IMLEM Meet #2
December, 2018

Intermediate Mathematics League of Eastern Massachusetts

CLUSTER COORDINATORS - A reminder to all students of some of the rules and of appropriate behavior during this meet:

- **Many of you are guests in someone else's school – please be respectful of the classrooms and spaces you are using. Any "out of control" behavior in the halls or during a round is not acceptable. If an adult deems your behavior disrespectful or inappropriate, your score may not be counted.**
- **No calculators (or only scientific calculators allowed for meets #4, #5)**
- **Everyone take a moment to turn off any electronic devices that you want to have with you during the rounds. No electronic devices may be on during the rounds. Use of these devices during the rounds will result in a disqualification.**

Category 1
Mystery
Meet #2 - December, 2018

1) There was a flood at the school. Some of the tiles on the floor were damaged and need to be replaced, as shown in this picture. How many tiles need to be replaced if they are the same size as the undamaged tiles surrounding the empty space?

2) Three consecutive whole numbers have a sum of 141. If five times the largest of the three numbers is added to half of the smallest and twice the middle number, then what is that total?

3) Larry and his daughter went for a walk. She took three steps for every five steps that he took. In all, they took a total of 1296 steps. Larry took how many more steps than his daughter?

Answers

1) _____

2) _____

3) _____

Solutions to Category 1
Mystery
Meet #2 - December, 2018

Answers
1) 12
2) 357
3) 324

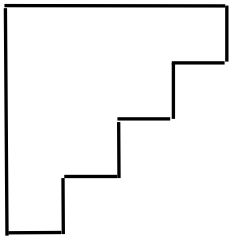
1) Counting the missing tiles yields 12 missing tiles.

2) One non-algebra approach is to guess and check. Another is to realize that the middle of the three consecutive whole numbers is one-third of the sum. Dividing 141 by 3 yields the value of that middle number, or 47. So, the three consecutive numbers are 46, 47, and 48. Five times the largest = (5)(48), or 240. Half of the smallest is (1/2)(46), or 23. Twice the middle number is (2)(47), or 94. The total sum is 240 + 23 + 94, or 357.

3) Together, Larry and his daughter took 8 steps combined for every three she took and for every five that he took. Dividing 1296 by 8 yields the number of 8-step sets they walked together in all, or 162 sets. Larry took (5)(162), or 810 steps while his daughter took (3)(162), or 486 steps. To find how many more steps Larry took than his daughter, subtract 486 from 810. That is 810 - 486, or 324 steps.

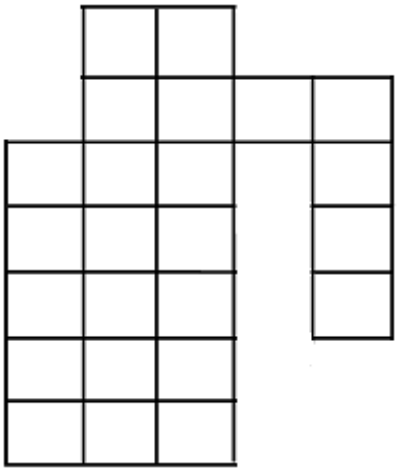
Category 2
Geometry
Meet #2 - December, 2018

1) In this polygon, all angles are right angles. Each of the short segments is three inches long. How many inches are in the perimeter of the polygon?



2) The rectangle to the right is 24 centimeters (cm) long and 16 centimeters wide. How many square centimeters are in the total area of the black triangles?

3) The diagram below consists of identical squares "glued" together at the edges. The total area of the resulting polygon is 216 square feet. How many feet are in its perimeter?



Answers

1) _____ inches

2) _____ sq cm

3) _____ feet

Solutions to Category 2
Geometry
Meet #2 - December, 2018

Answers
1) 48
2) 192
3) 90

1) The sum of the 3" vertical lengths is (4)(3), or 12". The vertical side opposite those 3" lengths is also 12" as is the total lengths of the four horizontal 3" lengths and the long horizontal side opposite those 3" lengths. So, the perimeter is (4)(12), or 48".

