 + H) + 2H + C$$

Answers

1) _____

2) _____

3) _____

Solutions to Category 5
Algebra
Meet #1 - October, 2024

1) $6(-5) + 9(3) + 2(10) - 8(7)$
$= -30 + 27 + 20 - 56$
$= -39$

2) $3N + 5(2N - 7) = 4N + 37$
$3N + 10N - 35 = 4N + 37$
$13N - 35 = 4N + 37$
$9N = 72$
$N = 8$

3) $5(H + 2) + 11 = 3(8 + H) + 2H + C$
$5H + 10 + 11 = 24 + 3H + 2H + C$
$5H + 21 = 5H + 24 + C$
$21 = 24 + C$
$C = -3$

Answers

1) - 39

2) 8

3) - 3

Category 6
Team Round
Meet #1 - October, 2024

Each of the following NINE problems is worth <u>four</u> points.

1) I wrote the first 100 positive integers and then erased every digit "1" that I had written. How many digits did I erase?

2) The product of four fours is equal to the sum of how many fours?

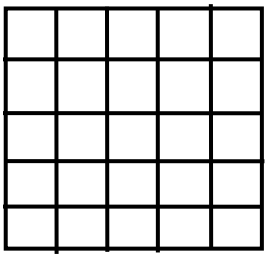
3) On Monday, the number of ants on the Tennessee Ant Hill was 20. The number of ants doubled each day. On day X, the number of ants first exceeded 2024. If Monday = 1, Tuesday = 2, Wednesday = 3, and so on, then what is the value of X ?

4) The ages of five Sequoia trees, in years, are consecutive even integers. If the sum of the trees' ages is 4,440 years, then how many years old is the oldest of the five Sequoia trees?

5) If the mean (average) of A, B, and C is 16 and the mean of A and B is 12, then what is the value of C ?

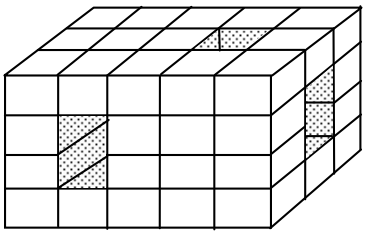
6) This coming Halloween, Gomez plans to scare twice as many people as Morticia. Morticia plans to scare three times as many people as Uncle Fester. In all, they plan to scare, at most, 2,024 people. If no one is scared more than once, then, at most, how many people does Morticia plan to scare?

7) Counting every possible square from a 1x1 to a 5x5, what is the total number of squares in this grid?



8) How many of the fifteen positive factors of 400 are divisible by 4 ?

9) How many of the smallest cubes are in this figure? All holes extend thru the figure.



<u>ANSWERS</u>

1) _____

2) _____

3) _____

4) _____

5) _____

6) _____

7) _____

8) _____

9) _____

Solutions to Category 6
Team Round
Meet #1 - October, 2024

ANSWERS	
1)	21
2)	64
3)	1
4)	892
5)	24
6)	606
7)	55
8)	9
9)	44

1) Counting every "1" that got erased:
from 1-10: 2 from 11-20: 10 (the 11 has two ones)
from 21-100: 9 (one for every span of ten numbers, plus an extra "1" for the number 100).
Total: 21

2) $(4)(4)(4)(4) = 4X$
$(4)(4)(4) = X$
$64 = X$

3) Mon: 20 Tues: 2(20) = 40 Wed: 2(4) = 80
Thurs: 2(80) = 160 Fri: 2(160) = 320
Sat: 2(320) = 640 Sun: 2(640) = 1280
Mon: 2(1280) = 2560
So, Monday is the first day when the number first exceeds 2,024. Monday = 1.

4) $X + (X + 2) + (X + 4) + (X + 6) + (X + 8) = 4440$
$5X + 20 = 4440$
$5X = 4420$
$X = 884$
The oldest tree is X + 8 years old, so $X + 8 = 892$.

5) If the average of A, B, and C is 16, then their sum is (3)(16), or 48.
If the average of A and B is 12, then their sum is (2)(12), or 24.
$A + B + C = 48$
$A + B = 24$
Subtracting the two equations yields $C = 24$.

6) Fester = X; Morticia = 3X; Gomez = 2(3X), or 6X
$X + 3X + 6X < 2024$
$10X < 2024$
$X < 202.4$
If we use X = 202, then the three people scare a total of (10)(202), or 2020 people. If we use X = 203, then they scare (10(203), or 2030 people, which exceeds 2024.
So, Morticia plans to scare (3)(202), or 606 people.

NEXT PAGE FOR SOLUTIONS TO PROBLEMS #7-9

7) Number of squares:

1x1:	25
2x2:	16
3x3:	9
4x4:	4
5x5:	1
Total:	55

8) The fifteen positive factors of 400:
1, 2, 4, 5, 8, 10, 16, 20, 25, 40, 50, 80, 100, 200, 400
The factors divisible by 4: 4, 8, 16, 20, 40, 80, 100, 200, 400
How many is that? 9

Alternate solution:
The prime factorization of 400 is 2 x 2 x 2 x 2 x 5 x 5
There are six factors of 400 that are NOT divisible by 4:
1, 2, 5, 5x5, 2x5, and 2 x 5 x 5.
So, subtract the number of factors that are not divisible by 4 from the total number of factors: 15 - 6 = 9.

9) If it were a solid block with no "tunnels," there would be 4 x 3 x 5, or 60 small blocks.
The two horizontal tunnels removed 6 + 10 - 2, or 14 blocks.
(The two horizontal tunnels overlap, so only two blocks had to be removed at their intersection.)
The tunnel starting at the right intersected with the vertical tunnel, so only 2 additional blocks were removed.
60 - (14 + 2) = 44 total blocks.