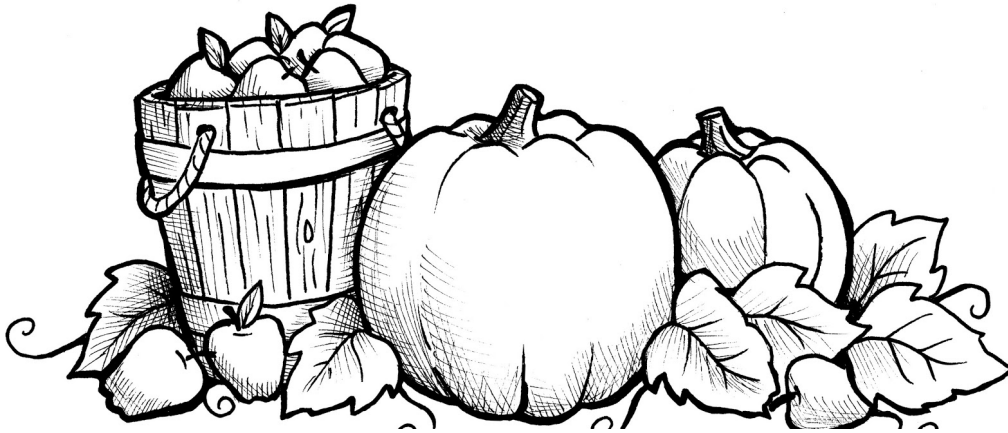


IMLEM Meet #1  
October, 2021

Intermediate  
Mathematics League  
of  
Eastern Massachusetts



**CLUSTER COORDINATORS - A reminder to all students of some of the rules and of appropriate behavior during this meet:**

- **No calculators (or only scientific calculators allowed for meets #4, #5)**
- **Everyone take a moment to turn off any electronic devices that you want to have with you during the rounds. No electronic devices may be on during the rounds. Use of these devices during the rounds will result in a disqualification.**



**Solutions to Category 1  
Mystery  
Meet #1 - October, 2021**

1) How many MINUTES! At 200 pages per 4 HOURS, Reid can read 700 pages in  $700 / (200 / 4)$ , or 14 hours. Convert 3.5 hours to minutes:  $(14)(60) = 840$  minutes.

2) Students may inadvertently figure that the pen costs a dollar and the pencil costs ten cents. But then the pen would only cost 90 cents more than the pencil. A quick finagling of the numbers should yield that the pen costs \$1.05 and the pencil just 5 cents.

3) Making an organized list may help. For the range of numbers from 1-99, the teens alone produce the digit 1 eleven times. For the rest, one more than a multiple of 10 (1, 21, 31, 41, etc.) produces another nine ones, for a total of 20 ones. The same should hold true for the other ranges of 100 numbers:

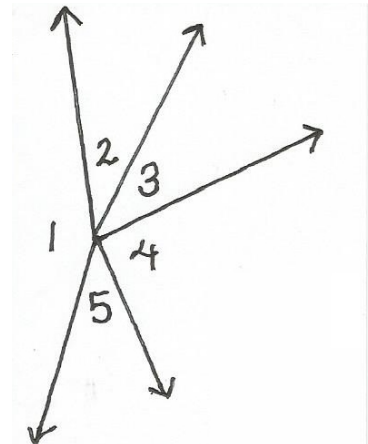
<u>range</u>	<u>number of ones digits</u>
1-99	20 includes the extra 1 for the number 11
100-199	120 (20, plus an extra 100 ones for the hundreds digits)
200-299	20
300-399	20
400-430	13

The total number of ones is  $20 + 120 + 20 + 20 + 13$ , or 193.

