

IMLEM Meet #2
December, 2016

Intermediate Mathematics League of Eastern Massachusetts



Category 1

Mystery

Meet #2 - December, 2016

- 1) Each member of the Tiny Tumblers gymnastics team wears a different 2-digit number between 1 and 100 on her uniform. Each number is a multiple of 6. What is the greatest number of girls that belong to the Tiny Tumblers?
- 2) What is the units digit (the "ones" place) of 3^{75} ?
- 3) Becca Pizzi of Belmont, Massachusetts, recently set a world record in the World Marathon 7-7-7 Challenge, running seven marathons (each 26+ miles) in seven consecutive days on all seven continents. Her times for the seven marathons - in hours, minutes and seconds - are listed in the chart below. How much faster, in seconds, was her fastest time than her slowest time? (Note: Becca pulled a muscle during the first mile of that slowest race, yet completed the marathon for the win.)

<u>Answers</u>	
1)	_____
2)	_____
3)	_____

<u>Continent</u>	<u>hours</u>	<u>minutes</u>	<u>seconds</u>
Antarctica	3	57	19
South America	3	44	18
North America	3	41	20
Europe	3	48	59
Africa	3	50	47
Asia	4	14	41
Australia	4	8	51

Solutions to Category 1

Mystery

Meet #2 - December, 2016

Answers

1) 15

2) 7

3) 2001

1) The largest multiple of 6 less than 100 is $(6)(16)$, or 96. So, there are 16 multiple of 6 between 1 and 100, except that the first one, 6, only has one digit. So, there are 15 two-digit multiples of 6 between 1 and 100.

2) $3^1 = 3$, $3^2 = 9$, $3^3 = 27$, $3^4 = 81$, $3^5 = 243$, and then the units digit repeats in this pattern, starting with the very first term:
3 9 7 1 3 9 7 1 ... So, the pattern repeats after every four digits.

Divide 75 by 4 to get how many groups of four digits there are, which is 18 groups of four digits, with a remainder of 3. The 72nd power of three has a units digits of 1. The next few end in 3, 9, and 7.

3^{75} has a units digit of 7.

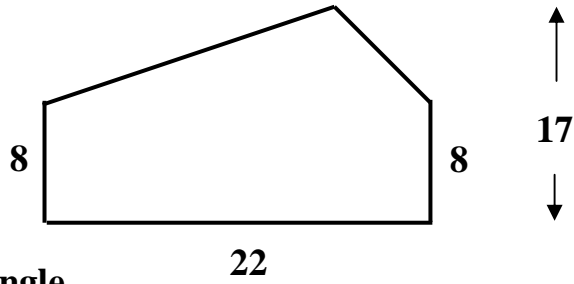
3) Becca's fastest time of $3:41:20 = 3(3600) + 41(60) + 20$ seconds, or $10,800 + 2460 + 20$, or 13,280 seconds.

Becca's slowest time of $4:14:41 = 4(3600) + 14(60) + 41$ seconds, or $14,400 + 840 + 41 = 15,281$ seconds.

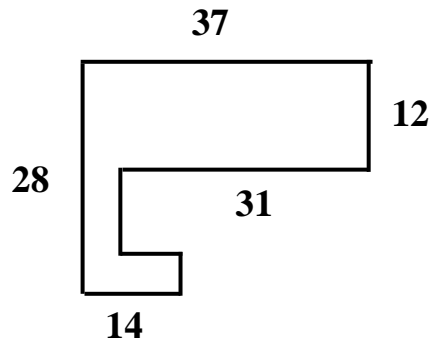
The difference between the fastest and slowest times = $15,281 - 13,280$ or 2001 seconds.

Category 2
Geometry
Meet #2 - December, 2016

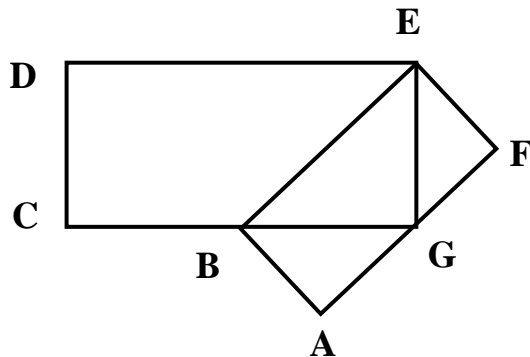
- 1) How many square inches are in the area of this pentagon? All measurements are in inches. Each of the base angles is a right angle.



- 2) How many centimeters are in the perimeter of this polygon? All angles are right angles and all segment lengths are in centimeters.



- 3) Rectangles CGED and ABEF intersect at points B, G, and E. B is the midpoint of CG. How many square inches are in the area of quadrilateral BCDE? The area of triangle ABG is 24 square feet and the area of triangle GFE is 38 square feet.



