IMLEM Meet #5
April, 2021

Intermediate Mathematics League of Eastern Massachusetts

Calculator Meet

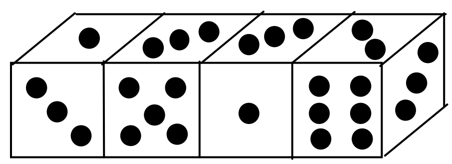
• CALCULATORS: only *scientific calculators* allowed for meets #4 & #5) • Everyone take a moment to turn off any electronic devices that you want to have with you during the rounds. No electronic devices may be on during the rounds. Use of these devices during the rounds will result in a disqualification.

Category 1
Mystery
Meet #5 - April, 2021

Calculator Meet

1) When Allen Iverson and Shaquille O'Neal faced off in the NBA basketball finals, O'Neal weighed twice as much as Iversen. Their weights totaled 495 pounds. How many pounds did O'Neal weigh?

2) Four standard cubical dice, with faces numbered 1, 2, 3, 4, 5, and 6, are arranged as shown below with nine of the faces visible from this perspective. What is the total number of dots (called "pips") that are not visible in this view of the diagram, including the pips on the faces of the dice that touch one another?



3) How many degrees are in the angle formed by the two hands of a standard 12-hour clock at 2:50 if that angle is between 180 and 360 degrees? Keep in mind that both hands move continuously and at different rates. Express your answer as a decimal.

Answers

1) ________

2) ________

3) ________

Solutions to Category 1
Mystery
Meet #5 - April, 2021

Answers	
1)	330
2)	57
3)	215

1) Since the weights are in the ratio 2:1, divide 495 by 3 to get the value of one part, or 165. O'Neal weighed (2)(165), or 330 pounds.

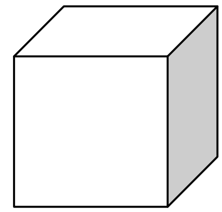
2) The sum of the number of pips on any one die is 1 + 2 + 3 + 4 + 5 + 6 = 21. The sum on all four dice is 4 x 21, or 84. The sum of the pips shown is 1 + 3 + 3 + 5 + 3 + 1 + 2 + 6 + 3 = 27. The total number of pips on the faces hidden from view is 84 - 27, or 57.

3) The angle that the minute hand from the 10:00 position to the 12:00 position is 60 degrees. The angle of the hour hand from the 12:00 position to its position at 2:50 is (2)(30) + (5/6)(30), or 60 + 25, or 85 degrees. Therefore, the obtuse angle between the hands is 60 + 85, or 145 degrees. On the "flip side" of the angle, the difference between 360 and 145 is 360 - 145, or 215 degrees.

Category 2
Geometry
Meet #5 - April, 2021

Calculator Meet

1) The total surface area of a cube is 384 square meters. How many cubic meters are in its volume?



2) Friendly Fred made a clay model of an ice cream cone, topped by a hemisphere (half-sphere) of ice cream whose diameter is the same as that of the top of the cone. He painted the entire project green, the color of his favorite flavor, pistachio. The cone has a height of 5.2 inches. The diameter of its circular top is 2.7 inches. How many square inches of green paint are used to cover Fred's project? Use $\pi \approx 3.14$. Round your final answer to the nearest tenth of a square inch. (Note: The only rounding should occur in the final step, when rounding your final answer to the nearest tenth.)

3) A sphere fits tightly inside a right circular cylinder, tangent to the two circular bases as well as to the wraparound surface. The volume of the cylinder is X % of the volume of the sphere. What is the value of X to the nearest tenth of a percent? Use $\pi \approx 3.14$.

Answers

1) ________ cu. m

2) ________ sq. in.

3) ________ %

Solutions to Category 2
Geometry
Meet #5 - April, 2021

Answers
1) 512
2) 34.2
3) 150

1) There are six identical square surfaces. Divide 384 by 6 to get 64, the area of one square surface. Square root 64 to get 8, the length of one side. Cube 8 to get the volume of 512 cubic meters.

