IMLEM Meet #3
January, 2021

Intermediate Mathematics League of Eastern Massachusetts

Category 1
Mystery
Meet #3 - January, 2021

1) Connor started watching his favorite movie at 10:48 A.M. The movie ended at 1:23 P.M. How many minutes long was the movie?

2) Laverne, Shirley, and Squiggy all live on the same side of Main Street, with Squiggy's house somewhere between the two others. Shirley walks toward Squiggy's house at 3 miles per hour for 40 minutes at the same time that Laverne rides her bike toward Squiggy's house at 15 miles per hour for 28 minutes. At the conclusion of the girls' trips, they meet at the same time at Squiggy's house. How many miles apart are Laverne's and Shirley's houses?

3) Jacqueline has twice as many candy bars as Victoria. If Jacqueline gave 24 candy bars to Victoria, then Victoria would have eight more candy bars than Jacqueline. How many candy bars did Jacqueline have originally?

Answers

1) _____

2) _____

3) _____

Solutions to Category 1
Mystery
Meet #3 - January, 2021

1) From the start of the movie until 12:00 is 72 minutes. From 12:00 until the end of the movie is 83 minutes for a total of 72 + 83, or 155 minutes.

2) Rate x Time = Distance.
Shirley: 3 mph x 40/60 hr = 2 miles.
Laverne: 15 mph x 28/60 hr = 7 miles.
So, Laverne's and Shirley's houses are 2 + 7, or 9 miles apart.

3) Students should have adequate time for guessing and checking. Students with fundamental skills with algebraically identifying unknowns while writing and solving an equation may do so more expediently.

Let X = the number of Victoria's candy bars before the exchange
2X = the number of Jacqueline's candy bars before the exchange
2X - 24 = the number of Jacqueline's candy bars after the exchange
X + 24 = the number of Victoria's candy bars after the exchange.

Equation: $X + 24 = 2X - 24 + 8$
$X + 24 = 2X - 16$
$40 = X$
$80 = 2X$

Therefore, Jacqueline originally had 80 candy bars.

Answers

1) 155

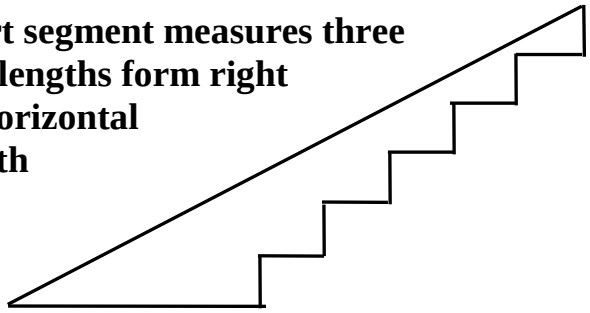
2) 9

3) 80

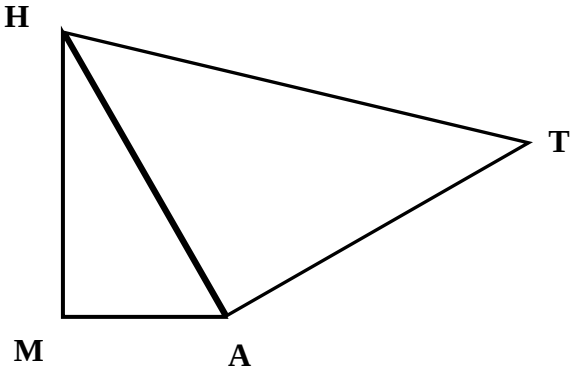
Category 2
Geometry
Meet #3 - January, 2021

1) The bottom of a 26-foot long ladder is placed on the ground, ten feet from the wall of a building. The top of the ladder is how many feet up the wall from the ground?

2) In the figure to the right, each short segment measures three inches. All vertical and horizontal lengths form right angles with adjacent vertical and horizontal lengths. The longer horizontal length measures nine inches. How many inches long is the diagonal length?



3) In the figure below and to the right, all measurements are in centimeters. Triangles AHM and AHT are both right triangles. HM = 12. AM = 9. HT = 39. How many square centimeters are in the area of quadrilateral MATH?



