IMLEM Meet #4
February, 2024

Intermediate Mathematics League of Eastern Massachusetts

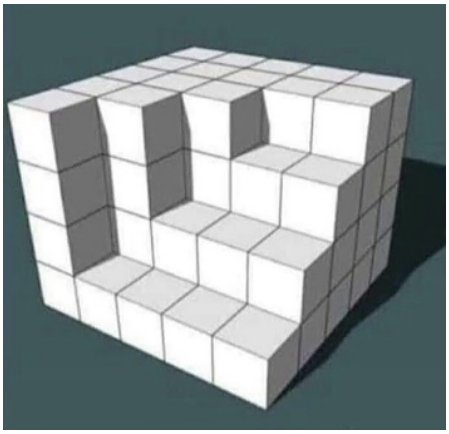
Calculator Meet

CLUSTER COORDINATORS - A reminder to all students of some of the rules and of appropriate behavior during this meet: • Many of you are guests in someone else's school – please be respectful of the classrooms and spaces you are using. Any "out of control" behavior in the halls or during a round is not acceptable. If an adult deems your behavior disrespectful or inappropriate, your score may not be counted. • CALCULATORS: only *scientific calculators* allowed for meets #4 & #5) • Everyone take a moment to turn off any electronic devices that you want to have with you during the rounds. No electronic devices may be on during the rounds. Use of these devices during the rounds will result in a disqualification.

Category 1
Mystery
Meet #4 - February, 2024

Calculator Meet

1)



This figure consists of many individual cubes, such that each column is supported from below, all the way to the bottom of the structure. How many of the individual cubes are there?

2) What value of A makes the following equation a true statement?

$$\sqrt{A+15} + \sqrt{A} = 15$$

3) The symbol $ represents a specific set of operations, such that

1 $ 4 = 5
2 $ 5 = 12
3 $ 6 = 21
4 $ 7 = 32

What is the value of 15 $ 18 ?

Answers

1) _____

2) _____

3) _____

Solutions to Category 1
Mystery
Meet #4 - February, 2024

Answers	
1)	80
2)	49
3)	285

1) Bottom layer = (5)(5) = 25
Layer above has 4 fewer = 21
Next layer up has 3 fewer = 18
Top layer has 2 fewer = 16
Total = 25 + 21 + 18 + 16 = 80 cubes.

2) $\sqrt{A+15} + \sqrt{A} = 15$

After some guessing and checking, students can quickly come to the conclusion that A = 49.
Then sqrt (49 + 15) + sqrt (49)
= sqrt (64) + 7
= 8 + 7
= 15.

3) Solution 1: A $ B = A + AB. **1 $ 4 = 1 + (1)(4) = 5**
2 $ 5 = 2 + (2)(5) = 12
And so on. Then 15 $ 18 = 15 + (15)(18) = 15 + 270 = 285.

Solution 2: The outputs show a pattern of increasing by consecutive odd numbers:

5 12 21 32 45 60 77 96 117 140
+7 +9 +11 +13 +15 +17 +19 +21 +23

165 192 221 252 285
+ 25 +27 +29 +31 +33

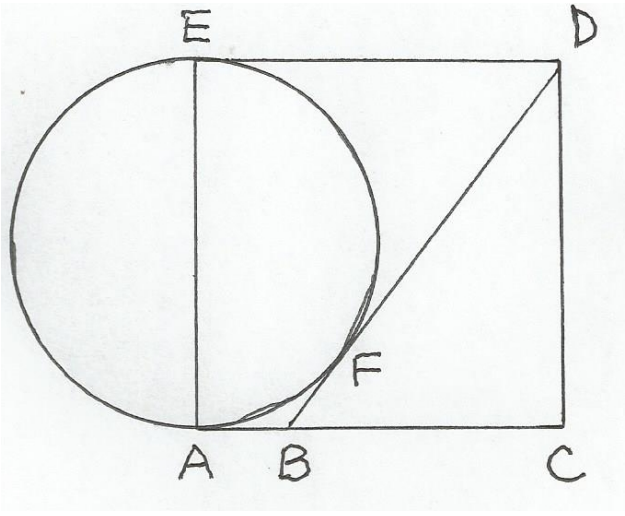
Category 2
Geometry
Meet #4 - February, 2024

Calculator Meet

1) Jasmine has a Hula Hoop with a diameter of 110 centimeters. How many centimeters are in its circumference? Ignore the thickness of the Hula Hoop. Use 3.14 as an approximation for the number pi.

2) A particular foreign coin has a radius of 15 millimeters. Jonas arranged a collection of these coins on a rectangular sheet of paper into rows and columns. The outermost coins are aligned with all four edges of the paper. All coins in any row or column touch each other. The paper measures 210 millimeters by 360 millimeters. How many coins are there?