Answers

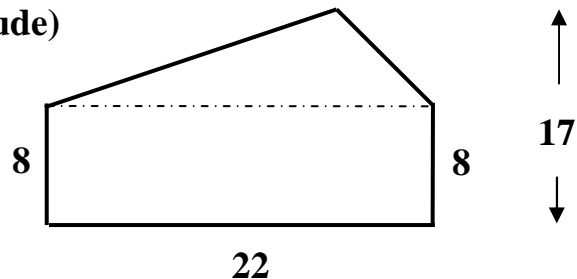
- 1) _____
 2) _____
 3) _____

Solutions to Category 2
Geometry
Meet #2 - December, 2016

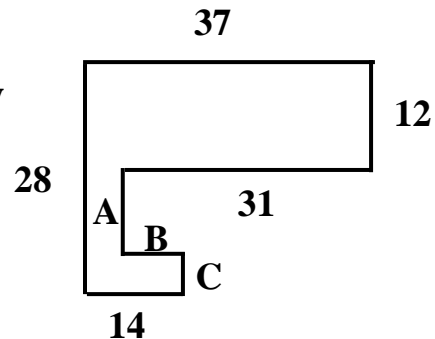
<u>Answers</u>	
1)	275
2)	146
3)	26,784

- 1) Divide the pentagon into a triangle and a rectangle with a horizontal line parallel to the base, making a 22 x 8 rectangle and a triangle whose base is 22 and whose altitude is (17 - 8), or 9. The total area is

$$\begin{aligned}
 & \text{rectangle} + \text{triangle} \\
 &= (\text{base})(\text{height}) + \frac{1}{2} (\text{base})(\text{altitude}) \\
 &= (22)(8) + \frac{1}{2} (22)(9) \\
 &= 176 + 99 \\
 &= 275 \text{ square inches.}
 \end{aligned}$$



- 2) There are three unlabelled segments. $A + C + 12$ is the same length as the segment marked 28. The width of the skinny column on the left = $37 - 31$, or 6. $14 - 6 = B = 8$. So, the perimeter = $(14 + 28 + 37 + 31) + B + (A + C + 12)$ = $110 + 8 + (28)$ = 146 centimeters.



- 3) Triangle BGE is half of rectangle ABEF. The sum of triangles ABG and GFE is also half of rectangle ABEF. So, the area of triangle BGE = $24 + 38 = 62$ square feet. With B as the midpoint of CG, triangle BGE is $\frac{1}{4}$ of rectangle CGED. So, quadrilateral BCDE is the other $\frac{3}{4}$ of rectangle CGED . . . or, another way, rectangle CGED is $3(\text{area of triangle BGE}) = 3(62) = 186$ square feet. The question asks, "How many square INCHES are in the area of quadrilateral BCDE," so we must convert 186 square feet to square inches: $186 \text{ square feet} \times 144 \text{ square inches per square foot} = 26,784 \text{ square inches.}$

Category 3

Number Theory

Meet #2 - December, 2016

- 1) Juan sees Thieu every 12 days and he sees Thrye every 28 days. Juan, Thieu, and Thrye are having lunch today. In how many days will they next see each other ?
- 2) $2520 = 2^3 \times 3^2 \times A \times B$ where A and B are prime numbers. What is the value of $A + B$?
- 3) Find the smallest whole number that is a common multiple of 18 and 40 and is also a perfect square.

Answers

1) _____

2) _____

3) _____

Solutions to Category 3
Number Theory
Meet #2 - December, 2016

<u>Answers</u>	
1)	84
2)	12
3)	3600

- 1) Find the LCM of 12 and 28. The numbers are small enough that the answer can be achieved by listing multiples of 12 and 28:

multiples of 12: 12 24 36 48 60 72 84

multiples of 28: 28 56 84

- 2) The prime factorization of $2520 = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7$.
 $A + B = 5 + 7 = 12$.

- 3) $18 = 2 \times 3 \times 3$

$40 = 2 \times 2 \times 2 \times 5$

Their LCM is $2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$.

The smallest multiple of this LCM must have an even number of like factors, so $= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 = 3600$.

Category 4
Arithmetic
Meet #2 - December, 2016

1) If $\frac{N+3}{28} = \frac{5}{7}$ then what is the value of N ?

2) What whole number is 60% of $\frac{5}{9}$ of 0.6 of 780 ?

3) If the repeating decimal $3.81111\dots$ were increased by a value of T, the total would be the mixed numeral $4\frac{2}{3}$. Express T as a common fraction.

