

IMLEM Meet #3
January, 2017

Intermediate Mathematics League of Eastern Massachusetts



Category 1

Mystery

Meet #3 - January, 2017

1) In a row of 21 seats numbered consecutively from 1 to 21 at the movie theatre, I sit in seat #13. All seats in the row are occupied. The ratio of the number of people to my left to the number on my right is $A : B$ where $A < B$ and the ratio is in lowest terms. What is the value of B ?

2) If $F \text{ 🐱 } G$ means $\frac{F+G}{F-G}$, then what is the value of

$(6 \text{ 🐱 } 4) \text{ 🐱 } 3$?

3) A basic recipe for a delicious "punch" drink has the following ingredients:

- 1/2 gallon of apple juice
- 3 cups of lemon-lime soda
- 64 fluid ounces of pineapple juice
- 2 quarts of water
- 1 cup of lemon juice

1 gallon = 4 quarts = 8 pints = 16 cups = 128 fluid ounces.
How many quarts of punch will this recipe make ?

Answers

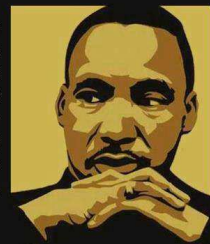
1) _____

2) _____

3) _____

Rarely do we find men who willingly engage in hard, solid thinking. There is an almost universal quest for easy answers and half-baked solutions. Nothing pains some people more than having to think.

• Martin Luther King, Jr.



Solutions to Category 1
Mystery
Meet #3 - January, 2017

1) If I sit in seat #13 out of 21 seats, then there are 8 seats to my left and 12 seats to my right, since $A < B$. The ratio $A:B = 8:12 = 2:3$. Therefore, $B = 3$.

2) $(6 \text{ 😈 } 4) = \frac{6+4}{6-4} = \frac{10}{2} = 5$.
 $(5 \text{ 😊 } 3) = \frac{5+3}{5-3} = \frac{8}{2} = 4$.

Answers	
1)	3
2)	4
3)	7

3) One strategy is to convert all quantities to fluid ounces:

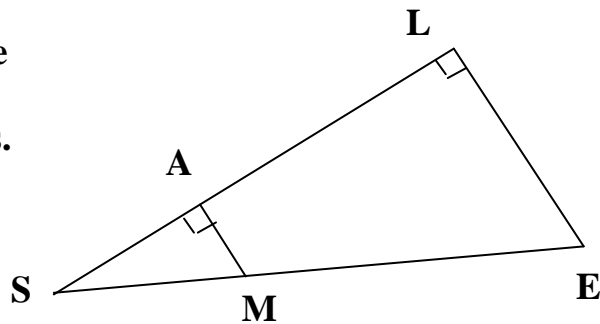
- 1/2 gallon of apple juice = $(1/2)(128) = 64$ fluid ounces.
- 3 cups of lemon-lime soda = $(3)(128/16) = 24$ fluid ounces.
- 64 fluid ounces of pineapple juice = 64 fluid ounces.
- 2 quarts of water = $(2)(128/4) = 64$ fluid ounces.
- 1 cup of lemon juice = $(1)(128/16) = 8$ fluid ounces.

The total number of fluid ounces is $64 + 24 + 64 + 64 + 8 = 224$.
This will make $224 / (128/4)$, or 7 quarts of punch.

Category 2
Geometry
Meet #3 - January, 2017

1) How many inches long is a rectangle whose diagonal is 20 inches and whose width is 12 inches?

2) How many centimeters are in the perimeter of triangle SLE ? All measurements are in centimeters. AS = 4, SM = 5, and LE = 9.



3) How many more diagonals does a convex 15-sided polygon have than a 12-sided regular polygon?

<u>Answers</u>	
1)	_____
2)	_____
3)	_____

Solutions to Category 2
Geometry
Meet #3 - January, 2017

- 1) The diagonal of the rectangle forms the hypotenuse of a right triangle. The width and length of the rectangle form the legs of the right triangle. Use the Pythagorean Theorem to find the length:

$$\begin{aligned}(\text{length})^2 + (\text{width})^2 &= (\text{hypotenuse})^2 \\(L)^2 + (12)^2 &= (20)^2 \\(L)^2 + 144 &= 400 \\L^2 &= 256 \\L &= 16\end{aligned}$$