2) Since s is equal to the slant "height," one must calculate s by using the Pythagorean Theorem, so s = the square root of the sum of the squares of 1.35 and 5.2, or the square root of 1.8225 + 27.04, or the square root of 28.8625 which is 5.37238 . . .
The total surface area is (pi)(r)(s) + (1/2)(4)(pi)(r squared)
= (3.14)(1.35)(5.37238 . . .) + (1/2)(4)(3.14)(1.35 squared))
= 22.7735188 . . . + 11.4453
= 34.218818 . . .
= 34.2 when rounded to the nearest tenth.

3) Write the formulas for the volume of a sphere and the volume of a cylinder, each with radius = R, then express as a ratio. Reduce to lowest terms then express as a percent:

$$\frac{\text{volume of cylinder}}{\text{volume of sphere}} = \frac{\pi(R^2)h}{\frac{4}{3}\pi(R^3)} = \frac{h}{\frac{4}{3}R} = \frac{2R}{\frac{4}{3}R} = (2)\left(\frac{3}{4}\right) = 1.5 = 150\%$$

Therefore, X = 150.

Category 3
Number Theory
Meet #5 - April, 2021

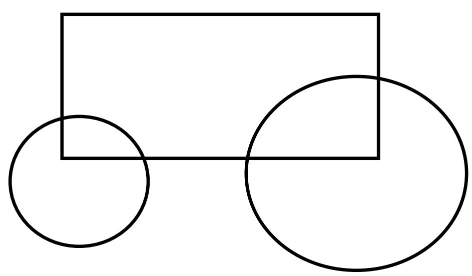
Calculator Meet

1) Set A = { multiples of 4 between 1 and 99 }
Set B = { multiples of 6 between 1 and 99 }
What is the sum of the numbers in $A \cap B$?
(What is the sum of the numbers in the intersection of sets A and B?)

2) Of the 517 students at the Amesbury Middle School,
- **248 own at least one dog,**
- **193 own at least one cat,**
- **76 own at least one dog and one cat.**

The rest have either a different pet or no pet at all.
How many students have either a different pet or no pet at all?

3) Santana arranged some of the toys that his school donated to Toys for Tots into roped areas as shown. All 824 trucks are in the rectangle. The 342 orange toys are in the smaller circle while the 487 red toys are in the larger circle. There are 72 red trucks and 119 orange trucks. If 2265 toys were donated in all, then how many of the toys were outside of the areas that Santana had roped off?



Answers

1) ________

2) ________

3) ________

Solutions to Category 3
Number Theory
Meet #5 - April, 2021

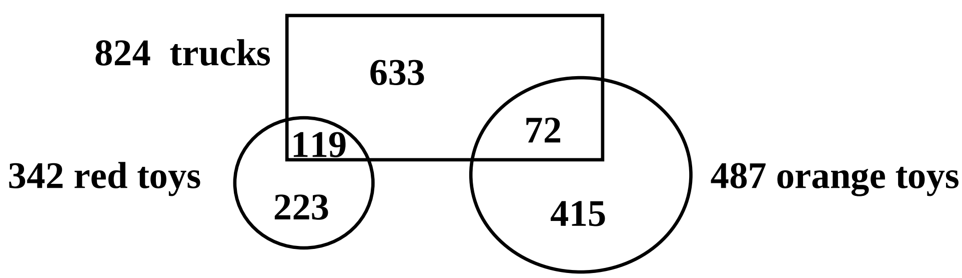
Answers
1) 432
2) 152
3) 803

1) The intersection of sets A and B consists of the LCM of the numbers in sets A and B
= { 12, 24, 36, 48, 60, 72, 84, 96 }
The sum = 12 + 24 + 36 + 48 + 60 + 72 + 84 + 96
= 432.

2) 517 - [(248 + 193) - 76]
= 517 - [441 - 76]
= 517 - 365
= 152.

3) The numbers in this diagram reflect how many toys have the assigned characteristics:

2265 = total number of toys



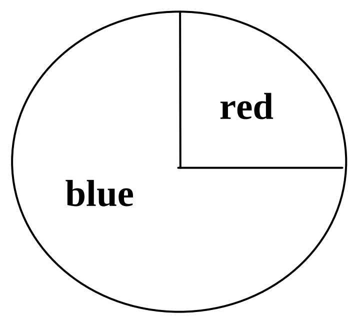
The number of toys that lie outside the roped area is
2265 - (633 + 72 + 119 + 415 + 223)
= 2265 - 1462
= 803

Category 4 *Calculator Meet*
Arithmetic
Meet #5 - April, 2021

1) A letter is chosen at random from the word ARITHMETIC. What is the probability that it is the letter T ? Once the answer expressed as a fraction in lowest terms, what is the sum of the numerator and denominator?

