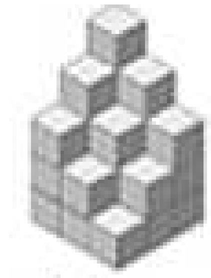
IMLEM Meet #2
November, 2017

Intermediate Mathematics League of Eastern Massachusetts

Category 1
Mystery
Meet #2 - November, 2017



1) Fewer than 50 individual wooden cubes of the same size are stacked to form this 3-D figure. Assume that all cubes in columns are supported from below, all the way to the base. How many of the individual wooden cubes comprise the figure?

2) Long ago and far away, the Boston Patriots football team beat the New York Giants by 8 points. The two teams combined for a total score of 74 points. How many points did the Patriots score?

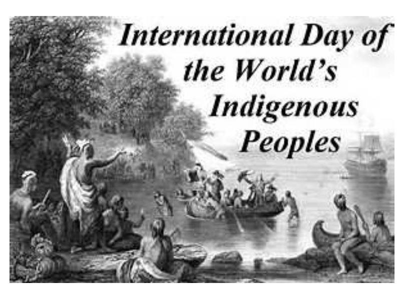
3) A "Mersenne prime" is a prime number of the form $2^P - 1$ where P is also a prime number. What is the greatest Mersenne prime that is less than 1000? Express your answer as a whole number.

<u>Answers</u>

1) __________

2) __________

3) __________



Solutions to Category 1
Mystery
Meet #2 - November, 2017

Answers	
1)	27
2)	41
3)	127

1) **The bottom layer contains 3x3, or 9 cubes. The layer above it has one less, or 8. Layer #3 has 3 less, or 6. Layer #4 has 3 while the top layer has 1.**
9 + 8 + 6 + 3 + 1 = 27.

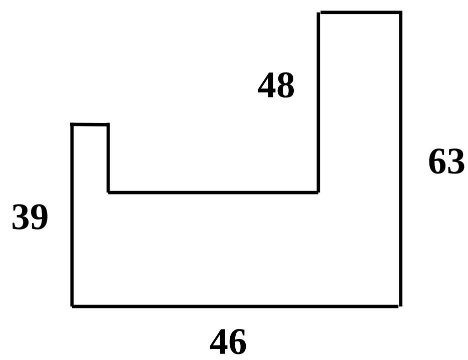
2) **One strategy, especially for students who do not yet have the necessary algebra skills, is to "guess, check, and adjust," by taking two reasonable numbers that are 8 apart, adding them up, and checking the total versus 74 until the correct numbers are achieved. Those numbers eventually are 33 and 41, with the Patriots scoring the higher amount of 41 points.**

3) **To achieve a number greater than 1000, P must be at most 9, as 2^9 = 512.**
2^3 - 1 = 7 and 7 is prime.
2^5 - 1 = 31 and 31 is prime.
2^7 - 1 = 127 and 127 is prime.
So, the answer is 127, written as a whole number.

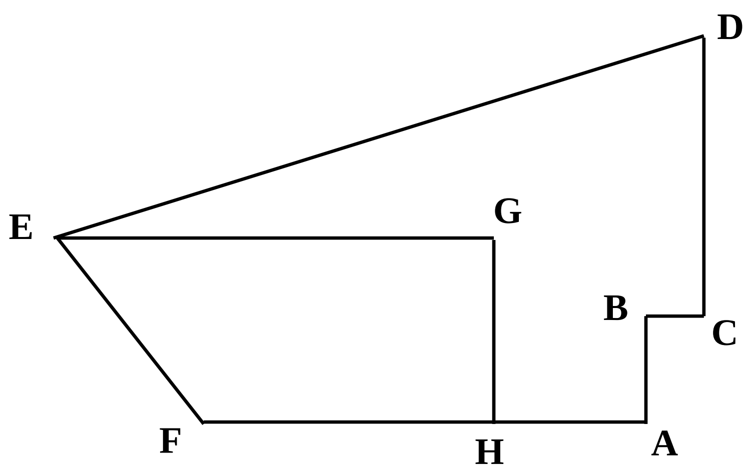
Category 2
Geometry
Meet #2 - November, 2017

1) A rectangle has a length of 12 centimeters (cm) and an area of 168 square centimeters. How many cm are in the width of the rectangle?