Answers	
1)	16
2)	36
3)	36

- 2) Triangle ASM is similar to triangle LSE. To establish the scaling factor, we need to know the lengths of a pair of corresponding sides. We can find the length of AM because it is a leg of right triangle ASM. Using the Pythagorean Theorem, we find that $AM = 3$. The side of triangle LSE that corresponds to AM is $LE = 9$, so the scale is $9:3$, or $3:1$. Then $LS = (3)(AS) = (3)(4) = 12$ and $SE = (3)(SM) = (3)(5) = 15$. The perimeter of triangle SLE is $9 + 12 + 15 = 36$ centimeters.
- 3) The number of diagonals in a convex polygon, where $S =$ the # of sides, is given by the expression $(S)(S - 3) / 2$. So, for the convex 15-gon, the number of diagonals is $(15)(12) / 2 = 90$ and the number of diagonals in the 12-gon is $(12)(9) / 2 = 54$. The difference is $90 - 54$, or 36 . Alternatively, at any vertex of the convex 15-gon, you can draw diagonals to all 15 vertices except itself and two neighbors, hence $15 - 3 = 12$ diagonals per vertex. There are 15 vertices so 15×12 diagonals. But that counts all diagonals twice, at each end. So there are $(0.5)(15 \times 12)$ diagonals to the 15-gon. Same argument for the 12-gon.

Category 3
Number Theory
Meet #3 - January, 2017



1) What is the base 10 value of the base 3 numeral 100110 ?

2) Using the symbol \times as a sign of multiplication, what is the value of the following expression? Express your answer in scientific notation.

$$(6 \times 10^8) \times (7 \times 10^{-5}) \times (5 \times 10^0) \times (4 \times 10^{12})$$

3) Assume that a rocket can average a speed of 1.4×10^3 miles per hour in space. How many days would it take for this rocket to reach the moon if the moon is 2.38×10^5 miles from Earth on this date? Use the rounded figure that there are 24 hours in a day. Ignore fluctuations in distances due to the fact that the moon's orbit is not circular. Round your final answer to the nearest tenth of a day (as a decimal, but not in scientific notation).

<u>Answers</u>	
1)	_____
2)	_____
3)	_____

**Solutions to Category 3
Number Theory
Meet #3 - January, 2017**

1) 100110 (base 3) = $243 + 9 + 3 = 255$.

2) $(6 \times 10^8) \times (7 \times 10^{-5}) \times (5 \times 10^0) \times (4 \times 10^{12})$
= $(6 \times 7 \times 5 \times 4) \times (10^{8+(-5)+0+12})$
= 840×10^{15}
= 8.4×10^{17} (answer in scientific notation)

- 3) Divide the distance to the moon in miles by the average speed in mph to get the number of hours it takes to get to the moon. Divide that answer by 24 to get the number of days.

$$\frac{2.38 \times 10^5}{1.4 \times 10^3 \times 24} = \frac{1.7 \times 10^2}{24} = \frac{170}{24} = 7.083... \approx 7.1$$

So, it would take about 7.1 days (rounded to the nearest tenth of a day) for the rocket to reach the moon.

Answers

1) 255

2) 8.4×10^{17}

3) 7.1

Category 4
Arithmetic
Meet #3 - January, 2017

1) Evaluate: $7^0 + 6^1 + 5^2 + 4^3 + 3^4 + 2^5 + 1^6 + 0^7$

2) If $72 + \sqrt{N-3} = 78$, then what is the value of $N^2 - 17$?

3) What is the value of $\left(\sqrt{\left(\sqrt[3]{26} \right)} \cdot \sqrt{3^4} \cdot \sqrt[6]{1^4} \right)^3$?

ANSWERS

1) _____

2) _____

3) _____

Solutions to Category 4
Arithmetic
Meet #3 - January, 2017

Answers

$$\begin{aligned} 1) \quad & 7^0 + 6^1 + 5^2 + 4^3 + 3^4 + 2^5 + 1^6 + 0^7 \\ & = 1 + 6 + 25 + 64 + 81 + 32 + 1 + 0 \\ & = 210 \end{aligned}$$

$$\begin{aligned} 2) \quad & 72 + \sqrt{N-3} = 78 \\ & \sqrt{N-3} = 6 \\ & N-3 = 36 \\ & N = 39 \end{aligned}$$

$$\text{So, } N^2 - 17 = (39)^2 - 17 = 1521 - 17 = 1504.$$

$$\begin{aligned} 3) \quad & \left(\sqrt{\sqrt[3]{26}} \cdot \sqrt{34} \cdot \sqrt[6]{14} \right)^3 \\ & = \left(\sqrt{2^{6/3} \times 9 \times 1} \right)^3 \\ & = \left(\sqrt{2^2 \times 9} \right)^3 \\ & = (2 \times 9)^3 \\ & = (18)^3 \\ & = 5832 \end{aligned}$$

