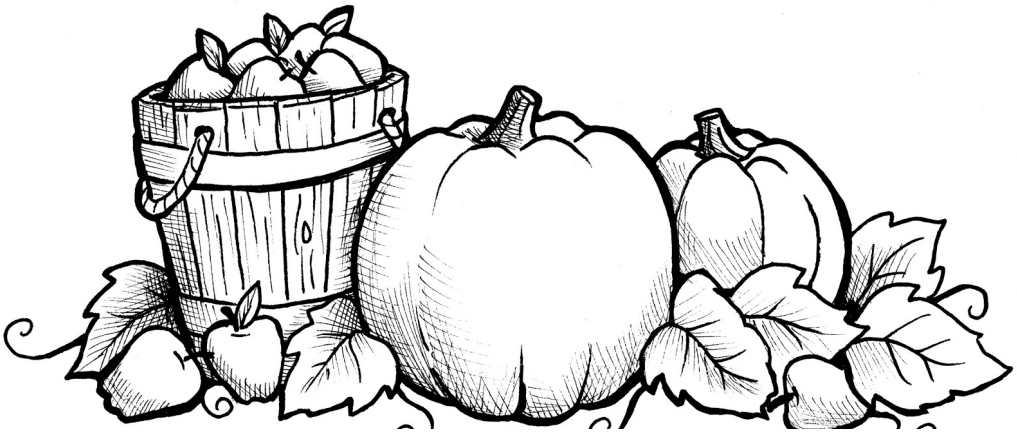
IMLEM Meet #1
October, 2019

Intermediate Mathematics League of Eastern Massachusetts



CLUSTER COORDINATORS - A reminder to all students of some of the rules and of appropriate behavior during this meet:

- **Many of you are guests in someone else's school – please be respectful of the classrooms and spaces you are using. Any "out of control" behavior in the halls or during a round is not acceptable. If an adult deems your behavior disrespectful or inappropriate, your score may not be counted.**
- **No calculators (or only scientific calculators allowed for meets #4, #5)**
- **Everyone take a moment to turn off any electronic devices that you want to have with you during the rounds. No electronic devices may be on during the rounds. Use of these devices during the rounds will result in a disqualification.**

Category 1
Mystery
Meet #1 - October, 2019

1) This pattern repeats continuously:

O C T O B E R O C T O B E R O C . . .

What is the 75th letter in the pattern?

2) In this addition, each letter represents a different digit somewhere from 1 through 9, inclusive. If E = 6 and T = 5, then what is the value of N ?