2) The bases of the black triangles have a total base length that is equal to the base of the rectangle, or 24 cm. Their combined areas are the same as that of one triangle with a base of 24 cm and an altitude equal to the width of the rectangle, or 16 cm. Therefore, the total area of the black triangles is (1/2)(base)(altitude) = (1/2)(24)(16) = 192 square cm.

3) There are 24 of the small squares. The area of one such square is equal to the total area of the squares divided by the number of squares, or 216 / 24, or 9 square feet. Each side of one square is the square root of 9, or 3 feet. The perimeter of the polygon consists of 30 of these 3' segments and is equal to (30)(3), or 90 feet.

Category 3
Number Theory
Meet #2 - December, 2018

1) **The GCF (greatest common factor) of A and B is the largest whole number that is a factor of both A and B.**
   X = the GCF of 12 and 20.
   Y = the GCF of 15 and 35.
   Z = the GCF of 8 and 21.
   What is the sum of X + Y + Z ?

2) **The prime factorization of 11,220 is 5 x A x B x C x 3 x 11. What is the sum of A + B + C ?**

3) **The GCF of F and G is 6. The LCM (lowest common multiple) of F and G is 72. F = 18. What is the value of G ?**

Answers

1) _____

2) _____

3) _____

Dec. 8 is "Pretend to be a Time Traveler" Day.

Solutions to Category 3
Number Theory
Meet #2 - December, 2018

Answers	
1)	10
2)	21
3)	24

1) $X = GCF(12, 20) = 4.$
$Y = GCF(15, 35) = 5.$
$Z = GCF(8, 21) = 1.$
$X + Y + Z = 4 + 5 + 1 = 10.$

2) The prime factorization can be accomplished by dividing 11,220 by increasing values of its prime factors.

$11{,}220 = 2 \times 5{,}610$
$= 2 \times 2 \times 2{,}805$
$= 2 \times 2 \times 3 \times 935$
$= 2 \times 2 \times 3 \times 5 \times 187$
$= 2 \times 2 \times 3 \times 5 \times 11 \times 17.$

Although many students will opt for what is familiar, as above, the problem gives several of the factors. Dividing 11,220 by $(5 \times 3 \times 11)$ gives 68. Prime factoring 68 will give the remaining prime factors: $68 = 2 \times 2 \times 17$. Therefore, $A + B + C = 2 + 2 + 17 = 21$.

3) Students can guess and check their way to the answer. However, some may know this relationship: GCF (A, B) x LCM (A, B) = A x B.
Substituting the given values gives $(6)(72) = (18)(G)$

$$432 = 18G$$
$$432 / 18 = G$$
$$24 = G$$

Category 4
Arithmetic
Meet #2 - December, 2018

1) When 48% is written as a fraction and then reduced to lowest terms, what is the value of the new denominator?

2) For how many whole number values of J is $\frac{7}{J}$ greater than $\frac{1}{4}$ but less than $\frac{1}{3}$?

3) C ☺ D is defined to be equal to $\frac{\frac{1}{C}-\frac{1}{D}}{\frac{1}{C}+\frac{1}{D}}$. Find the value of 8 ☺ 12.

Express your answer as a fraction in lowest terms.

Answers

1) _____

2) _____

3) _____

Solutions to Category 4
Arithmetic
Meet #2 - December, 2018

1) **48% = 48/100 = 24/50 = 12/25. Denominator is 25.**

2) **1/4 = 7/28. Also, 1/3 = 7/21. With 7 as a numerator, the fractions whose values lie between 1/4 and 1/3 have a denominator of 22, 23, 24, 25, 26, or 27. There are six such values for the new denominator.**

3) **8 ☺ 12** $= \dfrac{\frac{1}{8}-\frac{1}{12}}{\frac{1}{8}+\frac{1}{12}} = \dfrac{\frac{3}{24}-\frac{2}{24}}{\frac{3}{24}+\frac{2}{24}} = \dfrac{\frac{1}{24}}{\frac{5}{24}} = \frac{1}{5}$

Answers

1) **25**

2) **6**

3) $\frac{1}{5}$

Category 5
Algebra
Meet #2 - December, 2018

1) There are three positive integers, such that one is four more than another, while the third is twice the sum of the other two. The sum of all three integers is 54. What is the largest of the three integers?

