

IMLEM Meet #5
April, 2018

Intermediate Mathematics League of Eastern Massachusetts



Calculator Meet

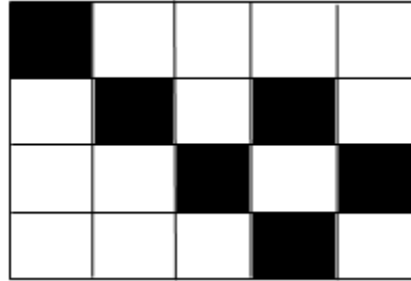
Category 1

Mystery

Meet #5 - April, 2018

Calculator Meet

- 1) A rectangle is divided into identical smaller rectangles, as shown. What percent of the smaller rectangles is shaded?



- 2) Eight dogs eat eight bowls of food at the same rate in eight minutes. How many minutes should it take four dogs to eat four bowls of food?
- 3) Two cars are heading in the same direction. The lead car is travelling at a rate of 48 miles per hour. The trailing car is going 60 miles per hour. How many miles apart are the two cars 45 minutes before the fast car reaches the slower car?

Answers

1) _____ %

2) _____

3) _____

Solutions to Category 1

Mystery

Meet #5 - April, 2018

- 1) Six of the 20 squares are shaded, so $\frac{6}{20}$ of the rectangle is shaded. Dividing 6 by 20 yields a decimal answer of 0.3. Convert to 30%.**
- 2) As long as the ratio of dogs to bowls of food remains constant - in this case, 1:1 - then it should take eight minutes. The number of dogs and the number of food in the question are in a ratio of 4:4, or simply 1:1, so it should take 8 minutes.**
- 3) When the two cars are one hour apart, they are 12 miles apart, as indicated by the difference in their rates ($60 - 48 = 12$). So, in 45 minutes, or $\frac{3}{4}$ of an hour, the two cars will be $\frac{3}{4}$ of 12 miles apart, or 9 miles.**

| <u>Answers</u> | |
|----------------|--------|
| 1) | 30 (%) |
| 2) | 8 |
| 3) | 9 |

Category 2
Geometry
Meet #5 - April, 2018



Calculator Meet

- 1) A cube has a volume of 512 cubic feet. How many feet are in the length of one edge?

- 2) A pyramid has a rectangular base with an area of 119 square inches. The pyramid has the same base as the rectangular solid and is half as tall as the rectangular solid. The altitude of the rectangular solid is 10 feet. How many cubic inches are in the volume of the pyramid?

- 3) Quaykah wants to paint the inside of a cylindrical storage silo that is 63 feet high and whose circular floor has a diameter of 19 feet. One gallon can of paint covers 50 square yards. How many gallon cans of paint must Quaykah purchase in order to complete the job? Notes: The store will only sell full cans of paint. The ceiling of the silo does not get painted, but the floor does get painted. Use $\pi \approx 3.14$.

Answers

- 1) _____ feet

- 2) _____ cu. in.

- 3) _____ cans

Solutions to Category 2
Geometry
Meet #5 - April, 2018

- 1) The cube root of 512 is 8 and represents the length of an edge of the cube.
- 2) The volume of the rectangular solid is (area of base) x (altitude), or (119)(10) = 1190 cubic inches. A pyramid with the same altitude and base as the rectangular solid would have 1/3 its volume. In this case, the altitude is given in FEET, so must be converted to (12)(10), or 120 inches. The altitude of the pyramid, being half that of the rectangular solid, is 60 inches. The dimensions of the pyramid, then are: area of base = 119 square inches, altitude = 60 inches. So, the volume of the pyramid is (1/3)(119)(60), or 2,380 cubic inches.

- 3) The surface area to be painted = (area of floor) + (area of vertical cylinder)
- $$= \pi r^2 + 2\pi rh$$
- $$\approx (3.14)(9.5)^2 + (2)(3.14)(9.5)(63)$$
- $$\approx (3.14)(90.25) + (3,758.58)$$
- $$\approx (283.385) + (3,758.58)$$
- $$\approx 4,041.965 \text{ square feet.}$$

One gallon of paint covers 50 square yards, or 450 square feet, as one square yard = 9 square feet. Divide the surface area to be painted (4,041.965) by the number of square feet that one gallon covers (450) to get 8.982 . . . cans of paint. Therefore, 9 cans of paint must be purchased.

