

IMLEM Meet #2
December, 2020

Intermediate Mathematics League of Eastern Massachusetts



CLUSTER COORDINATORS - A reminder to all students of some of the rules and of appropriate behavior during this meet:

- **No calculators (or only scientific calculators allowed for meets #4, #5)**
- **Everyone take a moment to turn off any electronic devices that you want to have with you during the rounds. No electronic devices may be on during the rounds. Use of these devices during the rounds will result in a disqualification.**
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Category 1

Mystery

Meet #2 - December, 2020

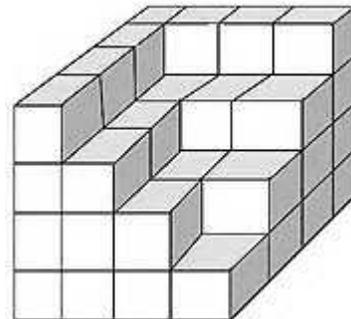


- 1) The letters in the word **ALGORITHM** are written repeatedly. What is the 200th letter?

A L G O R I T H M A L G O R I T

- 2) A machine can fill 120 boxes of candy in 8 minutes. Each box contains 100 pieces of candy. How many boxes can be filled in one hour?

- 3) How many of the smallest cubes compose this figure?



Answers

1) _____

2) _____

3) _____

**Solutions to Category 1
Mystery
Meet #2 - December, 2020**

Answers

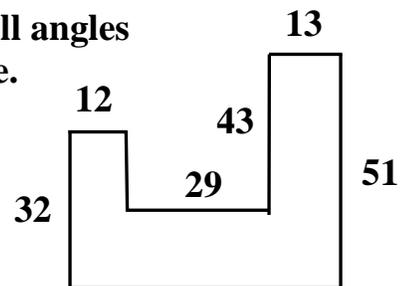
- 1) **The word ALGORITHM contains nine letters. Dividing 200 by 9 yields a quotient of 22 with a remainder of 2. So, there are 22 complete words with two remaining letters, namely A and L. The 200th letter is L.**
- 2) **The problem contains an irrelevant piece of information - that there are 100 pieces of candy in each box (critical reading). Divide 60 minutes by 8 to yield a quotient of 7.5. So, multiply 7.5 by 120 to get the number of boxes that can be filled in one hour, or 900 boxes.**
- 3) **Count the number of cubes in each tier and then find the sum:**
- | | |
|---------------------|--|
| bottom tier: | $4(4) = 16$ |
| tier 2: | $16 - 1 = 15$ |
| tier 3: | $16 - 4 = 12$ |
| top tier: | $16 - 9 = 7$ |
| total: | $7 + 12 + 15 + 16 = 50.$ |
- 1) **L**
- 2) **900**
- 3) **50**

Category 2
Geometry
Meet #2 - December, 2020

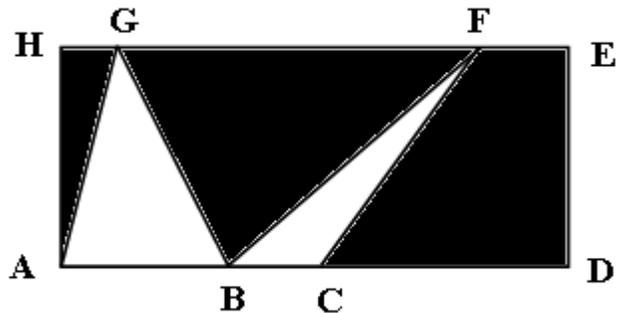


1) A rectangle has an area of 48 square feet and a perimeter of 32 feet. If its length is longer than its width, then how many feet are in its length?

2) How many centimeters (cm) are in the perimeter of this figure? All measurements are in centimeters. All angles are right angles. The figure is not drawn to scale.



3) Given rectangle ADEH. C is the midpoint of AD. AH = 14 inches and HE = 40 inches. Point B is somewhere between A and C. Point G is somewhere between H and F. Point F is somewhere between G and E. How many square inches are in the area of the shaded region?



<u>Answers</u>	
1)	_____ feet
2)	_____ cm
3)	_____ sq. in.

Solutions to Category 2

Geometry

Meet #2 - December, 2020

- 1) If the perimeter is 32 feet, then half the perimeter is 16 feet, which is the sum of the length and width.

If the area is 48 square feet, then we are seeking two numbers whose sum is 16 and whose product is 48. Simple guessing and checking gives us 4 and 12 as those two numbers. So, the length is 12 feet.

- 2) There are two missing lengths on the figure. The short vertical length can be found by subtracting 43 from 51 to get the width of the bottom "bar," or 8 cm.. Subtract 8 from the height of the left "tower" to get 24 cm. The length of the bottom of the figure is the sum of the horizontal pieces: $12 + 29 + 13 = 54$ cm. The perimeter is the sum of all eight lengths:
Perimeter = $54 + 32 + 12 + 24 + 29 + 43 + 13 + 51 = 258$ cm.