1) 210

2) 1504

3) 5832

Category 5

Algebra

Meet #3 - January, 2017

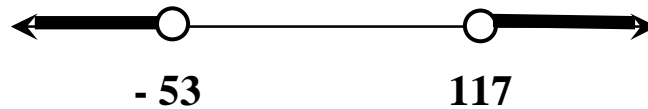
- 1) What is the sum of the two different values of X that make this absolute value equation true?

$$|2x + 6| = 28$$

- 2) Solve this inequality: $-4(3C - 5) < 92$

- 3) The graph below represents all values of A that make the accompanying absolute value inequality true. What is the absolute value of the difference $B - C$?

$$|A - B| > C$$



ANSWERS

1) _____

2) _____

3) _____

**Solutions to Category 5
Algebra
Meet #3 - January, 2017**

Answers

1) **- 6**

2) **$C > - 6$
or
 $- 6 < C$**

3) **53**

1) **Either $2x + 6 = 28$ or $2x + 6 = -28$
 $2x = 22$ or $2x = -34$
 $x = 11$ or $x = -17$**

Then the sum is $11 + (-17) = -6$

2) **$-4(3C - 5) < 92$
 $-4(3C) - (-4)(5) < 92$
 $-12C + 20 < 92$
 $-12C < 72$
 $C > -6$**

**original inequality
Distribute the -4.
having multiplied
subtracting 20 from both members
Dividing both members by -12
reverses the sense of order.**

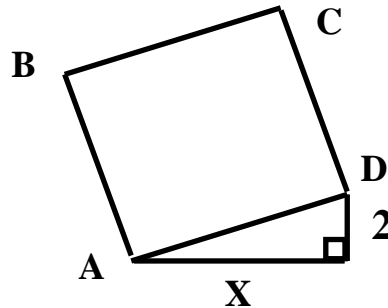
- 3) **The midpoint of -53 and 117 is $(-53 + 117) / 2$, or $64 / 2$, or 32.
That midpoint is 85 units from 117 and is also 85 units from -53.
The absolute value inequality can be stated thusly: "The distance between any point, A, and 32, is greater than or equal to 85 units." In symbols, this inequality looks like this: $|A - 32| > 85$. So, $B = 32$ and $C = 85$.
The absolute value of the difference $B - C$ is 53!**

Category 6
Team Round
Meet #3 - January, 2017

Each of the following nine problems is worth four points.

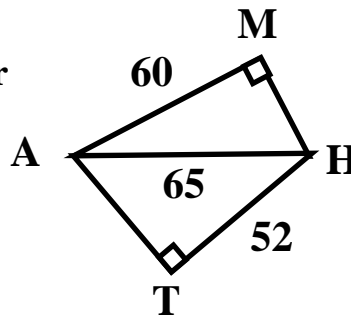
- 1) Caitlin has 2.75 pounds of candy. She puts the candy into bags that can each hold 3 ounces. How many bags can Caitlin completely fill ?
- 2) Samantha saved \$2.80 by purchasing an 8-ticket deal to the bowling alley for \$44.80. What is the regular cost for one ticket ?

3) In terms of X , what is the area of square ABCD ? The answer should not include a radical.



4) What is the 2017th digit to the right of the decimal point in the decimal representation of the fraction $3/7$?

5) How many inches are in the perimeter of quadrilateral MATH ? All measurements are in inches.



ANSWERS

1) _____

2) \$ _____

3) _____

4) _____

5) _____

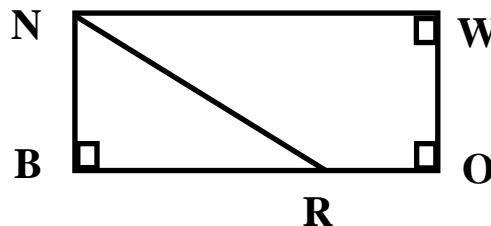
6) _____

7) _____

8) _____

9) _____

6) If the area of triangle NBR is 60 square feet, then how many feet are in the perimeter of trapezoid WORN ?
 $BR = 15$ and $NW = 25$.



