IMLEM Meet #1
October, 2017

# Intermediate Mathematics League of Eastern Massachusetts

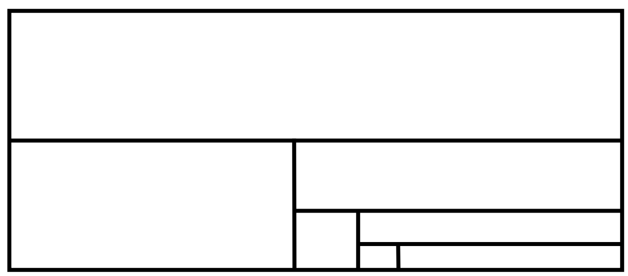
**Category 1**
**Mystery**
**Meet #1 - October, 2017**

**1) The letters in this sequence are repeated according to the following pattern:**

**O C T O B E R O C T O B E R O C T . . .**

**What is the 226th letter in the sequence?**

**2) A rectangle is a quadrilateral (4-sided polygon) with four right angles. This sheet of paper is an example of a rectangle. Also, every square is a rectangle. How many rectangles of any size are in the figure below? All angles are right angles.**



**3) Onopods have seven legs while tryptopods have 13 legs. Ant Sally has onopods and tryptopods in a shoe box. She counts a total of 168 legs. How many tryptopods are there?**

**Answers**

1) ______

2) ______

3) ______

**Solutions to Category 1**
**Mystery**
**Meet #1 - October, 2017**

| Answers |
|---|
| 1) C |
| 2) 13 |
| 3) 7 |

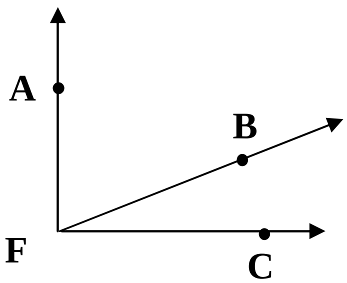
**1) There are seven letters in the word October. The number of times OCTOBER appears in 226 letters is 226 / 7, or 32 times, with two letters left over, namely O and C. So, the 226th letter is C.**

**2) There is one of the largest rectangle, two of the second-largest, two of the third-largest, two of the 4th largest, one of the square, one of the 5th largest, two of the 6th largest, one smallest square, and the remaining small rectangle, for a total of 1 + 2 + 2 + 2 + 1 + 1 + 2 + 1 + 1 = 13.**

**3) At first glance, students may think there is not enough information to solve this problem, especially those with enough mathematical expertise to seek an algebraic solution. However, noting that there must be an integer number of bodies and an integer number of legs, guessing and checking would yield that 13(7) + 7(11) gives a total of 91 + 77, or 168 legs. All other combinations do not give a total of 168 legs. Therefore, the number of tryptopods is 7. The question involved critical reading, requiring only that the number of TRYPTOPODS (13-legged critters) be found.**

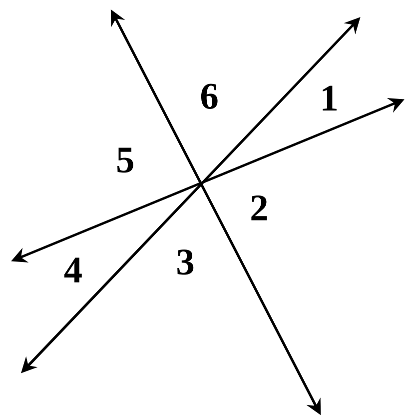
**Category 2**
**Geometry**
**Meet #1 - October, 2017**

**1)**

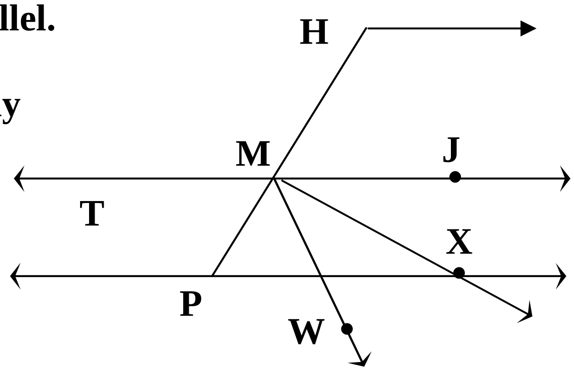



**Angle AFC is a right angle. Angle BFC measures 38 degrees. How many degrees are in the measure of the supplement of angle AFB ?**