- 3) The area of rectangle ADEH = $(14)(40) = 560$ square inches. The area of the unshaded regions is the sum of the areas of the two unshaded triangles:
 $(1/2)(\text{base})(\text{height}) = (1/2)(AB + BC)(14) = (1/2)(20)(14) = 140$ square inches.

The shaded area is the difference between the area of the rectangle and the sum of the areas of the unshaded triangles = $560 - 140 = 420$ square inches.

Answers

1) 12

2) 258

3) 420

Solutions to Category 3
Number Theory
Meet #2 - December, 2020

<u>Answers</u>	
1)	12
2)	17
3)	2042

1) **Factors of 36: 1, 2, 3, 4, 6, 9, 12, 18, 36**
Factors of 48: 1, 2, 3, 4, 6, 8, 12, 16, 24, 48
The GCF is 12.

2) **Divide 471,240 by the product of its known factors $3 \times 2 \times 5 \times 2$ to get the product of the remaining factors $A \times B \times C \times D \times E$.**

$$\begin{aligned} & 471,240 / (3 \times 2 \times 5 \times 2) \\ = & 471,240 / 60 \\ = & 7,854. \end{aligned}$$

$$\begin{aligned} \text{Prime factor } 7,854 &= 2 \times 3927 \\ &= 2 \times 3 \times 1309 \\ &= 2 \times 3 \times 7 \times 187 \\ &= 2 \times 3 \times 7 \times 11 \times 17. \end{aligned}$$

The largest among these last five prime numbers is 17.

3) **To find the least common multiple, or LCM, of 24, 42, and 56:**

$$\begin{aligned} 24 &= 2 \times 2 \times 2 \times 3 \\ 42 &= 2 \times 3 \times 7 \\ 56 &= 2 \times 2 \times 2 \times 7 \\ \text{LCM} &= (2 \times 2 \times 2) \times 3 \times 7 \\ &= 8 \times 21 \\ &= 168. \end{aligned}$$

Adding 168 days to October 31:

$$\begin{aligned} & \text{November} + \text{December} + \text{January} + \text{February} + \text{March} + \text{a few more days} \\ = & 30 + 31 + 31 + 28 + 31 + \text{a few more days} \\ = & 151 + \text{a few more days.} \end{aligned}$$

151 + 17 = 168, so the "few extra days" is 17. The date that the four people get together next is on April 17, 2021.

$$4 + 17 + 2021 = 2042.$$

Category 4
Arithmetic
Meet #2 - December, 2020



- 1) The fraction $\frac{24}{42}$ is simplified, or reduced to lowest terms. How much larger is the denominator than the numerator in the simplified fraction?

- 2) The repeating decimal $0.2444444 \dots$ in which the four repeats continuously, is converted to its fraction form and then reduced to lowest terms. How much less is the numerator than the denominator of that fraction?

- 3) When 30 % of 260 is added to 68 % of 75, the result is N % of 645. What is the value of N ?

Answers

1) _____

2) _____

3) _____

Solutions to Category 4
Arithmetic
Meet #2 - December, 2020

- 1) The fraction $24 / 42$ reduces to $4 / 7$ so that the denominator is 3 larger than the numerator.
- 2) Let $X = 0.244444 \dots$ and let $10X = 2.44444 \dots$
Then $10X - X = 2.44444 \dots - 0.244444 \dots$
So, $9X = 2.2$ and $X = 2.2 / 9$ or $22 / 90$.
Reducing to lowest terms yields $11 / 45$.
The numerator is 34 less than the denominator.
- 3) Shortcuts may help. For example, 30 % of 260 is $(3)(10 \%$ of 260), or $(3)(26) = 78$.
Also, 68 % of 75 = 75 % of 68, or $3 / 4$ of 68 = 51.
Their sum, or $78 + 51 = 129$, is N % of 645.
 $129 = N \%$ of 645
 $129 = (N / 100)(645)$
 $N = (129)(100) / 645$
 $N = 20$.

Answers

- 1) 3
2) 34
3) 20

Category 5

Algebra

Meet #2 - December, 2020



- 1) A rectangle and an equilateral triangle have the same perimeter. One side of the triangle measures 24 inches. The length of the rectangle is three times as long as its width. How many inches long is the rectangle?

- 2) If $A + 2B = 17$ and $C + 3D = 18$, then what is the value of $4A + 8B - (3C + 9D) - 12 + 8$?

- 3) Mathematics and science are loaded with fun and useful formulas to help discern information from given data. The formula that relates a temperature in degrees Fahrenheit, F , to degrees Celsius, C , is

$$F = \frac{9}{5}C + 32$$

The record high temperature for Boston in the month of December occurred on December 7, 1998 and was 76 degrees Fahrenheit. The record low occurred on December 29, 1933 and was 17 degrees below zero, Fahrenheit. What is the positive difference, in degrees Celsius, between the record high and low temperatures for December? Round your answer to the nearest whole number.