2) How many millimeters (mm) are in the perimeter of this polygon? All angles are right angles and all segment lengths are in mm.



3) In the figure below, points F, H, and A are collinear (in a straight line). Segment EG is parallel to FA. EG = 25, GH = 12, FA = 22, HA = 4, AB = 5, and CD = 15. BC = 3. Angles A, ABC, and C are right angles. GH is perpendicular to FA. How many square units are in hexagon ABCDEF? The figure is not drawn to scale.



Answers

1) ______

2) ______

3) ______

Solutions to Category 2
Geometry
Meet #2 - November, 2017

Answers

1) 14

2) 266

3) 455

**1) For any rectangle, length x width = area.
12 W = 168, so the width = 168 / 12, or 14 cm.**

2) The three horizontal unlabelled segments have a total length of 46 mm, the same as the length of the bottom segment. To find the length of the only unlabelled vertical segment, subtract 48 from 63 to find the width of the horizontal bar at the bottom of the figure. That bar width is 15. Then, the length of the unlabelled vertical segment is the difference between 39 and 15, or 24. So, the perimeter is the sum: 46 + 46 + 48 + 63 + 39 + 24, or 266 mm.

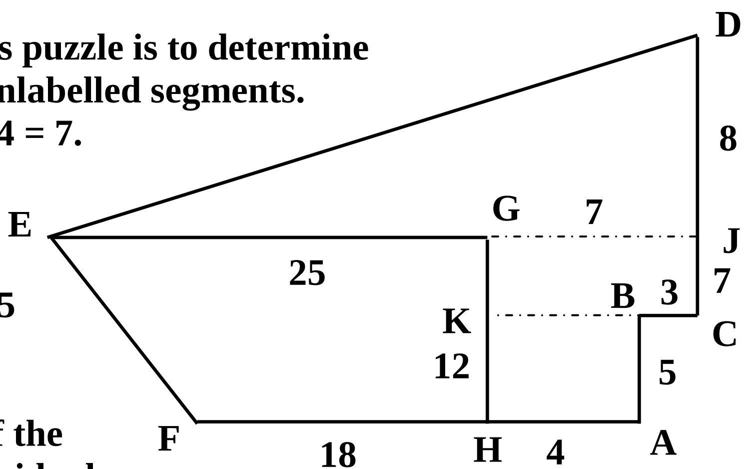
3) The key to solving this puzzle is to determine the lengths of some unlabelled segments.

GJ = HA + BC = 4 + 4 = 7.

KB = HA = 4.

FH = FA - HA = 22 - 4 = 18.

JC = GH - BA = 12 - 5 = 7.



The total area of the hexagon is the sum of the areas of the four individual parts as drawn:

Triangle EJD + trapezoid EGHF + rectangle GJCK + rectangle HKBA

= (1/2)(32)(8) + (1/2)(12)(25 + 18) + (7)(7) + (4)(5)

= 128 + 258 + 49 + 20

= 455 square units.

Category 3
Number Theory
Meet #2 - November, 2017

1) When the fraction $\frac{24}{42}$ is reduced to lowest terms, the result is $\frac{4}{7}$. What is the greatest common factor (GCF) of 24 and 42 ?

2) Radio station WAVE plays my favorite song every 12 hours. Station WIFE plays it every 18 hours while station WEEK plays it every 30 hours. My favorite song is playing at 4:00 P.M. on a Thursday on all three stations. At what time and on what day of the week will my favorite song play next on all three stations? You must include A.M or P.M. in your answer.

3) The prime factorization of 405,000 is $2^3 \times 3^N \times 5^P$.

Find the value of 7N + P.

Answers

1) ___________

2) ___________ (time)

___________ (day)

3) ___________

Solutions to Category 3
Number Theory
Meet #2 - November, 2017

Answers
1) 6
2) 4:00 A.M. Friday
3) 32

1) The simplest clue is to note that the fraction was reduced by a factor of 6. Otherwise, students might list the factors of 24 and 42 to get their GCF, or else use the prime factorization method.