**2) Three lines intersect at a common point. Angles 4 and 6 are complementary. Angle 3 measures 78 degrees. How many degrees are in the measure of angle 2 ?**




**3) The three horizontal lines are parallel. Angle H measures 117 degrees. Angle JMP is trisected. How many degrees are in angle XMT ?**




**Answers**

1) ______

2) ______

3) ______

**Solutions to Category 2**
**Geometry**
**Meet #1 - October, 2017**

**Answers**

**1) 128**

**2) 90**

**3) 141**

**1) Since angle AFC is a right angle, its component parts are complementary. So, angle AFB measures 90 - 38, or 52 degrees. Its supplement measures 180 - 52, or 128 degrees.**

**2) Since angles 4 and 6 are complementary, their sum is 90 degrees. Therefore, angle 5 measures 180 - 90, or 90 degrees. Angle 2 is vertical to angle 5 and has the same measure, or 90 degrees.**

**3) Since angle H measures 117 degrees, then angle MPW is its supplement: 180 - 117 = 63. Angle JMP = angle H, as corresponding angles are congruent. Since angle JMP is trisected, then each of the trisected angles measures 117 / 3, or 39 degrees. Angle PMX measures 2(39), or 78 degrees. Angle TMP = angle MPW = 63 degrees, as alternate interior angles are congruent. Angle XMT = angle TMP + angle PMX = 63 + 78 = 141 degrees.**

**Category 3**
**Number Theory**
**Meet #1 - October, 2017**

**1) Find the sum of all composite numbers between 37 and 47.**

**2) What is the smallest number that is divisible by five different prime numbers?**

**3)**  **is divisible by 3.**

 **has an odd number of factors.**

 **is divisible by 4.**

**What is the sum of all possible values of**  **between 1 and 1000 ?**

**Answers**

1) ______

2) ______

3) ______

**Solutions to Category 3**
**Number Theory**
**Meet #1 - October, 2017**

| Answers |
|---|
| 1) 294 |
| 2) 2310 |
| 3) 1980 |

**1) The composites between 37 and 47 are 38, 39, 40, 42, 44, 45, and 46. The only primes in that range are 41 and 43. The sum of the composites is**
**38 + 39 + 40 + 42 + 44 + 45 + 46 = 294.**

**2) 2 x 3 x 5 x 7 x 11 = 2310.**

**3) Whole numbers with an odd number of factors are perfect squares. Those between 1 and 1000 that are also divisible by 4 and 3 (multiples of 12) are the following:**

**36 = 4 x 9**
**144 = 4 x 4 x 9**
**324 = 4 x 9 x 9**
**576 = 8 x 8 x 9**
**900 = 4 x 9 x 25**

**36 + 144 + 324 + 576 + 900 = 1980**

**Category 4**
**Arithmetic**
**Meet #1 - October, 2017**

**1) What is the value of** $75 - 3\,(58 - 39) + 4 \cdot 8^2$ **?**

**2) The stem and leaf plot below indicates how many passes that New England Patriots quarterback, Tom Brady, completed for touchdowns during his past 17 regular seasons, excluding playoffs and Super Bowls. The number in the left-hand column is the tens digit. The number in the right-hand column is the units, or ones digit. What is the mean (average) number of touchdown passes per season that Tom Brady threw over the course of his 17 regular seasons? Round your answer to the nearest whole number. Include the two seasons when Tom did not complete any passes.**

| | |
|---|---|
| **0** | **0, 0** |
| **1** | **8** |
| **2** | **3, 4, 5, 6, 8, 8, 8, 8** |
| **3** | **3, 4, 6, 6, 9** |
| **5** | **0** |

**3) The mean of ten different whole numbers is 14. Each number is greater than three. Six of the numbers are 6, 9, 14, 21, 23, and 29. What is the largest possible value that one of the remaining four numbers could be?**

**Answers**

1) ______

2) ______

3) ______

**Solutions to Category 4**
**Arithmetic**
**Meet #1 - October, 2017**

| Answers |
|---|
| 1) 274 |
| 2) 27 |
| 3) 22 |

**1)** $75 - 3\,(58 - 39) + 4 \cdot 8^2$
= 75 - 3(19) + 4 x 8 x 8
= 75 - 57 + 256
= 18 + 256
= 274

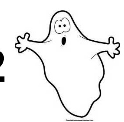
**2)** To find the mean (average), add the seventeen season totals of touchdown passes, then divide by 17, then round to the nearest whole number:
(0+0+18+23+24+25+26+28+28+28+28+33+34+36+36+39+50) / 17
= (456) / 17
= 26.82 . . . . that rounds to 27.