3) A circle has a diameter AE that measures 12 feet. ACDE is a square. How many square feet are in the area of triangle BCD ? Segment DB is tangent to the circle at point F.



Answers

1) _____ cm

2) _____ coins

3) _____ sq. ft.

Solutions to Category 2
Geometry
Meet #4 - February, 2024

Answers	
1)	345.4
2)	84
3)	54

1) Circumference = (pi)(diameter)
C = (3.14)(110)
C = 345.4 centimeters

2) If each coin has a radius of 15 mm, then its diameter is (2)(15), or 30 mm.
Dividing the length by 30 gives 360 / 30 = 12 coins.
Dividing the width by 30 gives 210 / 30 = 7 coins.
The rectangular array of coins is 12 rows with 7 coins in each row. Multiply (7)(12) to get the total number of coins, or 84 coins.

3) Tangents to a circle from an external point are congruent, so that DE = DF = 12. Also, BA = BF = let's say X.
Now focus on triangle BCD as a right triangle where the Pythagorean Theorem can be used to calculate various lengths.
BC, the base of the right triangle, is now 12 - X.
CD, the altitude of the right triangle, is one side of the square, or 12.
BD, the hypotenuse of the right triangle, is X + 12.
Pythagorean Theorem: $(12 - X)^2 + (12)^2 = (X + 12)^2$
$$144 - 24X + X^2 + 144 = X^2 + 24X + 144$$
Subtract X^2 and 144 from both sides, yielding
$$144 - 24X = 24X$$
$$144 = 48X$$
$$3 = X$$
Then BC = 12 - X = 12 - 3 = 9.
Finally, the area of triangle BCD = (1/2)(base)(altitude)
= (1/2)(9)(12)
= 54 square feet.

Category 3
Number Theory ***Calculator Meet***
Meet #4 - February, 2024

1) A fictional street, Curtis Avenue in Sudbury, has houses numbered in the following arithmetic sequence: 3, 10, 17, 24, and so on. What is the number on the 38th house on Curtis avenue?

2) Natasha has a birthday around the middle of the month. It is currently around the middle of the month of February. If she is exactly 451 months old, then what is the number of the month that was Natasha born if January = 1, February = 2, March = 3, and so on?

3) $\sum_{K=1}^{n} K^2 = \frac{(n)(n+1)(2n+1)}{6}$ **is the formula for adding consecutive square numbers in the following series:** $1^2+2^2+3^2+\ldots+n^2$.

For example, $\sum_{K=1}^{4} K^2 = 1^2+2^2+3^2+4^2 = 1+4+9+16 = 30$

We can achieve the same result by substituting 4 for n into the formula: $\frac{(n)(n+1)(2n+1)}{6} = \frac{(4)(4+1)(2(4)+1)}{6} = \frac{(4)(5)(9)}{6} = \frac{180}{6} = 30.$

*** * Important - this formula works for series that begin with K = 1.**

Find the value of the following sum: 64 + 81 + 100 + 121 + . . . + 3249.

Answers

1) _____

2) _____

3) _____

Solutions to Category 3
Number Theory
Meet #4 - February, 2024

Answers	
1)	262
2)	7
3)	63,225

1) A formula for the Nth term in this sequence is 7N - 4, as there is a difference of seven between any two consecutive terms and the first term is 3 less than 7. So, the value of the 38th term is 7(38) - 4, or 266 - 4, or 262.

2) Divide 451 by 12 to find the number of years we must count into the past: 451 / 12 = 37 years with remainder 7. So, thirty-seven years ago, it was the month of February. Now either count back 7 months or else count ahead 5 months to the month of Natasha's birthday, which is July, the 7th month. Answer: 7.

3) To find the required sum, subtract the sum of the squares of the integers from 1 through 7, inclusive, from the sum of the squares of the integers from 1 through 57, inclusive.

$$\left(\sum_{K=1}^{57} K^2\right) - \left(\sum_{K=1}^{7} K^2\right) = \left(\frac{(57)(57+1)(2\cdot 57+1)}{6}\right) - \left(\frac{(7)(7+1)(2\cdot 7+1)}{6}\right)$$

$$= \left(\frac{(57)(58)(115)}{6}\right) - \left(\frac{(7)(8)(15)}{6}\right) = \left(\frac{380{,}190}{6}\right) - \left(\frac{840}{6}\right)$$

= 63,365 - 140 = 63,225.

Category 4
Arithmetic
Meet #4 - February, 2024

Calculator Meet

1) Lynn Gueeny bought an electric bicycle a few years ago for $680. The bicycle has since lost 35% of its value. How many dollars is Lynn's bicycle worth now?