Answers

1) _____ feet

2) _____ inches

3) _____ sq cm

Solutions to Category 2
Geometry
Meet #3 - January, 2021

Answers	
1)	24
2)	30
3)	324

1) Using the Pythagorean Theorem, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

$$A^2 + B^2 = C^2$$
$$10^2 + B^2 = 26^2$$
$$100 + B^2 = 676$$
$$B^2 = 576$$
$$B = 24$$

So, the top of the ladder is 24 feet up the wall from the ground.

2) Invert the right angles to create a right triangle whose base is 9 + 5(3), or 24 inches long and whose height is 6(3), or 18 inches long. These two legs of 18 and 24 are in a scale of six times the legs 3 and 4 of a similar right triangle whose hypotenuse is 5. So, scaling the 5 by a factor of 6 yield a hypotenuse length of 30 inches for the given figure.

3) The Pythagorean Theorem, as applied to triangle AHM, results in AH = 15. The Pythagorean Theorem, as applied to triangle AHT, results in AT = 36. The area of quadrilateral MATH is the sum of the areas of right triangles AHM and AHT: (1/2)(base)(height) + (1/2)(base)(height)
= (1/2)(9)(12) + (1/2)(36)(15)
= 54 + 270
= 324 square centimeters.

Category 3
Number Theory
Meet #3 - January, 2021

1) Write the base 3 numeral 10100 as a base 10 numeral.

2) If $A = 7.382 \times 10^4$ **and** $B = 9.4613 \times 10^7$

then the product $AB = C \times D^N$ **where** $1 \le C < 10$ **and D = 10**

and N is an integer. What is the value of N?

3) The numerators and denominators of the fractions below are written in scientific notation. Compute. Express your answer as a standard numeral, or whole number.

$$\frac{4.2 \times 10^7}{8.1 \times 10^{-2}} \times \frac{10.8 \times 10^{-4}}{2.8 \times 10^3}$$

Answers

1) _____

2) _____

3) _____

Solutions to Category 3
Number Theory
Meet #3 - January, 2021

Answers	
1)	90
2)	12
3)	200

1) 10100 in base 3 converts to base 10 as follows: 1(81) + 0(27) + 1(9) + 0(3) + 0(1)
= 81 + 0 + 9 + 0 + 0
= 90.

2) At first glance, this problem appears to require a lot of long computing. However, since the question only asks for the value of N, then one need only approximate the value of C as somewhere between (7)(9) and (8)(10), or between 63 and 80. So, AB is some number between 6 and 8 times 10 then multiplied by 10 to the 11th power, or a number between 6 and 8 times 10 to the 12th power. Therefore, N = 12.

3) Computing the decimals yields a product of 2. Computing the powers of 10 yields a product of 10 squared. This outcome can be

Category 4
Arithmetic
Meet #3 - January, 2020

1) Evaluate: $6^0 + 5^1 + 4^2 + 3^3 + 2^4 + 1^5 + 0^6$

2) Evaluate: $2^{-1} + 2^{-2} + 2^{-3} + 2^{-4}$

Once your answer is expressed as a common fraction, reduced to lowest terms, what is the sum of the numerator and denominator?

3) Compute: $\sqrt[6]{(\sqrt[3]{8})^5 \cdot \sqrt[4]{16} \cdot \sqrt[5]{3125} \cdot (\sqrt{25})^5}$

ANSWERS

1) _____

2) _____

3) _____

Solutions to Category 4
Arithmetic
Meet #3 - January, 2021

Answers	
1)	66
2)	31
3)	10

1) $6^0 + 5^1 + 4^2 + 3^3 + 2^4 + 1^5 + 0^6$

= 1 + 5 + 16 + 27 + 16 + 1 + 0

= 66

2) $2^{-1} + 2^{-2} + 2^{-3} + 2^{-4}$

= 1/2 + 1/4 + 1/8 + 1/16

= 8/16 + 4/16 + 2/16 + 1/16

= 15/16

The sum of the numerator and denominator of the simplified fraction is 15 + 16, or 31.