Answers

1) 8

2) 2,380

3) 9

Category 3
Number Theory
Meet #5 - April, 2018



Calculator Meet

- 1) Set A = { multiples of 2 }
Set B = { multiples of 3 }
How many numbers between 20 and 40 are in the intersection of sets A and B ?
- 2) There are 24 students in homeroom 371. All 24 are required to sign up for at least one club. Two clubs are available - the Twilight Zone Club and the Dog Lovers Club. Fourteen students signed up for the Twilight Zone Club and 17 signed up for the Dog Lovers Club. How many students signed up for both clubs?
- 3) There are 360 students at the Lynn Breed Middle School.
- * 50% watch TV.
 - * 60% listen to radio.
 - * $\frac{1}{4}$ watch TV and go to movie theatres.
 - * 20% go to movie theatres and listen to radio.
 - * $\frac{2}{9}$ watch TV and listen to radio.
 - * 10% do all three activities.
 - * 17 students only went to movie theatres.
- How many students at the Lynn Breed Middle School do not participate in any of the three activities?

Answers

1) _____

2) _____

3) _____

Solutions to Category 3
Number Theory
Meet #5 - April, 2018

| <u>Answers</u> | |
|----------------|----|
| 1) | 3 |
| 2) | 7 |
| 3) | 27 |

- 1) Multiples of 2 and 3 are multiples of 6. Those that lie between 20 and 40 are 24, 30, and 36. That is three such multiples.
- 2) Since the sum of the numbers of students in each club exceeds the number of students in the homeroom, and every student must sign up for at least one club, then the number signing up for both clubs is $14 + 17 - B = 24$ where B = the number signing up for both. $31 - B = 24$, so $B = 7$.
- 3) Since all three activities overlap, the following Venn diagram can help organize the given data:

Checking all conditions:

* 50% watch TV = $46 + 44 + 36 + 54 = 180$ $180 = TV$

$216 = \text{radio}$

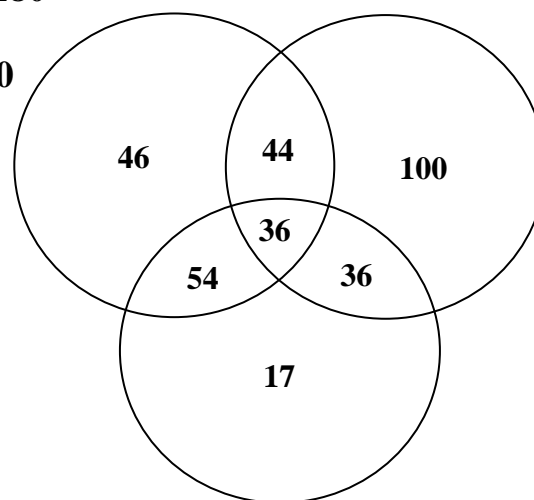
* 60% listen to radio = $44 + 36 + 36 + 100 = 216 = ((0.6)(360))$

* $1/4$ watch TV and go to theatres = $54 + 36 = 90 = (1/4)(360)$

* 20% go to theatres and listen to radio = $36 + 36 = 72 = (0.2)(360)$

* $2/9$ watch TV and listen to radio = $44 + 36 = 80 = (2/9)(360)$

* 10% do all three activities = $(0.1)(360) = 36$



The number of students who do not participate in any of the three activities is the difference between the total number of students and the sum of all the numbers in the Venn Diagram

$$= 360 - (46 + 44 + 36 + 54 + 36 + 100 + 17)$$

$$= 360 - 333$$

$$= 27.$$

Therefore, only 27 students did not participate in any of the three activities.

Category 4

Arithmetic

Meet #5 - April, 2018

Calculator meet

- 1) Elmer bought a bag of multi-colored organic carrots. Ten were yellow, seven were purple, five were red, and six were orange. If Elmer chose a carrot at random, what is the probability that it was purple? Express your answer as a common (simplified) fraction.

- 2) Two cubical dice, with faces numbered 1, 2, 3, 4, 5, and 6 are rolled. What is the probability that the sum of the numbers showing on the top surfaces of the two dice is greater than four? Express your answer as a common (simplified) fraction.