2) The LCM of the three numbers can be found by listing multiples of each until the first common multiple appears. Otherwise, prime factoring the numbers can produce the LCM as follows:

   12 = 2 x 2 x 3
   18 = 2 x 3 x 3
   30 = 2 x 3 x 5

   The LCM is 2 x 2 x 3 x 3 x 5 = 180.
   Converting to days, 180 hours is 180 / 24, or 7.5 days. Seven and a half days beyond 4:00 P.M. on a Thursday is 4:00 A.M. on a Friday. All answers must be correct in order for students to receive 2 points for this problem.

3) 405,000 = 405 x 1000
   = (5 x 3 x 3 x 3 x 3) x (2 x 2 x 2 x 5 x 5 x 5)
   The three twos are accounted for in the prime factorization $2^3 \times 3^N \times 5^P$.

   The remainder of the prime factorization is $3^4 \times 5^4$.
   So, N = 4 and P = 4.
   7N + P = 7(4) + 4 = 28 + 4 = 32.

Category 4
Arithmetic
Meet #2 - November, 2017

1) Express the following sum as an improper fraction in lowest terms:

$$\frac{1}{2} + 0.75 + 2\frac{1}{4}$$

2) Express this sum as a fraction in lowest terms: $0.2 + 0.22 + 0.2\%$

3) When the repeating decimal $0.436363636\ldots$ **is written as a common fraction, the result is** $\frac{A}{B}$. **What is the value of** $A + B$ **?**

<u>**Answers**</u>
1) _______
2) _______
3) _______

Solutions to Category 4
Arithmetic
Meet #2 - November, 2017

1) $1/2 + 0.75 + 2\ 1/4 = 0.5 + 0.75 + 2.25 = 3.5 = 7/2.$

2) $0.2 + 0.22 + 0.2\% = 0.2 + 0.22 + 0.002 = 0.422$

$= 422/1000 = 211/500.$

3) Let $X = 0.4363636\ldots$
$100X = 43.6363636\ldots$

Then
$$100X - X = (43.6363636\ldots) - (0.436363636\ldots)$$
$$99X = 43.2$$
$$X = 43.2 / 99$$
$$= 432 / 990$$
$$= 216 / 495$$
$$= 72 / 165$$

Finally, $72 + 165 = 237.$

Answers	
1)	$\frac{7}{2}$
2)	$\frac{211}{500}$
3)	237

Category 5
Algebra
Meet #2 - November, 2017

1) Tammy the turkey laid some eggs in September. She laid seven fewer in October than she laid in September. She laid four more eggs in November than she laid in September. She laid 162 eggs in all. How many eggs did Tammy lay in September?

2) The formula $V = 7 + 2n^2$ gives the value, V, of any term in the following sequence:

9 15 25 39 57 . . .

where n is the number of the term. For example, for the first term, 9, n = 1. For the second term, 15, n = 2, and so on. What is the number of the term whose value is 457 ?

3) If $2c + 5d = 38$ and $3e - 7f = 81$, then what is the value of

$28f - 12e - (10c + 25d)$?

Answers
1) ______
2) ______
3) ______

Solutions to Category 5
Algebra
Meet #2 - November, 2017

Answers
1) 55
2) 15
3) - 514

1) Let E = the number of eggs laid in September.
$E - 7$ = the number of eggs laid in October
$E + 4$ = the number of eggs laid in November.

Then $E + (E - 7) + (E + 4) = 162$
$$3e - 3 = 162$$
$$3E = 165$$
$$E = 55$$

Since E represents the number of eggs laid in September, and $E = 55$, then Tammy laid 55 eggs in September.

2)
$$V = 7 + 2n^2$$
$$457 = 7 + 2n^2$$
$$450 = 2n^2$$
$$225 = n^2$$
$$15 = n$$

Therefore, 457 is the 15th term in the sequence, and $n = 15$.

3) This problem involves the distributive property and its implications in values of literal expressions and their numeric multiples.

If $2c + 5d = 38$, then $5(2c + 5d) = 5(38)$, or 190.
If $3e - 7f = 81$, then $7f - 3e = -81$ and $4(7f - 3e) = 4(-81)$, or -324.