2) The length and width of a rectangle are doubled, thus increasing its area by X percent. What is the value of X ?

3) The formula for compound interest is $A = P(1+r)^t$ provided that

P = the principal, or amount invested,
r = the annual (yearly) rate at which the principal is invested,
t = the amount of time, in years, that the money is invested, and
A = the total amount of money, including the principal and the interest.

Saul found his great-grandfather's bank book in a box in the attic. His savings account on February 14, 1927 had a balance of $9.15. Assuming an average rate of 7% over the course of time, how many dollars is the savings account worth on February 14, 2024? Round your answer to the nearest dollar.

Answers

1) _____

2) _____

3) _____

Solutions to Category 4
Arithmetic
Meet #4 - February, 2024

Answers	
1)	442
2)	300
3)	6,481

1) The new value is (100% - 35%) of $680
= 65% of $680
= (0.65)(680)
= $442.

2) Area of original rectangle = (length)(width) = (LW)
Area of larger rectangle = (2L)(2W) = 4LW.
The amount of increase is 4LW - 1LW = 3LW.
The % of increase = (the amount of increase) divided by the original amount
= 3LW / 1LW = 3 = 300%.
Therefore, X = 300.

3) $A = P(1+r)^t$

A = (9.15)(1 + 0.04)^(2024 - 1927)
A = (9.15)(1.07)^97
A = (9.15)(708.31499 . . .)
A = 6,481.08219 . . .
Rounded to the nearest dollar, the savings account is worth $6,481 on February 14, 2024.

Category 5
Algebra
Meet #4 - February, 2024

Calculator Meet

1) Artie was born on February 15, 1954. His sister, Jane, was born on February 15, 1959. How many years older was Artie than Jane on their birthday in the year 2000?

2) Al Fraydo is 5 feet 8 inches tall casts a shadow that is 8 feet long. He stands in the vicinity of the tallest building in the world - the Burj Khalifa in Dubai - that is 2717 feet tall. To the nearest foot, how many feet long is the building's shadow?

3) Rob and Liz live 390 miles apart. They each leave their homes at the same time and head in each other's direction on the same road. Rob's average speed is 30 miles per hour less than Liz' average speed. After three hours on the road, they pass each other! How many miles per hour was Liz travelling?

ANSWERS

1) _____ years

2) _____ feet

3) _____ m.p.h.

Solutions to Category 5
Algebra
Meet #4 - February, 2024

Answers	
1)	5
2)	3,836
3)	80

1) Regardless of the year, Artie will always be 5 years older than Jane.

2) Be sure we are comparing similar units. Suggestion:
First, convert 5 feet 8 inches to inches:
(5)(12) + 8 = 68 inches.Then 8 feet = (8)(12), or 96 inches . . .
And 2717 feet = (2717)(12) = 32,604 inches.
Then set up a proportion, letting X = the length of the building's shadow:
height : shadow 68 : 96 = 32,604 : X
Cross products are equal: (68)(X) = (32,604)(96)

$$68X = 3{,}129{,}984$$
$$X = 3{,}129{,}984 / 68$$
$$X = 46{,}029.1765 \text{ inches}$$

Convert to feet: 46,029.1765 / 12 = 3,835.76471 feet
To the nearest foot, rounding gives us that the length of the building's shadow is about 3,836 feet long.

3) Rate x Time = Distance
Let S = Rob's rate of speed
then S + 30 = Liz' rate of speed
Since Rob and Liz travel toward one another, then by the time they pass each other, they will have travelled a total distance of 390 miles while taking the same amount of time to do so (3 hours).

$$\text{Rob's distance} + \text{Liz' distance} = 390$$
$$(\text{Rob's rate})(\text{Rob's time}) + (\text{Liz' rate})(\text{Liz' time}) = 390$$
$$(S)(3) + (S + 30)(3) = 390$$
$$3S + 3S + 90 = 390$$
$$6S + 90 = 390$$
$$6S = 300$$
$$S = 50$$
$$S + 30 = 80$$

Therefore, Liz' rate of speed was 80 miles per hour.

Category 6
Team Round
Meet #4 - February, 2024

Each of the following six problems is worth <u>six</u> points.