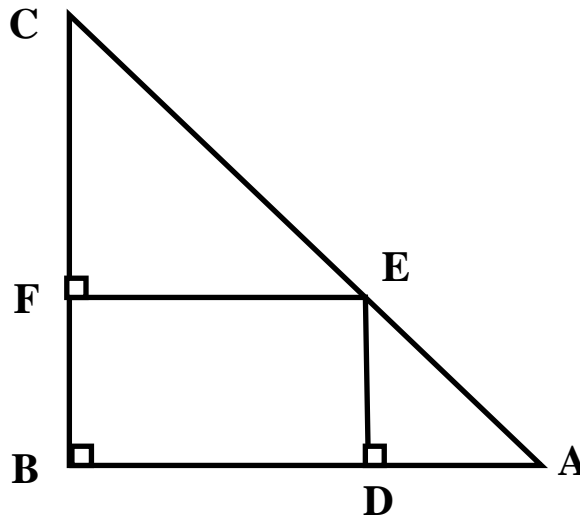
7) How many whole numbers are between $\sqrt[3]{152}$ and $\sqrt[4]{35,168}$?

*** Note: Questions #8 and #9 are on the next page.

8) How many integer values of Y make the following absolute value inequality true?

$$\left| \frac{17}{Y} \right| \geq 3$$

9) In the figure, $\frac{DE}{EF} = \frac{2}{3}$. If $AC = 30$, and $AB = BC$, then what is the length of EC ?



**Solutions to Category 6
Team Round
Meet #3 - January, 2017**

ANSWERS

- 1) 14
- 2) 5.95
- 3) $X^2 + 4$
- 4) 4
- 5) 176
- 6) 60
- 7) 8
- 8) 10
- 9) 18

- 1) There are 16 ounces in a pound, so 2.75 pounds = $(2.75)(16)$, or 44 ounces. $44 / 3 = 14.6666 \dots$ bags of candy, only 14 of which are full.
- 2) Add \$2.80 to \$44.80 to get the regular cost of 8 tickets = \$47.60. One ticket at that regular price is $\$47.60 / 8$, or \$5.95.
- 3) Use the Pythagorean Theorem to find the length of hypotenuse AD, so $AD = \sqrt{X^2 + 4}$. Then the area of square ABCD is the square of the length of AD, so is equal to $X^2 + 4$.
- 4) $3 / 7 = 0.428571428571428571 \dots$
The repeating pattern contains six digits, so divide 2017 by 6 to get how many blocks of six digits there are. The remainder is the critical piece of information, as it tells us how many digits to count past the digit 1. $2017 / 6 = 336$ with remainder 1. So, the 2017th digit is 4.

- 5) Use the Pythagorean Theorem to find the missing lengths MH and AT. People familiar with some of the basic Pythagorean triples may notice that, for triangle AHM, the given lengths are multiples (x5) of two sides of the basic triple 5-12-13, thus making $MH = (5)(5)$, or 25. Similarly for triangle ATH, the two given sides are multiples (x13) of the basic triple 3-4-5, thus making $AT = (13)(3)$, or 39. So, the perimeter of MATH is $60 + 25 + 52 + 39$, or 176 inches.
- 6) Area of triangle NBR = $(1/2)(\text{base})(\text{altitude}) = (1/2)(NB)(15) = 60$, so $NB = 8$. We need to find the length of NR as it is one side of the required trapezoid for which we will find the perimeter. Using the Pythagorean Theorem, $NR = 17$. $RO = 25 - 15$, or 10. The perimeter of WORN is, therefore, $25 + 8 + 10 + 17 = 60$ feet.

(NEXT PAGE FOR THE REST OF THE SOLUTIONS)

- 7) The approximate values of the radicals can be found through good approximation and number sense. The cube root of 152 will be found to be somewhere between 5 and 6, so is greater than 5. The fourth root of 35,168 is somewhere between 13 and 14, so is greater than 13. The integers in between are 6, 7, 8, 9, 10, 11, 12, and 13, so there are eight.
- 8) The values of Y that satisfy this inequality are -5, -4, -3, -2, -1, 1, 2, 3, 4, and 5, so there are ten integer values. (Students may inadvertently include the number zero as a solution because it fits the pattern. However, division by zero is a "mortal sin of mathematics.")
- 9) This problem is much easier than it appears at first sight. Once it is realized that there are three isosceles right triangles - ABC, EFC, and ADE. So, the hypotenuse AC of triangle ABC is divided into the ratio 2:3, thus making $AE = 12$ and $EC = 18$.