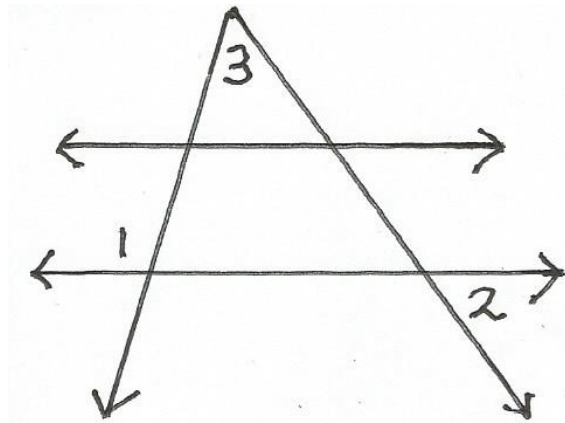
<u>Answers</u>	
1)	840
2)	5
3)	193

**Category 2**  
**Geometry**  
**Meet #1 - October, 2021**

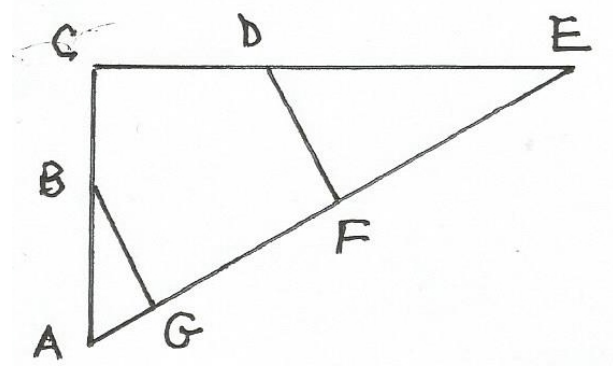
- 1) Five rays emanate from a common point, as shown. Angles 2 and 5 have the same measure. Angles 3 and 4 are complementary. Angle 1 measures 174 degrees. How many degrees are in the measure of angle 2 ?  
 (Figure not drawn to scale)



- 2) The two horizontal lines are parallel. Angle 1 measures 114 degrees. Angle 2 measures 71 degrees. How many degrees are in the measure of angle 3 ?



- 3) Angles C, DFE, and BGF are right angles. Angle FDE measures 72 degrees. What is the measure of angle CBG ?



**Answers**

1) \_\_\_\_\_

2) \_\_\_\_\_

3) \_\_\_\_\_

**Solutions to Category 2  
Geometry  
Meet #1 - October, 2021**

1) The sum of all five angles is 360 degrees. Since angles 3 and 4 are complementary, their sum is 90 degrees. The sum of angles 1, 3, and 4 is  $174 + 90$ , or 264 degrees. The remaining two angles, 2 and 5, are the same measure, so their sum is  $360 - 264$ , or 96 degrees. Half that is 48 degrees. So, angle 2 measures 48 degrees.

**Answers**

1) 48

2) 43

3) 162

2) Corresponding angles are congruent, so the corresponding angle above angle 1 measures 114 degrees and its adjacent collinear angle measures  $180 - 114$ , or 66 degrees. Also, vertical angles are congruent, the the angle vertical to angle 2 measures 71 degrees and its corresponding angle above it measures 71 degrees. The sum of the angles of the triangle is 180 degrees, so that angle 3 measures  $180 - (66 + 71)$ , or  $180 - 137$ , or 43 degrees.

3) Angle E measures  
 $180 - (DFE + (FDE))$   
 $= 180 - (72 + 90)$   
 $= 180 - 162$   
 $= 18$  degrees.

Angle A measures  
 $180 - (C + E)$   
 $= 180 - (90 + 18)$   
 $= 180 - 108$   
 $= 72$  degrees.

Angle ABG measures  
 $180 - (BGA + A)$   
 $= 180 - (90 + 72)$   
 $= 180 - 162$   
 $= 18$  degrees.

Therefore, angle CBG measures  $180 - ABG$   
 $= 180 - 18$   
 $= 162$  degrees.

**Category 3**  
**Number Theory**  
**Meet #1 - October, 2021**

1) What is the sum of all prime numbers between 10 and 20 ?

2) Find the value of C if

C is a whole number,

C is divisible by 6,

$C > 60$ ,

C does not have a factor of 5,

$C < 100$ ,

C is not divisible by 11, and

the sum of the digits of C is 12.

3) What is the largest prime factor of the product of all even numbers from 2 through 200, inclusive?

**Answers**

1) \_\_\_\_\_

2) \_\_\_\_\_

3) \_\_\_\_\_

**Solutions to Category 3**  
**Number Theory**  
**Meet #1 - October, 2021**

- 1) The sum of the prime numbers between 10 and 20 is  $11 + 13 + 17 + 19 = 60$ .
- 2) C is between 60 and 100. Those whose digits have a sum of 12 are 66, 75, 84, and 93. Only 66 and 84 are divisible by 6 but 66 is divisible by 11. So, the number is 84.
- 3) Clue: the largest prime number that is less than 100 is 97. Then  $(97)(2) = 194$ , the largest even number less than 200 that has 97 as a factor. When all the even numbers from 2 through 200 are multiplied, then 97 will be that product's largest prime factor.

