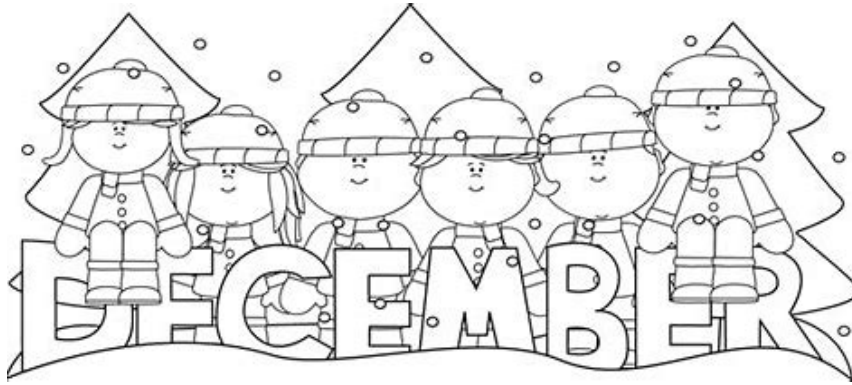


IMLEM Meet #2
December, 2022

Intermediate
Mathematics League
of
Eastern Massachusetts



CLUSTER COORDINATORS - A reminder to all students of some of the rules and of appropriate behavior during this meet:

- **No calculators (or only scientific calculators allowed for meets #4, #5)**
- **Everyone take a moment to turn off any electronic devices that you want to have with you during the rounds. No electronic devices may be on during the rounds. Use of these devices during the rounds will result in a disqualification.**
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Category 1

Mystery

Meet #2 - December, 2022

1) What is the value of the seventeenth number in the following pattern?

4 7 11 18 29 47 76 . . .

2) The sum of eight positive integers is 31. If no individual integer value can appear more than twice in the list of eight integers, then what is the greatest possible value that one of these eight integers can have?

3) There are 400 people at a soccer game. If there are 40 more adults than children and 40 more adult men than adult women, then how many adult men are at the game?

Answers

1) _____

2) _____

3) _____

Solutions to Category 1

Mystery

Meet #2 - December, 2022

Answers

1) 9349

1) Here are two ways to view the relationship among the terms:

2) 15

a) The pattern is similar to the Fibonacci sequence, in that any term is the sum of the two previous terms, or

3) 130

b) the difference between any two consecutive terms is equal to the value of the previous term.

The values of the first seventeen terms are as follows:

4 7 11 18 29 47 76 123 199 322 521 843 1364
2207 3571 5778 9349

So, the value of the seventeenth term is 9349.

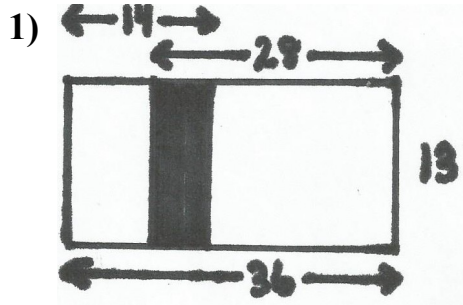
2) Minimize the value of as many numbers as possible, leaving only the final two numbers, one of which is as small as possible, leaving plenty of room for the largest value. Those first seven values are 1 1 2 2 3 3 4 and their sum is 16. So, the eighth number must be 31 - 16, or 15.

3) First, divide the 400 into two groups, adults and children, so that there are 40 more adults than children. There are 220 adults and 180 children. Now divide the 220 adults into adult men and adult women so that there are 40 more adult men than adult women. There are 130 adult men and 90 adult women. The question calls for the number of adult men, so there are 130.

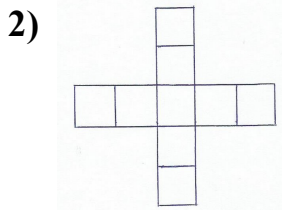
Category 2

Geometry

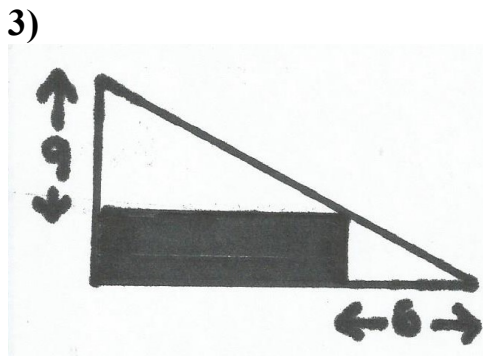
Meet #2 - December, 2022



Dimensions of various line segments are given as 14, 28, 13, and 36 inches, as shown. How many square inches are in the shaded area? All angles are right angles.