- 3) A bag of colored candy eggs contains eight red, eleven green, and five blue eggs. Foghorn selects three eggs at random and eats each one after selecting it from the bag. What is the probability that the three eggs are the same color? Express your answer as a decimal, rounded to the nearest thousandth.

ANSWERS

1) _____

2) _____

3) _____

Solutions to Category 4

Arithmetic

Meet #5 - April, 2018

1) Divide the number of purple carrots by the total number of all carrots. $7/(10 + 7 + 5 + 6) = 7/28 = 1/4$.

2) The grid below shows the face numbers 1, 2, 3, 4, 5, and 6 along the top row and also along the left-hand column. Each number in the interior of the grid is the sum of the number at the top of its column and the far left of its row. There are 30 sums that are greater than 4. The probability of getting one of these sums is $30/36$, or $5/6$.

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|----|----|----|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 6 | 7 | 8 | 9 | 10 | 11 | 12 |

3) The problem is equivalent to "draw without replacement."

$$\begin{aligned} & \text{all red} + \text{all green} + \text{all blue} \\ &= \left(\frac{8}{24} \cdot \frac{7}{23} \cdot \frac{6}{22}\right) + \left(\frac{11}{24} \cdot \frac{10}{23} \cdot \frac{9}{22}\right) + \left(\frac{5}{24} \cdot \frac{4}{23} \cdot \frac{3}{22}\right) \\ &= \left(\frac{336}{12,144}\right) + \left(\frac{990}{12,144}\right) + \left(\frac{60}{12,144}\right) \\ &= \left(\frac{1,386}{12,144}\right) \end{aligned}$$

Dividing 1,386 by 12,144 yields the decimal 0.11413...

Rounding to the nearest thousandth gives 0.114.

Therefore, the probability of three eggs of the same color are selected is about 0.114.

| <u>Answers</u> | |
|----------------|---------------|
| 1) | $\frac{1}{4}$ |
| 2) | $\frac{5}{6}$ |
| 3) | 0.114 |

Category 5

Algebra

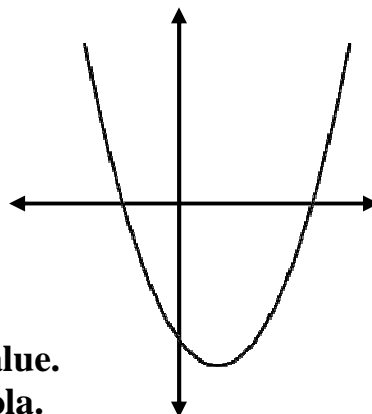
Meet #5 - April, 2018

Calculator Meet

- 1) There are two values of N that make the following quadratic equation true. What is the sum of those two values?

$$(N + 3)(N - 8) = 0$$

- 2) A quadratic equation of the form $Y = A(X - B)(X - C)$ is represented by a parabola that has X -intercepts at -3 and 7 and a Y -intercept at -42 . What is the Y -coordinate of the vertex of the parabola? The vertex is the lowest point of the parabola with the minimum Y -value. (X, Y) represents any point of the parabola. A , B , and C are constants.



- 3) The longer leg of a right triangle is 12 centimeters shorter than the hypotenuse. The shorter leg is 3 centimeters more than half the longer leg. How many centimeters are in the perimeter of the triangle?

ANSWERS

1) _____

2) _____

3) _____

Solutions to Category 5

Algebra

Meet #5 - April, 2018

Answers

- 1) If $(N + 3)(N - 8) = 0$, then either $N + 3 = 0$ or $N - 8 = 0$. So, either $N = -3$ or $N = 8$. Their sum is $-3 + 8$, or 5 .
- 2) The vertex of the parabola is on the symmetry line of the parabola. The X-coordinate of the vertex is the average of the X-coordinates of the two X-intercepts. The average $= (-3 + 7) / 2$, or 2 .

- 1) 5
2) -50
3) 240

Using the values of the two X-intercepts, we can write the equation as $Y = A(X + 3)(X - 7)$. To find the value of A, substitute the coordinates of the Y-intercept, $(0, -42)$, for X and Y:

$$-42 = A(0 + 3)(0 - 7)$$

$$-42 = A(3)(-7)$$

$$-42 = A(-21)$$

$$A = 2$$

$$\text{Now, the equation is } Y = 2(X + 3)(X - 7).$$

To calculate the Y-intercept of the vertex, substitute 2 for X:

$$Y = 2(2 + 3)(2 - 7)$$

$$Y = 2(5)(-5)$$

$$Y = -50$$

So, the Y-coordinate of the vertex is -50 .