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  O N E
+ O N E
-------
  T W O
```

3) What is the greatest number of consecutive positive whole numbers whose sum is 85 ?

ANSWERS

1) _____

2) N = ____

3) _____

Solutions to Category 1
Mystery
Meet #1 - October, 2019

Answers	
1)	B
2)	8
3)	10

1) There are 7 letters in the word OCTOBER. To identify the 75th letter, divide 75 by 7 = 10 with a remainder of 5. There are 10 complete words spelled OCTOBER. The remainder of 5 has us counting the next five letters as O C T O B. The 75th letter is B.

2) In the units column, where E = 6, 6 + 6 = 12, so the letter O has a value of 2. Carry the 1 into the tens column. Since T = 5, the tens digit must be 5, 6, 7, 8, or 9 to produce the carry of 1 in the hundreds column. We can eliminate 5 because the value of T is 5 (all digits must be different). We can eliminate 6, as O is 6. We can eliminate 7, as 7 + 7 + 1 = 15, so W cannot equal 5. We can eliminate 9, as 9 + 9 + 1 = 19, making N = W. Therefore, the final choice of 8 for N makes W = 7. The final addition, with each letter a different value, looks like this:

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  286
+ 286
  572
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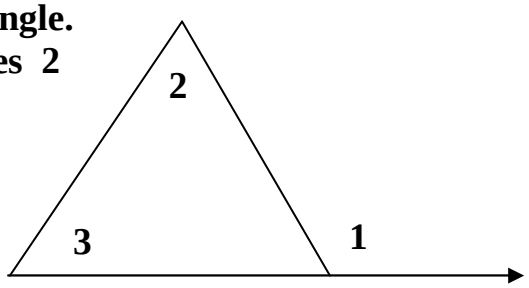
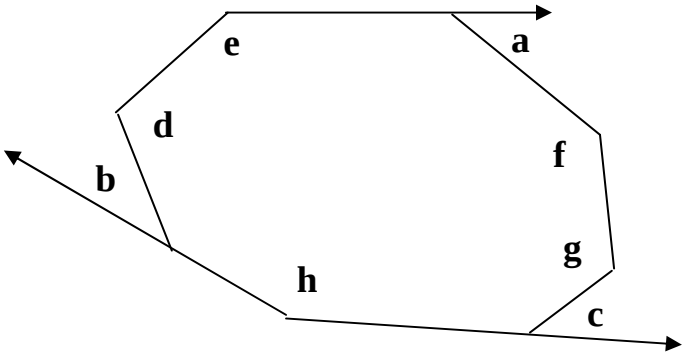
3) The sum 1 + 2 + 3 + . . . 10 + 11 + 12 = 78.
The sum 1 + 2 + 3 + . . . 10 + 11 + 12 + 13 = 91.
Experimenting with numbers in this range leads to this successful sum:
4 + 5 + 6 + . . . + 11 + 12 + 13 = 85. That is TEN consecutive numbers.

Category 2
Geometry
Meet #1 - October, 2019

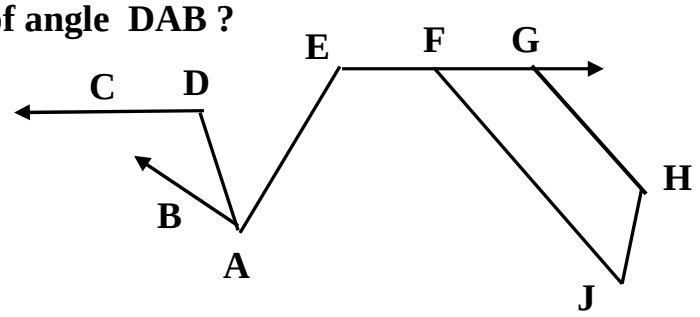
1) Angle 1 is an exterior angle of the triangle. Angle 1 measures 136 degrees. Angles 2 and 3 have the same measure. How many degrees are in angle 2 ?



2) Exterior angle a = exterior angle b = exterior angle c = 35 degrees. Interior angles d, e, f, g, and h are congruent and have the same measure. How many degrees are in the supplement of angle f ?



3) Ray DC is parallel to ray EG. In trapezoid FGHJ, angle H measures 138 degrees. Ray AB is perpendicular to segment AE. Angle CDA measures 117 degrees. Angles AEF and HJF are supplementary. How many degrees are in the measure of angle DAB ?



Answers

1) _____

2) _____

3) _____

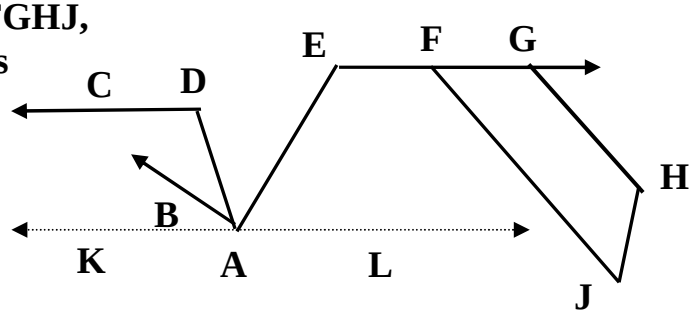
Solutions to Category 2
Geometry
Meet #1 - October, 2019

Answers	
1)	68
2)	51
3)	15

1) The interior angle supplementary to angle 1 is 180 - 136, or 44 degrees. The combined sum of angles 2 and 3 is 180 - 44, or 136 degrees. Students may know that an exterior angle of a triangle is equal to the sum of the two remote interior angles. Since angles 2 and 3 have the same measure, each one measures 136 / 2, or 68 degrees. Therefore, angle 2 measures 68 degrees.

2) Each of the three exterior angles of 35 degrees has a supplementary interior angle of 180 - 35, or 145 degrees. The total number of interior degrees of the octagon is (8 - 2) (180) or 1080 degrees. Subtract the sum of the three 145-degree angles from 1080 to get 645 degrees. Divide by 5 to get the number of degrees in each of the lettered interior angles: 645 / 5 = 129. So, angle f measures 129 degrees . . . but the question asks for its supplement: 180 - 129 = 51 degrees.