Then
$$\begin{aligned} & 28f - 12e - (10c + 25d) \\ = \ & 4(7f - 3e) - 5(2c + 5d) \\ = \ & 4(-81) - 5(38) \\ = \ & -324 - 190 \\ = \ & -514. \end{aligned}$$

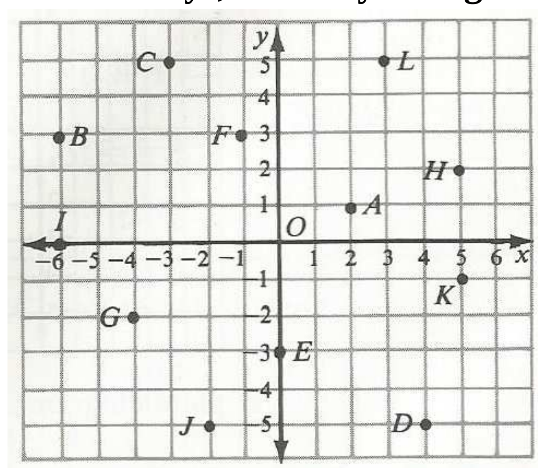
Category 6
Team Round
Meet #2 - November, 2017

Each of the following nine problems is worth <u>four</u> points.

1) What is the greatest common factor (GCF) of 12, 18, and 30 ?

2) If the perimeter of a square is 72 oodles, and a rectangle with the same area has a width of 12 oodles, then how many oodles are in the length of the rectangle?

3) Willy ate M&Ms every day for five consecutive days, each day eating six more than on the previous day. He ate 175 M&Ms in all. How many M&Ms did he eat on the fourth day?



4) In the grid to the right, points B, C, H, D, and G are connected, in that order, to form a pentagon. The coordinates of each point are integers. How many square units are in the area of pentagon BCHDG? (decimal)

<u>ANSWERS</u>

1) __________

2) __________

3) __________

4) __________

5) __________

6) __________

7) __________

8) __________

9) __________

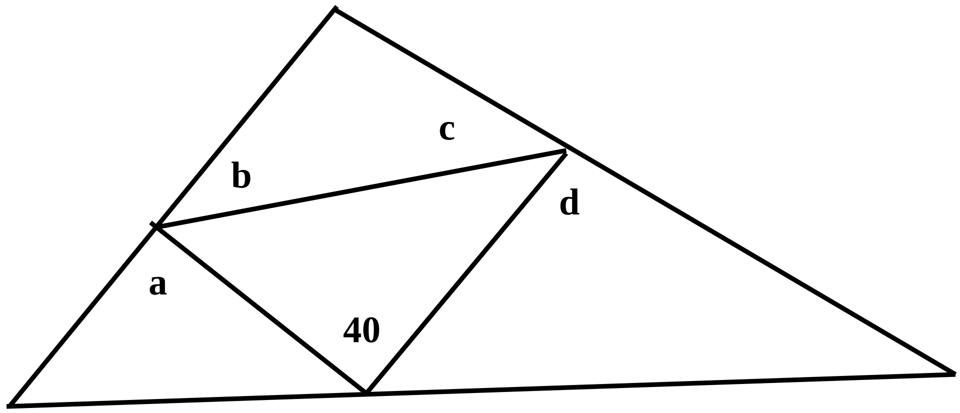
5) Let P be any prime number greater than 10. How many different prime factors does 25P have?

6) A piece of string is 13 inches long. If the string is cut into at least two pieces, then in how many different ways can the string be cut so that the length of each piece is a prime number of inches? To clarify: If A, B, and C were such lengths, then A-B-C and A-C-B and all other possible orders for the lengths A, B, and C are considered as equivalent, therefore not different. (The same prime number may be used more than once.)

7) Priscilla took some history tests and scored 72, 87, and 91. What is the mean of her next two test scores if her 5-test average is 88 ?

(Questions #8 and #9 are on the next page.)

8) Find the sum $a + b + c + d$ if each represents the number of degrees in its respective angle in the diagram at the top of the following page. There is one angle marked as 40 degrees.