- 3) Let $X =$ the length of the longer leg,
 $X + 12 =$ the length of the hypotenuse, and
 $0.5X + 3 =$ the length of the shorter leg.

Use the Pythagorean Theorem to find the value of X:

$$(0.5X + 3)^2 + X^2 = (X + 12)^2$$

$$0.25X^2 + 3X + 9 + X^2 = X^2 + 24X + 144$$

$$0.25X^2 - 21X - 135 = 0$$

$$X^2 - 84X - 540 = 0$$

$$(X - 90)(X + 6) = 0$$

$X = 90$ or $X = -6$. Only 90 makes sense for this problem, as lengths must be positive. Then the short leg = 48, the longer leg = 90, and the hypotenuse = 102. The perimeter is their sum: $48 + 90 + 102 = 240$ cm.

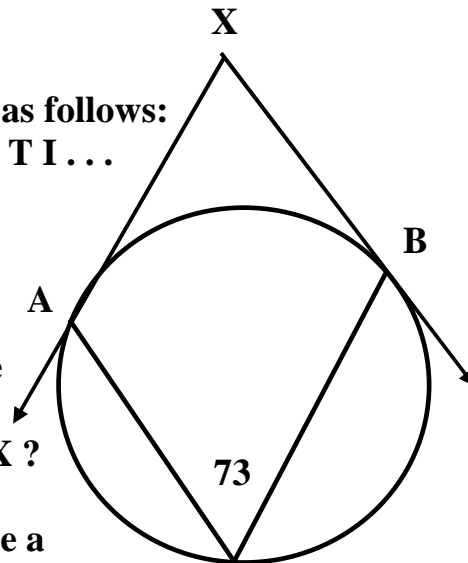
**Category 6
Team Round
Meet #5 - April, 2018**

Each of the following six problems is worth six points.

- 1) Twelve different diameters pass through the center of a circle. Through how many total points of the circumference of the circle will they pass?
- 2) A utility table in the teachers' work room has a rectangular surface four feet by eight feet and is supported by four legs at the corners that are each three feet long. The legs are flared away from the table, exposing an area on the floor beneath the table that has the same area as the surface of the table. What is the maximum number of one-inch cubes that can fit under the table?

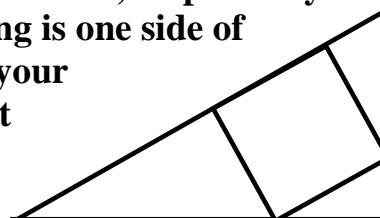
- 3) The word **NATICK** is written repeatedly as follows:
N A T I C K N A T I C K N A T I C K N A T I . . .
What is the 75th letter in this sequence?

- 4) For this circle, the inscribed 73-degree angle intersect two tangent lines at points **A** and **B**. The tangent lines form an angle exterior to the circle and measures **X** degrees. How many degrees are in angle **X**?



- 5) A solid green Gel-Oh cone fits tightly inside a solid red Gel-Oh right rectangular solid, displacing a volume inside the rectangular solid that is equal to the volume of the cone. The circular base of the cone is tangent to the four sides of the 14-inch by 14-inch base of the rectangular solid. Both solids are 22 inches high. What percent of the red Gel-O is the green Gel-Oh? Round your answer to the nearest whole percent. Use $\pi \approx 3.141$.

- 6) A square has its vertices on the perimeter of a right triangle, as shown. The two legs measure 21 and 52 inches, respectively. How many inches long is one side of the square? Round your answer to the nearest tenth of an inch.



ANSWERS

- 1) _____
- 2) _____
- 3) _____
- 4) _____
- 5) _____ %
- 6) _____

**Solutions to Category 6
Team Round
Meet #5 - March, 2018**

ANSWERS

- 1) 24
- 2) 165,888
- 3) T
- 4) 34
- 5) 35 (%)
- 6) 14.5

1) Each diameter intersects the circle in two points. Since the twelve diameters are different, they intersect the circle in 24 points.