The figure to the left shows an arrangement of nine squares, each with a side length of T . The perimeter of the figure is P centimeters and its area is A square centimeters. If $P = A$, then the value of T can be expressed as an improper fraction in lowest terms. What is the sum of the numerator and denominator of that fraction?



A rectangle is inscribed in a triangle, as shown. Two dimensions are given as 9 and 6 feet, respectively. How many square feet are in the shaded area?

<u>Answers</u>	
1) _____	sq. in.
2) _____	
3) _____	sq. ft.

Solutions to Category 2

Geometry

Meet #2 - December, 2022

- 1) The width of the shaded rectangle is $14 + 28 - 36$, or 6 inches. So, the shaded rectangle is 6 inches by 13 inches and its area is $(6)(13)$, or 78 square inches.
- 2) The perimeter, in terms of T , is $P = 20T$. The area is $A = 9(T)(T)$. Since $P = A$, then $20T = 9(T)(T)$. Divide both sides by T to get $20 = 9T$, so $T = 20/9$. The sum of the numerator is $20 + 9$, or 29.
- 3) The triangles on either side of the rectangle are similar. If X = the base of the larger triangle and Y = the altitude of the smaller triangle, then corresponding sides are in proportion. $9 / X = Y / 6$. Then cross products are equal, so $XY = (9)(6)$, or $XY = 54$. Since X and Y are the base and altitude of the shaded rectangle, then its area is XY or 54 square feet.

Answers

1) 78

2) 29

3) 54

Category 3

Number Theory

Meet #2 - December, 2022

- 1) The GCF of A and B is the greatest common factor of A and B.
The LCM of A and B is the lowest common multiple of A and B.

A = the GCF of 12 and 30

B = the LCM of 14 and 21

C = the GCF of 6 and 18

D = the LCM of 8 and 5

E = the GCF of 8 and 5

What is the value of the product ABCDE ?

- 2) How many of the following eleven numbers are factors of every positive whole number that is divisible by both 9 and 12 ?

3 4 6 8 18 24 27 36 45 72 108

- 3) Consider these factorizations:

$$C = 2 \times 3^2 \times N \quad \text{and} \quad D = 2^2 \times 5 \times N^2$$

If all of the lettered values are positive integers (whole numbers)
and $D = 1620$, then what is the value of C ?

Answers

1) _____

2) _____

3) _____

Solutions to Category 3
Number Theory
Meet #2 - December, 2022

- 1) $A = \text{GCF}(12, 30) = 6$
 $B = \text{LCM}(14, 21) = 42$
 $C = \text{GCF}(6, 18) = 6$
 $D = \text{LCM}(8, 5) = 40$
 $E = \text{GCF}(8, 5) = 1$
The product $ABCDE = (6)(42)(6)(40)(1)$
 $= 60,480.$

- 2) Whole numbers that are divisible by both 9 and 12 are multiples of their LCM, namely, 36. The only numbers in the list that are factors of 36 are 3, 4, 6, 18, and 36. So, there are five such numbers.

- 3) If $D=1620$, then $(N)(N) = 1620 / 20$, and

$81=N^2$ and $N = 9$. Then substituting 9 for N in the first equation,

$C=2 \times 3^2 \times 9$ and $C = 162.$

<u>Answers</u>	
1)	60,480
2)	5
3)	162

Solutions to Category 4

Arithmetic

Meet #2 - December, 2022

1) The fraction $7/20 = 35/100 = 35\%$
The percent of roses is $100\% - (18\% + 35\%)$
 $= 100\% - 53\% = 47\%$. So, $X = 47$.

2) Let $X = 0.54444\dots$
Then $10X = 5.444\dots$
Subtracting the first from the second yields
 $9X = 4.9$
 $X = 4.9 / 9 = 49 / 90$ which is in lowest terms.
The sum of the numerator and denominator
of $49 / 90$ is $49 + 90 = 139$.

3) The fraction of Evan's allowance that is set aside for savings is
 $1 - (2/9 + 20\%)$
 $= 1 - (2/9 + 1/5)$
 $= 1 - (10/45 + 9/45)$
 $= 1 - (19/45)$
 $= 26/45$
\$468 is $26/45$ of his yearly allowance, so his yearly allowance is
468 divided by $26/45$
 $= 468 \times 45/26$
 $= 810$.

Answers

1) 47

2) 139

3) 810

Category 5

Algebra

Meet #2 - December, 2022

1) What number added to 56 is equal to five times the original number?

2) Jethro has two more 4-pound watermelons than 6-pound watermelons. The watermelons weigh 158 pounds in all. How many watermelons does Jethro have?