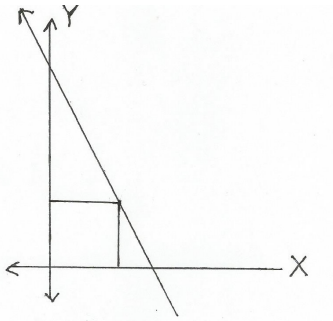
1) What is the value of N if $27^{17} + 27^{17} + 27^{17} = 3^{N}$

2) Coakley decided to save money on each of the prime-numbered dates during the month of January according to this pattern: He saved $5 on January 2, $10 on January 3, $15 on January 5, $20 on January 7, and so on, saving $5 more on each date than on the previous date. How many dollars did Coakley save in January?

3) If $2^{A} + 2^{B} + 2^{C} = 328$ **then what is the value of A + B + C ?**

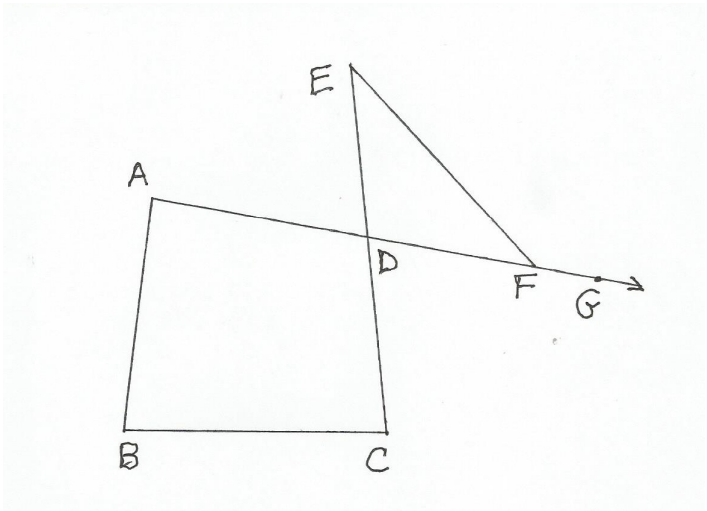
4) The equation of the diagonal line below, graphed on a standard X-Y plane, is

$Y = -\frac{5}{3}X + 10$ **How many square units are in the area of the square?**



5) If a circle's diameter is 26 inches, then how many more square inches are in its area than there are inches in its circumference? Use pi = 3.14.

6) In the diagram below, angles A, B, and C each have a measure of 80 degrees. DE = DF. How many degrees are in the measure of angle EFG?



ANSWERS

1) _____

2) _____

3) _____

4) _____

5) _____

6) _____

Solutions to Category 6
Team Round
Meet #4 - February, 2024

1) $27^{17} + 27^{17} + 27^{17} = 3^N$

$\left(3^3\right)^{17} + \left(3^3\right)^{17} + \left(3^3\right)^{17} = 3^N$

$3^{51} + 3^{51} + 3^{51} = 3^N$

$3\left(3^{51}\right) = 3^N$

$3^{52} = 3^N$

N = 52

ANSWERS	
1)	52
2)	330
3)	17
4)	14.0625
5)	449.02
6)	150

2) **Date in January:** 2 3 5 7 11
Number of dollars: 5 10 15 20 25

Date in January: 13 17 19 23 29 31
Number of dollars: 30 35 40 45 50 55

Total number of dollars: 5+10+15+20+25+30+35+40+45+50+55
= $330.

3) $Y = -\frac{5}{3}X + 10$ **No value of any variable can exceed 8, as 2^8 = 256 and 2^9 = 512. Experimenting with adding powers of 2 up to 2^8 will reveal that the sum of A, B, and C is 3 + 6 + 8 = 17 because 2^3 = 8, 2^6 = 64, 2^8 = 256 and 8 + 64 + 256 = 328.**

4) **The point where the square intersects the diagonal line has the coordinates (X, Y) where X = Y, since the figure is a square. So, we can substitute X for Y in the equation of the diagonal line and solve for X:**

$X = (-5/3)X + 10$
$X + (5/3)X = 10$
$(8/3)X = 10$
$8X = 30$
$X = 30 / 8 = 3.75$
so then Y also = 3.75

The area of the square is (3.75)(3.75) = 14.0625 square units.

Solutions. Category 6, Team Round - problems #5 & 6

5) Diameter of circle = 26 inches, so radius = 13 inches.
A = (pi)(square of radius) = (3.14)(13)(13) = 530.66 square inches.
C = (pi)(diameter) = (3.14)(26) = 81.64.
The numerical difference between the area and circumference is 530.66 - 81.64 = 449.02.

6) Regarding quadrilateral ABCD, angle ADC measures 360 - (3)(80), or 360 - 240, or 120 degrees. Its vertical (opposite) angle, EDF, also measures 120 degrees. Triangle EDF is isosceles because two of its sides have the same length, so the two base angles are congruent, with each measuring 30 degrees. So, the external angle, EFG, measures 180 - 30, or 150 degrees.