9) A unit fraction is a rational number written as a fraction where the numerator is the number 1 and the denominator is a positive integer. A unit fraction is, therefore, the reciprocal of a positive integer. How many unit fractions are there between $\frac{3}{16}$ and 62% ?

Solutions to Category 6
Team Round
Meet #2 - November, 2017

ANSWERS

1) 6

2) 27

3) 41

4) 68.5

5) 2

6) 8

7) 95

8) 220

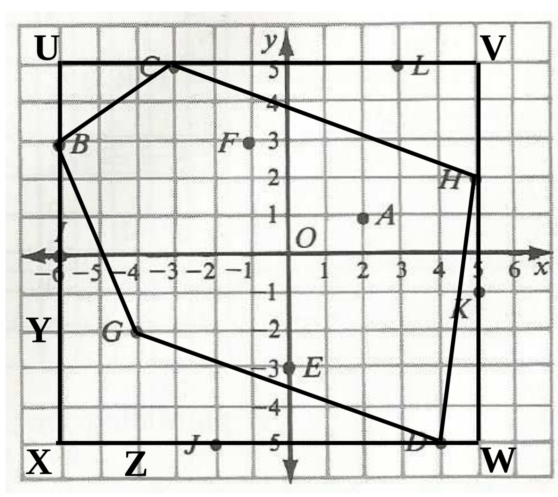
9) 4

1) 12 = 2 x 2 x 3
18 = 2 x 3 x 3
30 = 2 x 3 x 5
The GCF = 2 x 3 = 6.

2) If the perimeter of the square is 72 oodles, then one side is 72 / 4, or 18 oodles. Its area is (18)(18), or 324 square oodles. The rectangle's length is 324 / 12, or 27.

3) Let X = the number of M7Ms eaten on Day 1.
X + (X+6) + (X+12) + (X+18) + (X+24) = 175
5X + 60 = 175
5X = 115
X = 23
On the 4th day, Willy ate X+18 M&Ms, so he ate 23 + 18, or 41 M&Ms.

4) Connecting the five points of the pentagon is shown below. also shown is a rectangle that tightly contains the pentagon so that the pentagon's area can be thought of as
the difference between the area of the rectangle and the sum of the areas of the five right triangles plus the rectangle that lie outside the pentagon. The rectangle is 10 by 11 and has an area of 110 square units. The six areas outside the pentagon but contained by the rectangle are:



UBC = (1/2)(2)(3) = 3
CVH = (1/2)(3)(8) = 12
HWD = (1/2)(1)(7) = 3.5
DGZ = (1/2)(3)(8) = 12
GBY = (1/2)(5)(2) = 5
XYGZ = (2)(3) = 6

Continued onto the next page.

Area of pentagon = area of rectangle - sum of areas outside
= 110 - (3 + 12 + 3.5 + 12 + 5 + 6)
= 110 - 41.5
= 68.5 square units. (must be a decimal)

5) 25P = 5 x 5 x P where are three factors are prime The different prime factors of 25P are 5 and P, so there are two prime factors.

6) There are eight different ways that the 13-inch piece of string can be cut into a prime numbers of inches:

2 - 11
3 - 3 - 7
3 - 5 - 5
2 - 3 - 3 - 5
2 - 2 - 2 - 2 - 2 - 3
2 - 2 - 2 - 2 - 5
2 - 2 - 2 - 7
2 - 2 - 3 - 3 - 3

7) 72 + 87 + 91 + X + Y = 5(88)
250 + X + Y = 440
X + Y = 190
So, the mean of X and Y is 190 / 2, or 95.

8) The remaining two angles of the triangle containing the 40-degree angle have a sum of 140 degrees. Those two angles, plus angles a, b, c, and d form two straight angles with a total of 360 degrees.
360 - 140 = a + b + c + d = 220 degrees.

9) The unit fractions that lie between 3/16 and 62% are 1/5, 1/4, 1/3, and 1/2. Converting all to decimals can clarify that they all lie between 3/16, or 0.1875, and 62%, or 0.62.
1/5 = 0.2 1/4 = 0.25 1/3 = 0.333... 1/2 = 0.5
Therefore, there are four unit fractions that lie between 3/16 and 62%.