3) If $E \Psi H = \frac{E + H}{E - H}$ and $(12 \Psi 4) \Psi N = -0.5$ then what is the value of N?

Answers

1) _____

2) _____

3) _____

Solutions to Category 5
Meet #2 - December, 2022

1) Let $X =$ the original number

$$\text{Then } 56 + X = 5X$$

$$4X = 56$$

$$X = 14$$

2) Let $W =$ the number of 6-pound watermelons

$W + 2 =$ the number of 4-pound watermelons

$6W =$ the weight, in pounds, of the 6-pounders

$4(W + 2) =$ the weight, in pounds, of the 4-pounders

$$6W + 4(W + 2) = 158$$

$$6W + 4W + 8 = 158$$

$$10W + 8 = 158$$

$$10W = 150$$

$$W = 15 \quad \text{So, there are 15 6-pounders.}$$

$$W + 2 = 17 \quad \text{and there are 17 4-pounders}$$

and there are $15 + 17$, or 32 watermelons in all.

$$3) \quad (12 \Psi 4) = \frac{12+4}{12-4} = \frac{16}{8} = 2$$

$$(2 \Psi N) = \frac{2+N}{2-N} = -0.5.$$

$$-0.5(2 - N) = 2 + N$$

$$-1 + 0.5N = 2 + N$$

$$-3 = 0.5N$$

$$N = -6$$

Answers

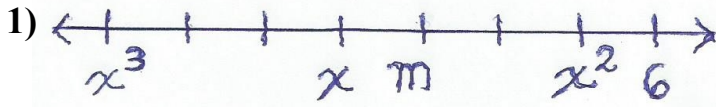
1) 14

2) 32

3) -6

Category 6
Team Round
Meet #2 - December, 2022

Each of the following twelve problems is worth three points.



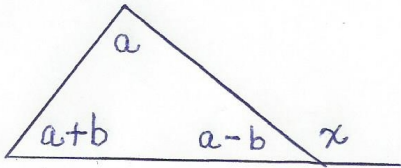
On this number line, all of the tick marks are equally spaced. What is the value of m ?

2) If $a < 0$, and the square of the quantity $a - 4$ is equal to 36, then what is the value of a ?

3) If $\frac{4X+3Y}{2Y} = \frac{5}{3}$ then what is the value of $\frac{Y}{X}$?

4) If $C - D = 5$ and $C^2 + D^2 = 15$ then what is the value of the product CD ?

5)



The interior angles of this triangle measure a , $a + b$, and $a - b$ degrees, respectively. An exterior angle measures x degrees. How many degrees are in the measure of angle x if $a + b = 83$?

6) Ninety teachers work at an Oz Middle School in Kansas. If the ratio of men teachers to women teachers is $7 : 8$, then how many more women teachers than men teachers work at Oz?

ANSWERS

1) _____

2) _____

3) _____

4) _____

5) _____

6) _____

**Solutions to Category 6
Team Round
Meet #2 - December, 2022**

1) Since x cubed is less than x , then x must be a negative number. Considering the equal spaces between x and x cubed, x must be -2 so that x cubed is -8 and making x squared equal to 4 . Counting backwards from 6 , m must be zero.

2) $(a - 4)^2 = 36$ so $(a - 4)$ is equal to either 6 or -6 . But $a < 0$, so $a - 4 = -6$ and $a = -2$.

3) $\frac{4X+3Y}{2Y} = \frac{5}{3}$ Cross products are equal, so
 $3(4X + 3Y) = 5(2Y)$
 $12X + 9Y = 10Y$
 $12X = Y$
 $12 = Y / X$

4) This is an exercise in reasoning in number sentences.

If $C - D = 5$, then the square of $C - D$ is 25 .

$$(C-D)^2 = C^2 - 2CD + D^2 = 25$$

Also, $C^2 + D^2 = 15$.

Subtracting one equation from the other yields $-2CD = 10$ and $CD = -5$.

5) The sum of the interior angles of a triangle is 180 degrees. So, $a + (a + b) + (a - b) = 180$ and $3a = 180$ so $a = 60$. $a + b = 83$, so $b = 23$ and $a - b = 37$. Angle x is supplementary to $a - b$, so $x = 180 - 37$, or 143 degrees.

6) Let X be the scaling factor so that $7X + 8X = 90$ and $15X = 90$ so that $X = 6$. Women teachers = $8X = 8(6) = 48$. Men teachers = $7X = 7(6) = 42$. Check: $48 + 42 = 90$ and $42 : 48$ simplifies to $7 : 8$. The difference in number between the women and men teachers is $48 - 42$, or 6 .

ANSWERS

- 1) 0
 2) -2
 3) 12
 4) -5
 5) 143
 6) 6