2) If 2E + 3H = 21 and 4M - J = - 31 then what is the value of 8E + 12H - (8M - 2J) + 7 ?

3) $F=\frac{9}{5}C+32$ **is the formula that converts a temperature in Celcius (C) degrees to Fahrenheit (F) degrees. The Massachusetts town of Concord set a record high temperature this past October with a temperature of 95 degrees F. What was that temperature in Celcius?**

Answers

1) _____

2) _____

3) _____

Solutions to Category 5
Algebra
Meet #2 - December, 2018

Answers
1) 36
2) 153
3) 35

1) **Let** X = the smallest integer.
$X + 4$ = the next larger integer
$2(X + X + 4)$ = the third integer

Then $(X) + (X + 4) + 2(X + X + 4) = 54$
$2X + 4 + 2X + 2X + 8 = 54$
$6X + 12 = 54$
$6X = 42$
$X = 7$
$7 + 11 = 18$
$2(7 + 11) = 36.$

Therefore, the largest of the three integers is 36.

2) **If $2E + 3H = 21$, then $8E + 12H = 4(2E + 3H) = 4(21) = 84.$**
If $4M - J = -31$, then $8M - 2J = 2(4M - J) = 2(-31) = -62.$
Then $8E + 12H - (8M - 2J) + 7 = 84 - (-62) + 7 = 84 + 62 + 7 = 153.$

3)

$$F = \frac{9}{5}C + 32$$

$$95 = \frac{9}{5}C + 32$$

$$63 = \frac{9}{5}C$$

$$63\left(\frac{5}{9}\right) = C$$

$$35 = C$$

Category 6
Team Round
Meet #2 - December, 2018

Each of the following nine problems is worth <u>four</u> points.

1) Let P be any prime number greater than 10. How many distinct (different) prime factors does the product 72P have?

2) The measures of the interior angles in a convex pentagon are in the ratio 2:3:4:5:6. The largest angle is how many degrees greater than the smallest angle?

3) What is the smallest positive whole number value of N, such that the product 120N is the square of a positive integer?

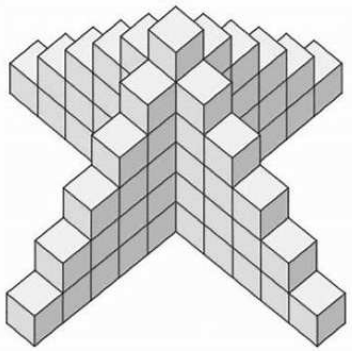
4) Thesaurus can speak 120 words in 1.5 minutes and can write 96 words in $2\frac{2}{3}$ minutes. How many more words can Thesaurus speak in ten minutes than he could write in ten minutes?

5) The sequence below is formed by the following rules: 1) if a number is even, then divide it by 2 to get the next number, and 2) if a number is odd, then multiply it by 3 and then add 1 to get the next number. What is the value of the 100th term in this sequence? 12 6 3 10 5 16 . . .

ANSWERS

1) _____
2) _____
3) _____
4) _____
5) _____
6) _____
7) _____
8) _____
9) _____

6) How many small cubes are in this figure? Each column of cubes sits on the floor, or rests on the same plane.