3) $\sqrt[6]{(\sqrt[3]{8})^5 \cdot \sqrt[4]{16} \cdot \sqrt[5]{3125} \cdot (\sqrt{25})^5}$

= $\sqrt[6]{2^5 \cdot 2^1 \cdot 5^1 \cdot 5^5}$

= $\sqrt[6]{2^6 \cdot 5^6}$

= (2) (5)

= 10.

Category 5
Algebra
Meet #3 - January, 2021

1) Evaluate this absolute value expression: $|0| + |42| + |-28|$

2) If $|X + 5| < 12$ **then either** $X < 7$ **or** $X > M$ **where M is a constant.**

What is the value of M ?

3)

The graph above is the solution set to the absolute value inequality

$$|X + N| < A$$

where A and N are integer constants and X is a variable representing the set of all real number values that make the inequality true. What is the value of N - A ?

Answers

1) _____

2) _____

3) _____

Solutions to Category 5
Algebra
Meet #3 - January, 2021

1) $|0| + |42| + |-28| = 0 + 42 + 28 = 70.$

2) This inequality can be translated as, "The distance X and - 5 is less than 12 units." The points on a number line that are 12 units from - 5 are 7 and - 17. Therefore M = - 17.

3) The midpoint of the solution set is - 3. The distance between -3 and either endpoint is 5 units. Therefore, the inequality is $|X + 3| < 5$ where N = 3 and A = 5. The value of N - A = 3 - 5, or - 2.
Note: The expression X - (- 3) is equivalent to X + 3.

Answers

1) 70

2) - 17

3) - 2

Category 6
Team Round
Meet #3 - January, 2021

Each of the following six problems is worth <u>six</u> points.

1) Find the sum of all solutions to this inequality if the replacement set for X (the only allowable values) is the set of { negative integers }:

$$-7 - 3(2X - 5) < 38$$

2) Find the measure of one exterior angle of a regular pentadecagon (15-sided polygon).

3) A pair of positive two-digit numbers, 6A and A2, has a product of 5576. What is the value of the digit A ?

4) The digits 0, 1, 2, and 3 are arranged to form a 4-digit number that is divisible by all of the smallest four prime numbers. What is that 4-digit number?

5) The sum of four consecutive prime numbers is 168. What is the product of the smallest and largest of these four prime numbers?

6) Save-a-Dog in Sudbury, Massachusetts is showing two types of dogs on its web site that need a good home. Of the 27 dogs, there are 25% more terriers than pugs. How many pugs are there?

<u>ANSWERS</u>

1) _____

2) _____

3) _____

4) _____

5) _____

6) _____

Solutions to Category 6
Team Round
Meet #3 - January, 2021

ANSWERS	
1)	- 10
2)	24
3)	8
4)	2310
5)	1739
6)	12

1) $-7 - 3(2X - 5) < 38$

$-7 - 6X + 15 < 38$

$-6X < 30$

$X > -5$

The sum of the solutions is

$-4 + -3 + -2 + -1 = -10.$

2) One exterior angle = 360 / (the number of exterior angles) = 360 / 15 = 24 degrees.

3) For the product of A and 2 to result in a units digit of 6, A could equal 3 or 8. The two possible two-digit numbers are either 63 and 32 or 68 and 82. The latter product results in 5576. Therefore, A = 8.

4) The four smallest prime numbers are 2, 3, 5, and 7. If the four-digit number is to be divisible by both 2 and 5, then it is divisible by 10, thus making the units digit of the four-digit number 0. The sum of the digits is 6, so the four-digit number is divisible by 3. The only permutation of the four digits that makes the resulting number divisible by 7 is 2310.

5) The average of the four consecutive prime numbers is about 168 / 4, or about 42. The prime numbers on either side of 42 are 37, 41, 43, and 47, the sum of which is 168. The product of the smallest and largest of these is (37) (47), or 1739.

6) Let P = the number of pugs and let 1.25P = the number of terriers.

Then $P + 1.25P = 27$

$2.25P = 27$

$P = 27 / 2.25$

$P = 12$

So, there are 12 pugs.