3) Drawing an auxiliary line through point A that is parallel to ray DC. In trapezoid FGHJ, since angle H measures 138 degrees, then angle HJF is its supplement and measures 42 degrees. Angles HJF and AEF are supplementary, so angle AEF measures 138 degrees. Due to parallel lines, angle EAL is the supplement of angle AEF and measures 42 degrees. Similarly, if angle CDA measures 117 degrees, then its supplement is angle DAK that measures 63 degrees. At the bottom of the picture, angle DAK + angle DAE + angle EAL = 180. So, 63 + DAE + 42 = 180 . . . . . 105 + DAE = 180 . . . . . and DAE = 75 degrees. Subtract 75 from 90 to get DAB = 15.

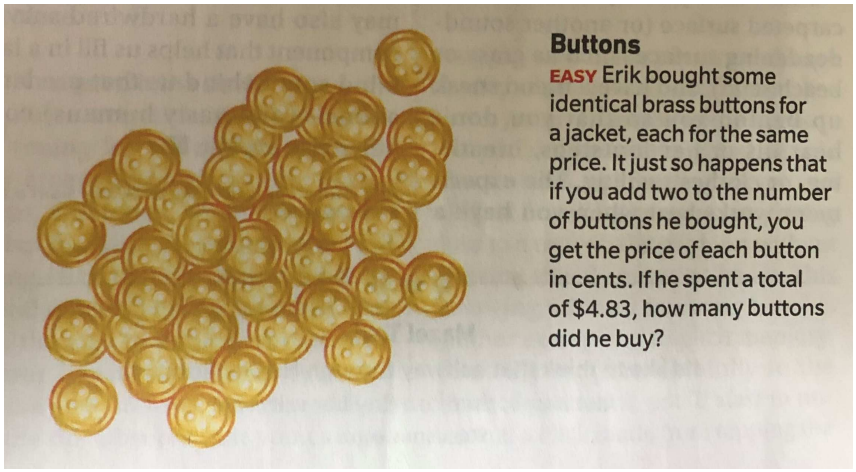


Category 3
Number Theory
Meet #1 - October, 2019

1) The factors of 6 are 1, 2, 3, and 6, as 6 is divisible by all four numbers. Find the sum of all factors of 12.

2) A = the sum of all prime numbers between 10 and 20
B = the sum of all composite numbers between 19 and 31
C = the sum of all numbers between 20 and 50 that have an odd number of factors
What is the value of A + B - C ?

3) The following problem appeared in a recent edition of Reader's Digest. Read the problem and then answer the question: How many buttons did Erik buy?



Buttons
EASY Erik bought some identical brass buttons for a jacket, each for the same price. It just so happens that if you add two to the number of buttons he bought, you get the price of each button in cents. If he spent a total of $4.83, how many buttons did he buy?

Answers

1) _____

2) _____

3) _____

Solutions to Category 3
Number Theory
Meet #1 - October, 2019

1) The factors of 12 are: 1, 2, 3, 4, 6, and 12. Their sum is 1 + 2 + 3 + 4 + 6 + 12 = 28.

2) A = 11 + 13 + 17 + 19 = 60
B = 20 + 21 + 22 + 24 + 25 + 26 + 27 + 28 + 30 = 223
C = 25 + 36 + 49 (all square numbers) = 110
A + B - C = 60 + 223 - 110 = 173.

3) The total cost of 483 cents is the product of the number of buttons multiplied by the cost, in cents, of each button. 483 = (3)(7)(23) = (21)(23). If you add 2 to 21, you get 23. So, the number of buttons is 21 and the price per button is 23 cents.

Answers

1) 28

2) 173

3) 21

Category 4
Arithmetic
Meet #1 - October, 2019

1) What is the value of $7(32 - 17) - (2)(3)(5) + (4)(6)(7)$?

2) Jen's five test scores were 81, 96, N, 83, and P. N = P. If her average of the five test scores was 86, then what is the value of N ?

3) If W is the median of the list of numbers below, all different, then what is the greatest possible difference between the highest and lowest possible values of W if W is a whole number?

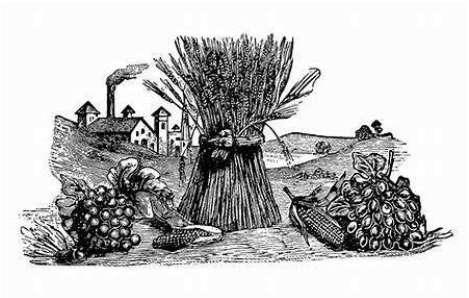
10 18 4 17 3 W 21

Answers

1) _____

2) _____

3) _____



Solutions to Category 4
Arithmetic
Meet #1 - October, 2019

1) $7(32 - 17) - (2)(3)(5) + (4)(6)(7)$
$= 7(15) - 30 + 168$
$= 105 - 30 + 168$
$= 243$

2) Jen's average score of 86 is computed as the sum of her five scores divided by 5,
$(81 + 96 + N + 83 + P) / 5 = 86$
Since $N = P$, substitute one for the other:
$(260 + 2N) / 5 = 86$
$260 + 2N = 430$
$2N = 170$
$N = 85$

3) Arrange the seven numbers in order: 3 4 10 W 17 18 21