2) A bag of NECCO wafers contains 15 red, 28 brown, and 12 green wafers. Three wafers are taken from the bag, at random. What is the probability that all three are the same color? Express your answer as a decimal, rounded to the nearest hundredth.

3) A Bernoulli trial is a probability experiment that has only two possible outcomes. Consider a game where a player must spin a circular spinner that stops on either red or blue, as shown. The angle at the center of the spinner is a right angle.



Let n = the number of spins
x = the number of successes
p = the probability of success (desired outcome) for one trial
f = the probability of non-success (failure) for one trial.
The probability of achieving x successes for n spins is given by this formula:

$$({}_{n}C_{x})(P^{x})(f^{n-x})$$

If Liz spins the spinner 20 times, then what is the probability that it will stop on blue exactly 13 times? Express your answer as a percent, rounded to the nearest whole percent.

<u>footnote</u>: Jacob Bernoulli was a Swiss mathematician (1655 - 1705)

<u>ANSWERS</u>

1) ________

2) ________

3) ________

Solutions to Category 4
Arithmetic
Meet #5 - March, 2014

Answers
1) 6
2) 0.15
3) 11

1) The word ARITHMETIC has ten letters. The letter T occurs twice. The probability that T is chosen is 2/10 or 1/5. The sum of the numerator and denominator of 1/5 is 1 + 5, or 6.

2) This is a classic example of "probability without replacement. One a wafer is selected, there is one fewer wafer in the bag. The probability of choosing three of the same color , with R = red, B = brown, and G = green:

(R)(R)(R) + (B)(B)(B) + (G)(G)(G)
= (15/55)(14/54)(13/53) + (28/55)(27/54)(26/53) + (12/55)(11/54)(10/53)
= (2730/157,410) + (19,656/157,410) + (1320/157,410)
= 23,706/157,410
= 0.1506 . . . .
Rounded to the nearest hundredth yields 0.15.

3) $(20C13)(0.75^{13})(0.25^{7})$

$\approx$ (77,520)(0.023760)(0.000061035)

$\approx$ 0.1124

$\approx$ **11 %**

Category 5 *Calculator Meet*

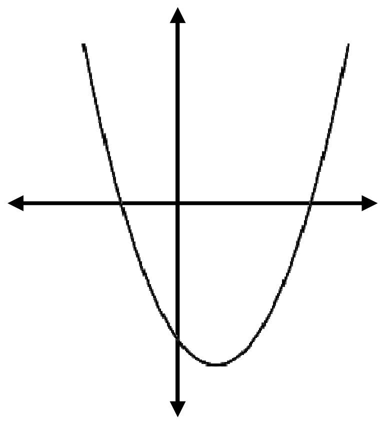
Algebra

Meet #5 - April, 2021

1) This quadratic equation, written in factored form, has two solutions. In other words, there are two distinct values of X that make the equation true. What is the sum of those two solutions?

$$(X + 5) (X - 8) = 0$$

2) The graph of a parabola intersects the X-axis at (- 3, 0) and (7, 0) and intersects the Y-axis at (0, - 84).
If the parabola represents the equation $y = ax^2 + bx + c$
then what is the value of a + b + c ?
(The diagram is not drawn to scale.)



3) Hoses are attached to two outdoor faucets so that a backyard swimming pool can be filled. If both faucets are used, it takes two hours to fill the pool. If either faucet were used alone, then one faucet would take three hours less than the other to fill the pool. What is the least amount of time required to fill the pool if only one faucet is used?

ANSWERS

1) ________

2) ________

3) ________

Solutions to Category 5
Algebra
Meet #5 - April, 2021

Answers	
1)	3
2)	- 96
3)	3

1) Using the theorem that if the product of two or more quantities is zero, then any of those quantities may be equal to zero:
X + 5 = 0, so X = - 5.
X - 8 = 0, so X = 8.
So, the sum of the two solutions is - 5 + 8, or 3.