Answers

1) _____

2) _____

3) _____

Solutions to Category 4
Arithmetic
Meet #2 - December, 2016

1) $\frac{N+3}{28} = \frac{5}{7}$ Scaling $5/7$ by a factor of $4/4$ yields

$$4(5) = N + 3$$

$$20 = N + 3$$

$$17 = N$$

2) 60% of $\frac{5}{9}$ of 0.6 of 780

$$= 0.6 \times \frac{5}{9} \times 0.6 \times 780$$

$$= \frac{6}{10} \times \frac{5}{9} \times \frac{6}{10} \times 780$$

After doing some "cancelling" to reduce the product to lowest terms, the final answer is 156.

3) $3.811111 \dots + T = 4\frac{2}{3}$

$$T = 4\frac{2}{3} - 3.811111 \dots$$

$$= 4.666666 \dots - 3.811111 \dots$$

$$= 0.855555 \dots$$

If $X = 0.855555 \dots$ and $10X = 8.55555 \dots$, then the difference is

$$9X = 7.7, \text{ and } X = 7.7 / 9 \text{ or } 77 / 90.$$

Answers

1) 17

2) 156

3) $\frac{77}{90}$

Category 5

Algebra

Meet #2 - December, 2016

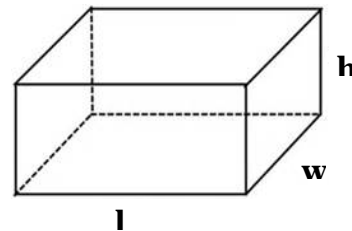
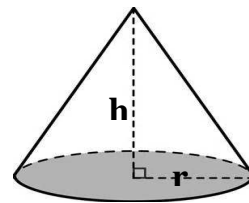
- 1) This puzzle was circulating throughout the Internet a few weeks ago. What is the value of the last line ?

$$\begin{array}{l} \text{🍒} \text{🍒} \text{🍒} = 27 \\ \text{🍀} \text{🍀} \text{🍀} \text{🍒} = 24 \\ \text{🍒} \text{🍀} \text{🍊} \text{🍊} = 96 \\ \text{🍊} \text{🍒} \text{🍀} = ??? \end{array}$$

- 2) Moe is three times as old now as Larry will be four years from now. If Larry is five years old now, then how many years old will Moe be in seven years?

- 3) The formula for the total surface area (TSA) of a cone is

$TSA = \pi r^2 + \pi r h$ where h = the height and r = the radius of the circular base. Find the smallest number of cubic inches in the volume of a rectangular box that tightly contains the cone whose total surface area is 77π square inches and the diameter of its circular base is 14 inches. The formula for the volume of a rectangular box is $V = l w h$ where l = the length of the base, w = the width of the base, and h = the height of the box. Assume that the circular base of the cone occupies the same plane as the bottom of the rectangular box.



Answers

1) _____

2) _____

3) _____

Solutions to Category 5

Algebra

Meet #2 - December, 2016

- 1) If three cherries = 27, then one cherry = 9.
If three clovers and a cherry = 24, then three clovers = 15 and one clover = 5. Two oranges, a clover, and a cherry make 96, so two oranges = 82 and one orange = 41. Finally, one orange, one cherry, and one clover make $41 + 9 + 5$, or 55.

Equally valid is a solution where the implied operation is multiplication, yielding one cherry = 3, one clover = 2, one orange = 4, and the final answer is $3 \times 2 \times 4 = 24$.

So, either answer, 55 or 24, is correct.

- 2) Start with Larry being 5 years old now. Then work to the right. Then go to Moe - now he is 4(9), or 36. Then work to the right. Nyuk nyuk!

	<u>Now</u>	<u>in 4 yrs</u>	<u>in 7 yrs</u>
Moe	$3(9) = 27$	31	34
Larry	5	9	12

- 3) Use the formula for the TSA of the cone to find the height of the cone. This will also be the height of the box. The length and width of the box are each the diameter of the cone.

$$\begin{aligned} \text{TSA (cone)} &= \pi r^2 + \pi r h = \pi(7^2) + \pi(7)(h) = 77\pi \\ &\pi(49) + \pi(7)(h) = 77\pi \end{aligned}$$

$$\begin{aligned} \text{Divide both sides by pi :} & \quad 49 + 7h = 77 \\ & \quad 7h = 28 \\ & \quad h = 4. \end{aligned}$$

The length of the rectangular box = the width = 14, and $h = 4$.
The volume of the rectangular box = $l w h = (14)(14)(4) = 784$ cubic inches.

Answers

1) 55

or

24

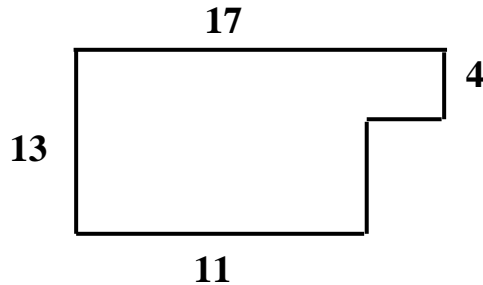
2) 34

3) 784

Category 6
Team Round
Meet #2 - December, 2016

Each of the following nine problems is worth four points.