For W to be the median and for all numbers to have different values, W can be any whole number from 11 through 16. The difference between the highest and lowest possible values of W is 16 - 11, or 5.

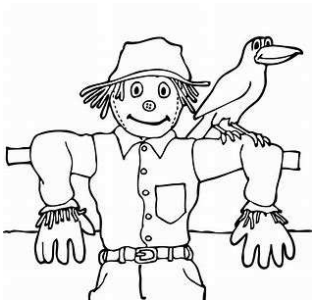
Answers

1) 243

2) 85

3) 5

Category 5
Algebra
Meet #1 - October, 2019



1) Simplify: 3C + 8E + 6C - 2E - 5C + E

2) If 3 + 2 = 44 and 5 (2 - 3) = 7 + 12, then what is the value of 100 - 5 + 8 ?

3) The 26 letters of the alphabet, { A, B, C, . . . , Z } correspond to the consecutive integers { - 11, - 10, - 9, . . . , 13, 14 }. For example, A = - 11, E = - 7, R = 6, and so on. The number zero is included among these integers. Find the sum of the numbers corresponding to the letters in the following word: AUTUMN

Answers

1) _____

2) _____

3) _____

Solutions to Category 5
Algebra
Meet #1 - October, 2019

1) 3C + 8E + 6C - 2E - 5C + E = 4C + 7E.

2)  **+ 2 = 44**
3  **= 42**
 **= 14**

5 (2  **- 3) = 7**  **+ 12**
10  **- 15 = 7**  **+ 12**
3  **= 27**
 **= 9**

So, 100 - 5  **+ 8**  **= 100 - (5)(14) + (8)(9)**
= 100 - 70 + 72
 **= 102**

3)

A	B	C	D	E	F	G	H	I	J	K	L	M
-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1

N	O	P	Q	R	S	T	U	V	W	X	Y	Z
2	3	4	5	6	7	8	9	10	11	12	13	14

A	U	T	U	M	N
-11	9	8	9	1	2

So, sum (AUTUMN) = - 11 + 9 + 8 + 9 + 1 + 2 = 18.

Answers

1) 4C + 7E
or
7E + 4C

2) 102

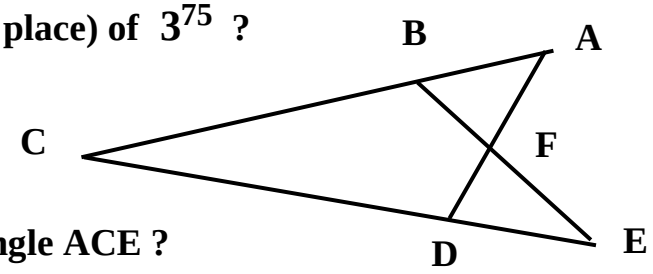
3) 18

Category 6
Team Round
Meet #2 - October, 2019

Each of the following nine problems is worth <u>four</u> points.

1) What is the units digit (ones place) of 3^{75} ?

2) Angle CAD = 38 degrees.
Angle CEB = 47 degrees.
Angle BFD = 124 degrees.
How many degrees are in angle ACE ?



3) If W is a whole number and W^3 (the cube of W) is between 9000 and 10,000, then what is the value of W ?

4) Travis entered the highway at mile marker 176 and exited at mile marker 512, travelling at a constant speed. At which numbered mile marker had he completed 75% of his trip on this highway?

5) Each of the first 1000 positive integers is raised to the fourth power. How many of their answers has either a 1 or a 6 as its units (ones) digit?

6) Paul and John were born on the same date but in different years, T years apart. Last year, Paul was 5 times as old as John. This year, Paul's age is the square of John's age. What is the value of T ?

7) There are three positive integers, F, G, and H, such that their average is 20 and $F \le G \le H$. If the median is F + 11, then what is the smallest possible value of H?

8) How many odd 3-digit integers greater than 400 are composed of three different non-zero digits?

9) Using the answers to problems #1-8 and substituting them for the capital letters as indicated in the answer box to the left, what is the value of the following expression?

4(3C - F) + 7DG - 5(2A + 7H) + 9BE

<u>ANSWERS</u>

1) _____ = A

2) _____ = B

3) _____ = C

4) _____ = D

5) _____ = E

6) _____ = F

7) _____ = G

8) _____ = H

9) _____

Solutions to Category 6
Team Round
Meet #1 - October, 2019

ANSWERS	
1)	7
2)	39
3)	21
4)	428
5)	800
6)	12
7)	25
8)	189
9)	349,219