**3)** The sum of the ten numbers is (10(14), or 140. The sum of the given numbers is 6+9+14+21+23+29 = 102. So, the sum of the remaining four numbers is the difference 140 - 102, or 38. Of these four numbers, all of which must be greater than three and all different, to get the highest number, three of the four numbers must be the smallest available, or 4, 5, and 7 (6 was already used). So, 4 + 5 + 7 + X = 38, and X = 22.

**Category 5**
**Algebra**
**Meet #1 - October, 2017**

1) If  = 11, and  = - 4, and  = 5, then what is the value of 3  - 7  + 8  ?

2) What value of  makes the following equation true?

17 - 3  = 5 ( 1 -  ) - 3 ( 4 - 2  )

3) If the following expression simplifies to $Ax + By$, then what is the value of $10A - 9B$ ?

$$5(4x + y) - 7(2x + 3y) - 6(x - 8y)$$

| **Answers** |
|---|
| 1) ______ |
| 2) ______ |
| 3) ______ |

**Solutions to Category 5**
**Algebra**
**Meet #1 - October, 2017**

| **Answers** | |
|---|---|
| 1) | 131 |
| 2) | 6 |
| 3) | - 288 |

**1)** For simplicity, substituting $X = 11$, $Y = -4$, and $Z = 5$, then $3Z - 7Y + 8X = 3(5) - 7(-4) + 8(11)$ $= 15 + 28 + 88 = 131$.

**2)** Again, for simplicity, substituting X for the ghost:

$$17 - 3X = 5(1 - X) - 3(4 - 2X)$$
$$17 - 3X = 5 - 5X - 12 + 6X$$
$$17 - 3X = -7 + X$$
$$24 = 4X$$
$$6 = X$$

**3)**
$$5(4x + y) - 7(2x + 3y) - 6(x - 8y)$$
$$= 20x + 5y - 14x - 21y - 6x + 48y$$
$$= 0x + 32y$$

So, $A = 0$ and $B = 32$. Then $10A - 9B = 10(0) - 9(32) = 0 - 288$ $= -288$.

**Category 6**
**Team Round**
**Meet #1 - October, 2017**

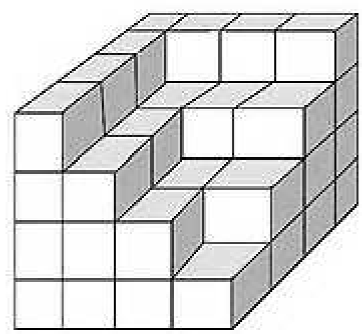
**1) What is the sum of all the prime numbers between 250 and 270?**

**2) If N = - 2, then find the value of** $N + N^2 + N^3 + N^4 + N^5 + N^6$ **.**

**3) The supplement of angle W is 14 degrees more than three times the complement of angle W. How many degrees are in the measure of the supplement of angle W ?**

**4) On an analog clock, both the minute hand and the hour hand rotate clockwise, with the minute hand rotating faster than the hour hand. At nine o'clock, the angle between the two hands is 90 degrees. How many degrees are in the angle formed by the two hands 25 minutes later if that angle is greater than 180 degrees?**

**5) Small wooden cubes are stacked to form this 3-D figure. Assume that all cubes in columns are supported from below, all the way to the base. How many of the small wooden cubes comprise the figure?**



**6) In the figure at the bottom of the page, AD = AE and AB = BF = CF = CE = DE. What is the value of x if it is the number of degrees in angle A ?**