Answers

1) _____

2) _____

3) _____

Solutions to Category 5

Algebra

Meet #2 - December, 2020

1) The perimeter of the equilateral triangle is $(3)(24)$, or 72 inches. If the perimeter of the rectangle is the same as that of the triangle, then half its perimeter is 36 inches, or the sum of its length and width. If the length is three times the width, then $W + 3W = 36$, $4W = 36$, and $W = 9$ and the length is 27 inches.

2) If $A + 2B = 17$, then multiplying both sides by 4 yields $4A + 8B = 68$.

If $C + 3D = 18$, then multiplying both sides by 3 yields $3C + 9D = 54$.

$$\begin{aligned}\text{Then } & 4A + 8B - (3C + 9D) - 12 + 8 \\ & = 68 - (54) - 12 + 8 \\ & = 14 - 12 + 8 \\ & = 2 + 8 \\ & = 10.\end{aligned}$$

3) The straightforward method is to convert the two Fahrenheit temperatures to Celsius and then subtract and round:

$$\text{record high: } F = 9/5 C + 32, \quad 76 = 9/5 C + 32, \quad 44 = 9/5 C, \quad C = 220 / 9.$$

$$\text{record low: } F = 9/5 C + 32, \quad -17 = 9/5 C + 32, \quad -49 = 9/5 C, \quad C = -245 / 9.$$

The difference is $220 / 9 - (-245 / 9) = 465 / 9 = 51.666\dots$ which, rounded to the nearest whole number, is 52 degrees Celcius.

Answers

1) 27

2) 10

3) 52

Category 6
Team Round
Meet #2 - December, 2020

Each of the following six problems is worth six points.

- 1) What is the smallest prime number that is greater than 140 ?
- 2) On a standard 12-hour clock, how many degrees are in the acute angle formed by the hour hand and minute hand at the time 2:16 ?
- 3) One-third of the flowers in Alice's garden are yellow, 40% are orange, and the rest are red in a garden that has 180 flowers. How many flowers are red?
- 4) What is the sum of two consecutive even integers whose squares differ by 148 ?
- 5) A Pythagorean Triple is an ordered triple of positive integers, (X, Y, Z) , such that $X^2 + Y^2 = Z^2$.

If $(X, 16, 20)$ and $(10, Y, 26)$ and $(16, 30, Z)$ are Pythagorean Triples, then what is the greatest prime factor of $X + Y + Z$?

- 6) Riley draws a rectangle whose length is 160 inches and whose width is 70 inches. Mason draws a rectangle whose length is 50 inches longer than Riley's rectangle and whose area is 50% larger than Riley's. What is the width of Mason's rectangle?

ANSWERS

1) _____

2) _____

3) _____

4) _____

5) _____

6) _____

Solutions to Category 6

Team Round

Meet #2 - December, 2020

- 1) For the odd numbers (potential prime numbers) greater than 140, 141 is divisible by 3, 143 is divisible by 11, 145 is divisible by 5, and 147 is divisible by 3. The next odd number, 149, is prime.
- 2) At 2:10, the minute hand is on the 16 and makes an angle with a hand pointing to 12 of $\frac{16}{60}$ of 360 degrees, or 96 degrees. The hour hand has rotated $\frac{16}{60}$ of 30 degrees, or 8 degrees from a hand pointing to 2, or $30 + 30 + 8$ degrees (68 degrees) from a hand pointing to 12. So, at 2:16, the hands make an angle that is the difference of the first two, or $96 - 68$, or 28 degrees.
- 3) Yellow = $\frac{1}{3}$ of 180 = 60 flowers. Orange = 40% of 180 = 72 flowers. Red = $180 - (60 + 72) = 180 - 132 = 48$ flowers.
- 4) Let X and $X + 2$ be two consecutive even integers. Then $(X+2)^2 - X^2 = 148$. Then $X^2 + 4X + 4 - X^2 = 148$ and $4X + 4 = 148$. So, $4X = 144$ and $X = 36$ and $X + 2 = 38$. The sum of the two consecutive even integers is $36 + 38$, or 74.
- 5) Students can grind out the calculations via the Pythagorean Theorem or recognize these triples as multiples of the familiar triples (3, 4, 5), (7, 12, 13), and (8, 15, 17), thus making $X = 12$, $Y = 24$, and $Z = 34$.
The sum $X + Y + Z = 70$. The greatest prime factor of 70 is 7.

ANSWERS

- | | |
|----|-----|
| 1) | 149 |
| 2) | 28 |
| 3) | 48 |
| 4) | 74 |
| 5) | 7 |
| 6) | 80 |

- 6) Riley's rectangle measures 160 by 70 inches and has an area $(160)(70)$, or 11,200 square inches. The length of Mason's rectangle is $160 + 50$, or 210 inches. The area of Mason's rectangle is $(1.5)(11,200)$, or 16,800 square inches. The width is the area divided by the length, or $16,800 / 210 = 80$ inches.