2) The x-intercepts allow us to write this quadratic equation in factored form: y = a(x + 3)(x - 7). Using the y-intercept to substitute for x and y the values of 0 and - 84, respectively, yields - 84 = a(0 + 3)(0 - 7) or - 84 = a(3)(- 7). Therefore, - 84 = a(- 21) and a = 4. Now we have y = 4(x + 3)(x - 7). Multiply out to get y = 4(x^2 - 4x - 21) or y = 4x^2 - 16x - 84. Now we have the values of a, b, and c as 4, -16, and -84, respectively. So, a + b + c = 4 + (- 16) + (- 84) or - 96.

3) (Rate) (Time) = Work.
Let X = the number of hours it would take for Hose A to fill the pool, if working alone. Let X + 3 = the number of hours it would take for Hose B to fill the pool, if working alone. So, the rate for Hose A is $\left(\frac{1}{X}\right)$ of a job per hour and the rate for Hose B is $\left(\frac{1}{X+3}\right)$ of a job per hour. $\left(\frac{1}{X}\right)(2)$ represents the amount of work that Hose A does in 2 hours. $\left(\frac{1}{X+3}\right)(2)$ represents the amount of work that Hose B does in 2 hours.

The amount of work of Hose A plus the amount of work of Hose B is equal to one job:

NEXT PAGE FOR THE REMAINDER OF THE SOLUTION TO #3

$$\left(\frac{1}{X}\right)(2) + \left(\frac{1}{X+3}\right)(2) = 1$$

$$\left(\frac{2}{X}\right) + \left(\frac{2}{X+3}\right) = 1$$

$$2X + 6 + 2X = X^2 + 3X$$

$$2(X + 3) + 2X = (X)(X + 3)$$

So, $X = 3$ or $X = -2$

We will take the positive solution, 3, as it represents the number of hours it takes for Hose A to fill the pool while working alone. Since Hose B takes 3 hours longer, the answer to the question is 3 hours (the least amount of time it takes when only one faucet is used).

Category 6
Team Round
Meet #5 - April, 2021

Each of the following nine problems is worth four points.

1) Auntie Ida turned 97 years old on February 1, 2021. In what year was she born?

2) The following quadratic equation has two solutions. What is the sum of those two solutions? $x^2 - 40 = 3x$

3) A cube and a rectangular solid have the same volume. The surface area of the cube is 864 square inches. The base of the rectangular solid measures 24 inches by 18 inches. How many inches are in the height of the rectangular solid?

4) It takes 16 seconds to inflate a spherical balloon that is 10 inches in diameter. How many seconds should it take to inflate a spherical balloon whose radius is 20 inches if inflated at the same rate?

5) If $7N - 37 = 208$, then what is the value of $\frac{23}{6 - 5N}$? Express your answer as a decimal, rounded to the nearest hundredth.

ANSWERS

1) __________

2) __________

3) __________

4) __________

5) __________

6) __________

7) __________

8) __________

9) __________

6) A group of friends is planning a trip. Each friend plans to contribute the same amount of money. The total cost of the trip is $800. Two of the friends, though, are unable to attend. However, the remaining friends agree to share equally in the cost. Doing so has increased the cost per friend by $20. How many friends were in the original group?

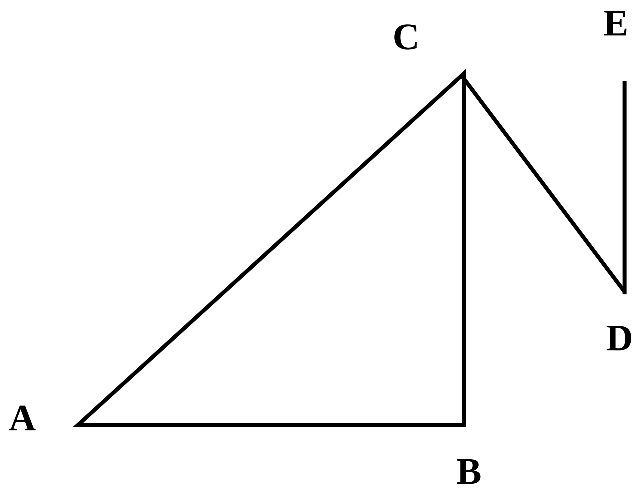
7) Between the years 1497 and 1500, Amerigo Vespucci made two voyages to the New World. One voyage lasted 43 days longer than the other. How many days did the longer voyage last if the two voyages lasted a total of 1003 days?