**ANSWERS**

**1) _______**

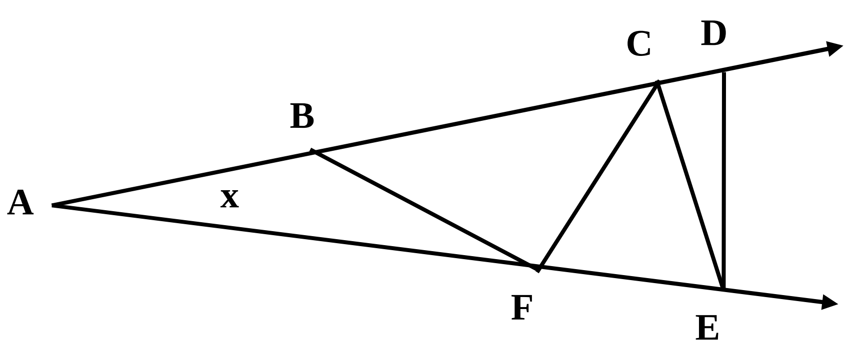
**2) _______**

**3) _______**

**4) _______**

**5) _______**

**6) _______**



**Solutions to Category 6**
**Team Round**
**Meet #1 - October, 2017**

| **ANSWERS** | |
|---|---|
| 1) | 1040 |
| 2) | 42 |
| 3) | 128 |
| 4) | 227.5 |
| 5) | 50 |
| 6) | 20 |

**1) 251 + 257 + 263 + 269 = 1040**
**All even numbers are divisible by 2.**
**253 = (11)(23); 261 = (3)(87); 267 = (3)(89).**
**259 = (7)(37); 255, 260, and 265 are divisible by 5.**

**2)** $N + N^2 + N^3 + N^4 + N^5 + N^6$
$= (-2) + (-2)^2 + (-2)^3 + (-2)^4 + (-2)^5 + (-2)^6$
$= -2 + 4 + (-8) + 16 + (-32) + 64 = 42$

**3) Let 90 - W = the complement of angle W**
**and 180 - W = the supplement of angle W**

$180 - W = 3(90 - W) + 14$
$180 - W = 270 - 3W + 14$
$2W = 104$
$W = 52$ **and**

**the supplement of W is 180 - W, or 180 - 52, which is 128 degrees.**

**4) The minute hand rotates 25 / 60 of 360 degrees (or 150 degrees) while the hour hand rotates 25 / 60 of 30 degrees (or 12.5 degrees), as each hour has 1/12 of 360, or 30 degrees. So, the 90-degree angle of the hands at 9 o'clock increases by 150 - 12.5, or 137.5 degrees. The new angle between the hands at 9:25 is 90 + 137.5 degrees, or 227.5 degrees.**

**5) The bottom level contains 4 x 4, or 16 cubes. The layer above it is missing one, so has 15 cubes. The layer above is missing four, so has 12 cubes. The top layer has 7 cubes. The total number of cubes is 16 + 15 + 12 + 7 = 50 cubes.**

**6) The short segments create a bunch of isosceles triangles whose base angles are congruent. Also, any exterior angle of a triangle is equal to the sum of its two remote interior angles. Labeling all angles in terms of x yields the following:**

**(NEXT PAGE)**

**a) Angle CBF is the exterior angle of triangle ABF, and equals**
**$x + x$, or $2x$ degrees.**
**b) Angle BFC is $180 - (2x + 2x)$, or $180 - 4x$ degrees.**
**c) Therefore, angle CFE and CEF each are equal to**

$$\begin{aligned} & 180 - (x + (180 - 4x)) \\ &= 180 - (180 - 3x) \\ &= 180 - 180 + 3x \\ &= 3x. \end{aligned}$$

**d) Angle FCE $= 180 - (3x + 3x)$, or $180 - 6x$.**
**e) Angles ECD and EDC each are equal to**

$$\begin{aligned} & 180 - (2x + (180 - 6x) \\ &= 180 - (180 - 4x) \\ &= 180 - 180 + 4x \\ &= 4x. \end{aligned}$$

**f) Angle CED is equal to $180 - (4x + 4x)$, or $180 - 8x$.**
**g) The base angles of isosceles triangle ADE are congruent, so**

$$\begin{aligned} 4x &= 3x + (180 - 8x) \\ 4x &= 180 - 5x \\ 9x &= 180 \\ x &= 20. \end{aligned}$$