2) Convert feet to inches, then multiple the three dimensions. 4 feet = (4)(12), or 48 inches. Eight feet = (8)(12), or 96 inches. Three feet = (3)(12), or 36 inches. Volume = (48)(96)(36), or 165,888 cubes.

3) Since there are six letters in the word NATICK, divide 75 by 6 to get 12 with remainder 3. There are 12 complete words spelled NATICK, consuming 72 letters, followed by the three letters N, A, and T. So, the 75th letter is T.

4) Strategy: Draw central angle AOB that has twice the measure of the inscribed angle. So, the central angle measure 146 degrees. The radii meet the tangents at points A and B at right angles, each measuring 90 degrees. The sum of the measures of quadrilateral AOBX is 360 degrees. So, the measure of angle X is $360 - (90 + 146 + 90)$, or $360 - 326$, or 34 degrees.

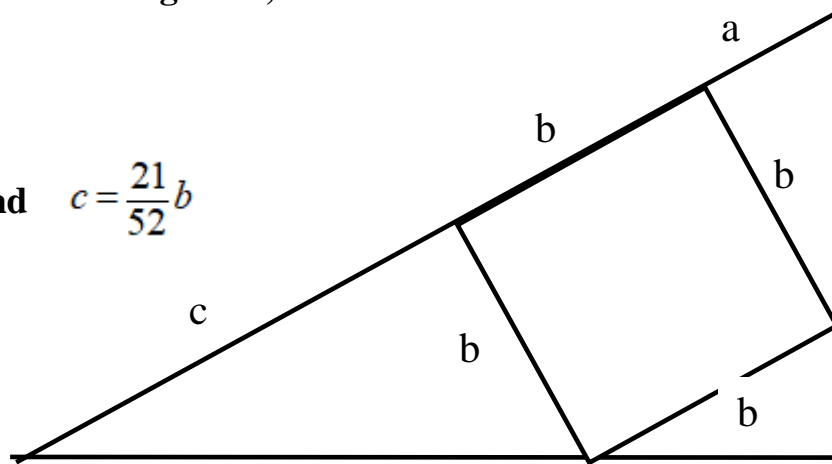
5) The volume of the rectangular solid, discounting the cone, is the product length x width x height, or $(14)(14)(22) = 4,312$ cubic inches. The circular base of the cone is pi times the square of the radius, or $(3.141)(7)(7)$, or 153.909 square inches. The volume of the cone is the product of the one-third the height and the area of the base, or $(1/3)(22)(153.909)$, or 1,128.666 cubic inches. This is the volume of the green Gel-Oh. The red Gel-Oh is the difference between the volume of the rectangular solid and the volume of the cone, or $4,312 - 1,128.666$, or 3,183.334 cubic inches. The question asks, "What percent of the red Gel-Oh is the green Gel-Oh?" Divide the volume of the green Gel-Oh by the volume of the red Gel-Oh to get 0.3545 . . . Rounding to the nearest whole percent yields 35%.

THE SOLUTION TO #6 IS ON THE NEXT PAGE.

- 6) One side of the square is parallel to the hypotenuse, forming a total of four similar right triangles. Using the labeled diagram below, ratios of corresponding sides are congruent, as follows:

$$\frac{b}{c} = \frac{a}{b} = \frac{21}{52}$$

So, $a = \frac{21}{52}b$ and $c = \frac{21}{52}b$



Then use the Pythagorean Theorem, substituting the above expressions for b :

$$21^2 + 52^2 = (a + b + c)^2$$

$$21^2 + 52^2 = \left(\frac{21}{52}b + b + \frac{52}{21}b \right)^2$$

$$21^2 + 52^2 = \left(b \left(\frac{21}{52} + 1 + \frac{52}{21} \right) \right)^2$$

$$441 + 2704 = b^2 \left(\frac{441}{1092} + \frac{1092}{1092} + \frac{2704}{1092} \right)^2$$

$$3145 = b^2 \left(\frac{4237}{1092} \right)^2$$

$$\frac{3145 \cdot 1092 \cdot 1092}{4237 \cdot 4237} = b^2$$

$$\sqrt{\frac{3145 \cdot 1092 \cdot 1092}{4237 \cdot 4237}} = b$$

$$b \approx 14.4535\dots$$

$$b \approx 14.5 \quad \text{rounded to the nearest tenth.}$$