**Answers**

1) 60

2) 84

3) 97

**Category 4**  
**Arithmetic**  
**Meet #1 - October, 2021**

1) What is the value of  $16 + 4[100 - (6 \times 20 \div 5 \times 2)]$  ?

2) For this set of numbers { 7, 12, 9, 3, 9, 20, 12, 6, 12 },  
A = the median,  
B = the mean, and  
C = the mode.

What is the value of  $A - B + C$  ?

3) The mean of five numbers is 7. The mean of seven other numbers is 11.  
The mean of yet another three numbers is 10. If the number 2 is thrown  
into the set, then what is the mean of all 16 numbers?

**Answers**

1) \_\_\_\_\_

2) \_\_\_\_\_

3) \_\_\_\_\_



**Solutions to Category 4**  
**Arithmetic**  
**Meet #1 - October, 2021**

1)  $16 + 4 [ 100 - ( 6 \times 20 / 5 \times 2 ) ]$   
 $= 16 + 4 [ 100 - ( 120 / 5 \times 2 ) ]$   
 $= 16 + 4 [ 100 - ( 24 \times 2 ) ]$   
 $= 16 + 4 [ 100 - 48 ]$   
 $= 16 + 4 [ 52 ]$   
 $= 16 + 208$   
 $= 224$

**Answers**

1) 224

2) 11

3) 9

2) For A, arrange the numbers in order, then select the middle value. 3 6 7 9 9 12 12 12 20  
The middle value is 9, so  $A = 9$ .

B = the sum of the nine numbers divided by 9.

$$= 90 / 9$$

$$= 10$$

C = the most frequent value = 12.

$$A - B + C = 9 - 10 + 12$$

$$= 11$$

3) The mean of all 16 numbers is equal to the sum of the 16 numbers divided by 16

$$= [ (5)(7) + (7)(11) + (3)(10) + 2 ] / 16$$

$$= [ 35 + 77 + 30 + 2 ] / 16$$

$$= 144 / 16$$

$$= 9$$

## Category 5

### Algebra

#### Meet #1 - October, 2021

1) If  $M = 17$ ,  $A = -3$ ,  $T = 6$ , and  $H = -5$ , then what is the value of

$$M - A - T - H ?$$

2) If  $3X - 8 = 19$

and  $16 - 2Y = 26$

and  $4(7Z + 3) = 180$

then what is the value of  $10XYZ$  ?

3) Find the value of  $N$  if

$$2(N + 1) + 3(4N - 5) - 6(7N + 8) - 9(10N - 11) = 12(13 - 14N) + 32$$

### Answers

1) \_\_\_\_\_

2) \_\_\_\_\_

3) \_\_\_\_\_

**Solutions to Category 5**  
**Algebra**  
**Meet #1 - October, 2021**

1)  $M - A - T - H$   
 $= 17 - (-3) - 6 - (-5)$   
 $= 17 + 3 - 6 + 5$   
 $= 19$

2)  $3X - 8 = 19$ , so  $3X = 27$  and  $X = 9$ .  
 $16 - 2Y = 26$  so  $2Y = -10$  and  $Y = -5$ .  
 $4(7Z + 3) = 180$  so  $7Z + 3 = 45$   
and  $7Z = 42$  and  $Z = 6$ .  
 $10XYZ = (10)(9)(-5)(6)$   
 $= -2700$

3)  $2(N + 1) + 3(4N - 5) - 6(7N + 8) - 9(10N - 11) = 12(13 - 14N) + 32$   
 $2N + 2 + 12N - 15 - 42N - 48 - 90N + 99 = 156 - 168N + 32$   
 $-118N + 38 = 188 - 168N$   
 $50N = 150$   
 $N = 3$