1) The first bunch of powers of 3 yield the following answers: 3, 9, 27, 81, 243, 729, 2187, 6561, 19,683. The pattern of the units' digits - 3, 9, 7, 1, 3, 9, 7, 1, 3 . . . repeats in the block of four digits 3, 9, 7, 1. Divide the exponent 75 by 4 to get 18 with remainder 3. There are 18 complete blocks of the four digits, followed by 3, 9, and then 7. The 75th digit is 7.

2) The 124-degree angle yields two supplements, namely angles AFB and DFE, each with a value of 56 degrees. Angle ABF is 180 - (38 + 56), or 86 degrees. Its supplement, angle EBC, is 94 degrees. Similarly, angle ADE is 180 - (56 + 47), or 77 degrees and its supplement, angle ADC, is 103 degrees. Quadrilateral CBFD now has three measured angles, so that angle ACE is 360 - (94 + 124 + 103), or 39 degrees.

3) Since the cube of 20 is 8000, then W is not much larger. In fact, the cube of 21 is 9261, the only cube between 9000 and 10,000.

4) Travis travelled 512 - 176, or 336 miles. 75% of 336 is 252. Adding 252 to 176 yields 428. So, Travis completed 75% of his trip at mile marker 428.

5) Considering only the units digit of fourth powers of the first ten positive integers, they are: 1, 6, 1, 6, 5, 6, 1, 6, 1, and 0. This is true for every "pack" of ten consecutive positive integers thereafter. How many such packs are there? 1000 / 10, or 100. So, (100)(8) = 800.

SEE NEXT PAGE FOR SOLUTIONS TO #6-9.

6) The first three sentences yield the following equations:

P - J = T P - 1 = 5 (J - 1) P = (J)(J)

Solving the second equation for P in terms of J: P = 5J - 4

Substitute 5J - 4 for P in the first and third equations:

5J - 4 - J = T 5J - 4 = (J)(J)

Solve the final equation for J: (J)(J) - 5J + 4 = 0

(J - 4)(J - 1) = 0

J = 4 or J = 1.

The solution J = 1 fails to meet all three conditions. The solution J = 4 yields a value of P = 16 in the second equation. Substituting J = 4 and P = 16 into the first equation makes T = 12. Now all three conditions are satisfied.

7) To make H as small as possible, it is necessary to make F as large as possible. Since the average is 20, then F + G + H = 60. Fiddling with the numbers yields the following: F G H = 12 23 25

Therefore, H = 25.

8) Listing all possibilities would be time-consuming but is an option. But consider a quick analysis of the 3-digit numbers from 401 - 500. Each must have a hundreds digit of 4. Now consider the important word ODD - that we only consider the ODD numbers from 401 through 999. Within the 401-499 range, we have 5 options for the units digit. Then, the tens digit can't be 4, 0, or the units digit, which leaves us 7 options. So there are (5)(7) numbers in this range. The same is true for the ranges 601-699 and 801-899. For the range 501-599, the hundreds digit is already odd. So, the units digit has 4 options while the tens digit can't be 5, 0, or the units digit, which leaves us 7 options. So, there are (4)(7) numbers in this range. The same is true for the ranges 701-799 and 901-999. In all, the total number of numbers is 35 + 35 + 35 + 28 + 28 + 28, or 189.

9) 4(3C - F) + 7DG - 5(2A + 7H) + 9BE

= 4((3)(21) - 12) + (7)(428)(25) - (5)((2)(7) + (7)(189)) + (9)(39)(800)

= 4(63 - 12) + 74,900 - (5)(14 + 1323) + 280,800

= 4(51) + 74,900 - (5)(1337) + 280,800

= 204 + 74,900 - 6685 + 280,800

= 349,219