8) What is the value of the following expression? Note: the two exclamation points are factorial symbols: $1 - 2 + (3!)^4 + 5 + 6!$

PROBLEM #9 IS ON THE NEXT PAGE.

9) **How many degrees are in the measure of angle BAC if**

- **ED is parallel to CB,**
- **AB is perpendicular to CB,**
- **angle CDE measures 48 degrees, and**
- **the measure of angle DCB is twice the measure of angle ACB?**



Solutions to Category 6
Team Round
Meet #5 - April, 2021

ANSWERS

1)	1924
2)	3
3)	4
4)	1024
5)	- 0.14
6)	10
7)	523
8)	2020
9)	66

1) Subtracting 97 from 2021 yields 1924.

2) Transpose the equation, then factor and use the "zero product" theorem:
$x^2 - 40 = 3x$ then $x^2 - 3x - 40 = 0$ then $(x - 8)(x + 5) = 0$ so $x = 8$ or $x = -5$. The sum of the solutions is $8 + (-5)$, or 3.

3) Divide the surface area of the cube by 6 to find the area of one surface = 144. Next, square root 144 to get the length of the cube = 12. So, the volume of the cube is (12)(12)(12), or 1728. This is also the volume of the rectangular solid. To find the height of the rectangular solid, divide its volume by the area of its base: 1728 / [(24)(18)] = 1728 / 432 = 4. So, the height of the rectangular solid is 4 inches.

4) The volume of the larger balloon is equal to the cube of the ratio of two corresponding linear measures. The diameter of the smaller balloon is 10 inches while the radius of the larger balloon is 20 inches, making its diameter 40 inches. so the ratio of two corresponding linear measures is 40:10 or 4:1. The cube of that ratio is 64:1. So, it should take 64 times longer to inflate the larger balloon:
(16 seconds)(64) = 1024 seconds.

5) $7N - 37 = 208$ so $7N = 245$ and $N = 35$. Substitute 35 for N in $23 / (6 - 5N)$ to get $23 / (6 - 175)$ or $23 / (-169)$ or $-0.13609\ldots$ which, when rounded to the nearest hundredth, is -0.14.

6) Let X = the number of friends, and
M = the amount of dollars that each friend would pay, initially.
Then X - 2 = the number of friends who will attend the trip, and
M + 20 = the amount of dollars that each friend WILL pay.

THE SOLUTIONS TO PROBLEMS #6-9 ARE ON THE NEXT PAGE.

Since the total cost is fixed at $800, we have two equations"
Equation #1: MX = 800
Equation #2: (M + 20)(X - 2) = 800

Use Equation #2 and then, at an opportune time, substitute 800 / X for M:

$$(M + 20((X - 2) = 800$$
$$MX - 2M + 20X - 40 = 800$$

Substitute 800 for MX:

$$800 - 2M + 20X - 40 = 800$$
$$-2M + 20X = 40$$
$$-2 (M - 10X) = 40$$
$$M - 10X = -20$$

Substitute 800 / X for M:

$$800 / X - 10X = -20$$
$$800 - 10X\text{^}2 = -20X$$
$$80 - X\text{^}2 = -2X$$
$$0 = X\text{^}2 - 2X - 80$$
$$0 = (X - 10(X + 8)$$

So, X = 10 or X = -8. The number of friends must be a positive number, so we accept 10 as the number of friends in the original g roup while rejecting -8.

7) Let V = the number of days of one voyage, and
V + 43 = the number of days of the other voyage.

Then $V + (V + 43) = 1003$

$$2V + 43 = 1003$$
$$2V = 960$$
$$V = 480$$
$$V + 43 = 523$$

So, the longer voyage lasted 523 days.

8) $1 - 2 + (3!)\text{^}4 + 5 + 6!$

$= 1 - 2 + [(3)(2)(1)]\text{^}4 + 5 + (6)(5)(4)(3)(2)(1)$

$= 1 - 2 + 6\text{^}4 + 5 + 720$

$= 1 - 2 + 1296 + 5 + 720$

$= 2020$

9) Angle DCB = 48 degrees, as alternate interior angles are congruent.
Angle ACB is half that, or 24 degrees. Angle A = 180 - (24 + 90).
So, angle A measures 66 degrees.