**Answers**

1) 19

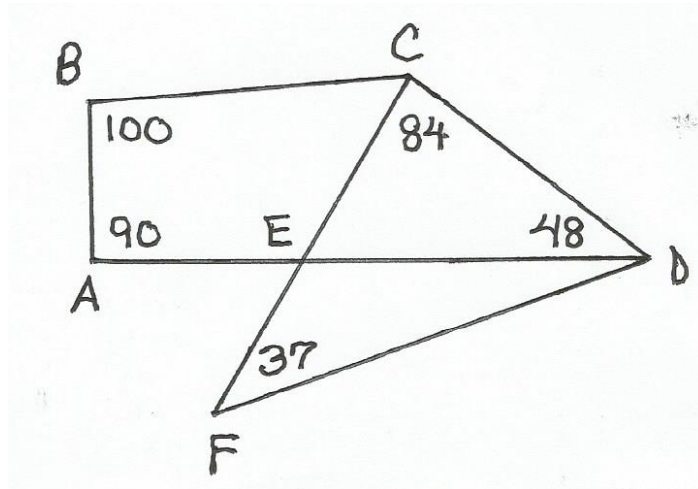
2) -2700

3) 3

**Category 6**  
**Team Round**  
**Meet #1 - October, 2021**

*Each of the following six problems is worth six points.*

- 1) What is the maximum number of points where a circle and a square can intersect?
- 2) A set of  $W$  consecutive positive integers has a sum of 45. What is the maximum value of  $W$  ?
- 3) How many degrees are in the measure of angle EDF ? Refer to the diagram at the bottom of the page.
- 4) How many 3-digit numbers greater than 500 consist of three different non-zero digits?
- 5) At my local farm stand, there were 60 more apples than peaches and five times as many apples as peaches. How many apples were there?
- 6) The answers to questions #1-5 are represented by the letters A, B, C, D, and E in the following expression.  
What is the value of  $3BD + 5AC - 7E$  ?



**ANSWERS**

- 1) \_\_\_\_\_
- 2) \_\_\_\_\_
- 3) \_\_\_\_\_
- 4) \_\_\_\_\_
- 5) \_\_\_\_\_
- 6) \_\_\_\_\_

**Solutions to Category 6  
Team Round  
Meet #1 - October, 2021**

**ANSWERS**

- 1) 8  
2) 9  
3) 11  
4) 280  
5) 75  
6) 7475

1) If a circle just intersects the four vertices of a square, then there are four points of intersection. shrink the circle a bit and there can be as many as 8 points of intersection.

2) There are several sets of consecutive positive integers whose sum is 45, for instance 14, 15, 16 and 7, 8, 9, 10, 11. But the set with the maximum number of numbers is 1, 2, 3, 4, 5, 6, 7, 8, 9. Therefore,  $W = 9$ .

3) The sum of the measures of the four angles of a quadrilateral is 360. ABCD has vertex angles of 100, 90, 48, and angle BCD. The sum of the three given angles is 238, so angle BCD must measure  $360 - 238$ , or 122. So, angle BCE

is  $122 - 84$ , or 38 degrees. Then angle AEC measures  $360 - (100 + 90 + 38)$ , or  $360 - 228$ , or 132 degrees. Its vertical angle, FED, also measures 132 degrees. The sum of the measures of triangle FED is 180 degrees, so that angle EDF =  $180 - (37 + 132)$ , or  $180 - 169$ , or 11 degrees.

4) from 501-520: 512, 513, 514, 516, 517, 518, 519. That's seven numbers.  
from 520-530: 521, 523, 524, 526, 527, 528, 529. That's seven numbers.  
The same holds true for each decade of numbers in the 500s, except for 550-560 where there are none, just like for 500-510. So, from 500-600, there are eight decades of seven numbers for a total of 56 numbers.  
The same holds true for the subsequent centuries of numbers from 600-1000 with no exceptions. There are five centuries in total, so there are  $(5)(56)$ , or 280 3-digit numbers greater than 500 consisting of three different non-zero digits. The permutation  $5 \times 8 \times 7$  also yields 280.

5) Let  $X$  = the number of peaches and  $X + 60$  = the number of apples.  
Then  $5X = X + 60$ ,  $4X = 60$ , and  $X = 15$ . So, the number of apples is  $15 + 60$ , or 75.

6)  $3BD + 5AC - 7E$   
=  $(3)(9)(280) + (5)(8)(11) - (7)(75) = 7560 + 440 - 525$   
= 7475