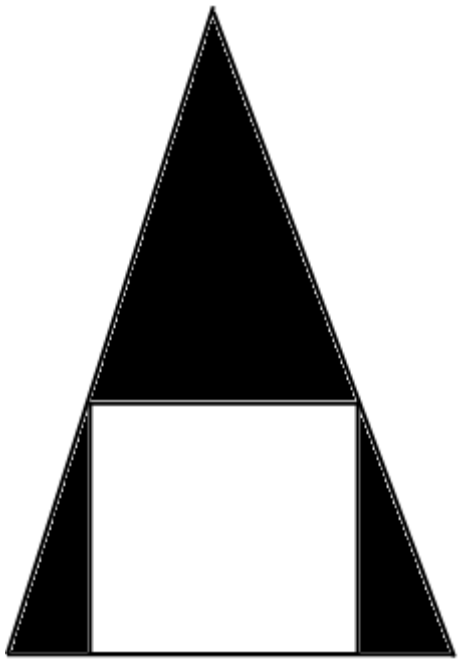


7) If $1000^4 = 10^{2N}$, then what is the value of N ?

8) What is the positive difference between 0.8 and $0.\overline{8}$? Express your answer as a fraction in lowest terms.

(Question #9 is on the next page.)

9) The two slanted sides of this isosceles triangle are congruent. The base is ten yards long and the altitude measures 15 yards. The perimeter of the inscribed square is 24 yards. How many square feet are in the shaded area?



Solutions to Category 6
Team Round
Meet #2 - December, 2018

ANSWERS	
1)	3
2)	108
3)	30
4)	440
5)	1
6)	66
7)	6
8)	$\frac{4}{45}$
9)	351

1) 72P = 2 x 2 x 2 x 3 x 3 x P. Since P is greater than 10, the there are three distinct prime factors: 2, 3, and P.

2) The total number of degrees of the interior angles is (180)(3), or 540 degrees.
$2X + 3X + 4X + 5X + 6X = 540$
$20X = 540$
$X = 27.$
The largest angle = (6)(27), or 162 degrees.
The smallest angle = (2)(27), or 54 degrees.
The difference is 162 - 54, or 108 degrees.

3) Prime factor 120, then "double up" the factors to produce a product that is a perfect square, or the square of a positive integer:
120 = 2 x 2 x 2 x 3 x 5. We need another factor of 2, another factor of 3, and another factor of 5 to produce a product = a perfect square:
2 x 2 x 2 x 2 x 3 x 3 x 5 x 5. Therefore,
P = 2 x 3 x 5, or 30. Check that (120)(30) is a perfect square . . . = 3600 which is the square of 60. Check!

4) 120 : 1.5 as X : 10. Cross multiplying, 1.5X = 1200 and X = 800.
Thesaurus can speak 800 words in 10 minutes.
96 : 2 2/3 as Y : 10. Cross multiplying, 2 2/3 Y = 960 and Y = 360.
Thesaurus can write 360 words in 10 minutes. The difference is 800 - 360, or 440.

5) Continue the sequence to find a clue:
12 6 3 10 5 16 8 4 2 1 4 2 1 4 2 1 4 2 1 . . .
The 8th term is 4, as is every 3rd term thereafter.
The 9th term is 2, as is every 3rd term thereafter.
The 10th term is 1, as is every 3rd term thereafter.
The 100th term is 90 terms after the 10th term, so the 100th term is 1.

Continued onto the next page.

6) The center column has 6 cubes. Each of the four extending column sets has 1 + 2 + 3 + 4 + 5, or 15 cubes. The total number of cubes is (15)(4) + 6, or 66.

7) $1000^4 = 10^{2N}$

$(10^3)^4 = (10)^{2N}$

$10^{12} = (10)^{2N}$

2N = 12

N = 6.

8) 0.8 = 8/10 while 0.888 . . . = 8/9.
8/9 - 8/10 = 80/90 - 72/90 = 8/90 = 4/45.

9) The perimeter of the square is 24 yards, so one side measures 24 / 4, or 6 yards, and its area is (6)(6), or 36 square yards.
The large triangle's area is (1/2)(base)(altitude) = (1/2)(10)(15) = 75 square yards. The shaded area = the large triangle - the square
= 75 - 36 = 39 square yards.
The question requires an answer in square FEET. There are nine square feet in a square yard, so converting 39 square yards to square feet: (39)(9) = 351 square feet.