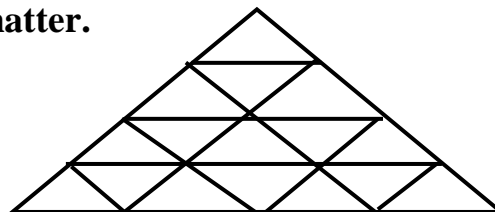
- 1) What is the greatest common factor (GCF) of 18, 42, and 72 ?
- 2) How many square feet are in the area of this figure? All angles are right angles and all lengths are in feet.



- 3) How many whole numbers are greater than 238 but less than 715 ?
- 4) Ninety-six days after Thursday is what day of the week ?
- 5) The chickens in Henry's hen house laid seven dozen dozen eggs last week. The ducks in Quimby's Quackery laid 3^6 eggs last week. How many more eggs were laid by the chickens than the ducks ?
- 6) How many triangles of any size are in the diagram at the bottom of the page ?

<u>ANSWERS</u>	
1)	_____
2)	_____
3)	_____
4)	_____
5)	_____
6)	_____
7)	_____
8)	_____
9)	_____

- 7) There are 42 people on a bus. At the first bus stop, three people get off and then two people get on. This happens at every bus stop. At which stop number is the bus empty ?
- 8) I have eight siblings. The sum of our ages is 116 years. In seven years, what will be the sum of our ages ?
- 9) How many different ways can six identical guitars be distributed to Crosby, Stills, and Nash so that each musician gets at least one guitar ? The order of the distribution does not matter.



**Solutions to Category 6
Team Round
Meet #2 - December, 2016**

ANSWERS

- 1) 6
- 2) 167
- 3) 476
- 4) Tuesday
- 5) 279
- 6) 27
- 7) 40
- 8) 179
- 9) 10

- 1) Because the answer is relatively small, it can be found by the "listing factors" method:
factors of 18: 1, 2, 3, 6, 9, 18
factors of 42: 1, 2, 3, 6, 7, 14, 21, 42
factors of 72: 1, 2, 3, 4, 6, 8, 9, 12, 24, 36, 72
common factors: 1, 2, 3, 6. So, GCF = 6.
- 2) The missing lower right-hand corner measures $(17 - 11) \times (13 - 4)$, or $6 \times 9 = 54$. The area of the given figure can be calculated by subtracting this area from (17×13) , or $221 - 54 = 167$ square feet.
- 3) There are $A - B - 1$ whole numbers between A and B. $715 - 238 - 1 = 476$.
- 4) Divide 96 by the number of days in a week (7), then take the remainder and add that number of days to Thursday. $96 / 7 = 13$ with remainder 5. Five days after Thursday is Tuesday.

5) Seven dozen dozen = $(7)(12)(12) = 1008$. $3^6 = 729$.
 $1008 - 729 = 279$.

- 6) There are 16 of the smallest triangle, 7 of the next size up, consisting of four tessellated triangles (Including the inverted one in the lower half of the figure), 3 of the next size up, consisting of nine tessellated triangles, and 1 of the largest one that contains all of the smallest triangles. $1 + 3 + 7 + 16 = 27$.

The remaining three solutions are on the next page.

7) This solution has an unexpected twist:

<u>Stop #</u>	<u># of people after 3 get off</u>	<u># of people after two get on</u>
1	39	41
2	38	40
3	37	39

This pattern continues, but it gets interesting as the bus becomes nearly empty. Although the following equation is, at first glance, appropriate for the problem, it does not acknowledge that the bus could be empty after 3 people get off, but before two more get on:

$$42 - 3X + 2X = 0$$

$$42 - X = 0$$

$$X = 42, \text{ indicating that the bus empties at stop \#42.}$$

However, the bus actually empties before stop #42, just after 3 people get off and just before 2 people are about to get on:

<u>Stop #</u>	<u># of people after 3 get off</u>	<u># of people after two get on</u>
37	3	5
38	2	4
39	1	3
40	0	

After stop #40, the bus is empty!

8) If I have 8 siblings, then there are 9 of us. If the sum of our current ages is 116, then in 7 years, the sum of our ages will be $116 + 7(9)$, or $116 + 63 = 179$ years.

9) A chart can be set up to organize the ways:

<u>Crosby</u>	<u>Stills</u>	<u>Nash</u>
1	1	4
1	2	3
1	3	2
1	4	1
2	1	3
2	2	2
2	3	1
3	1	2
3	2	1
4	1	1

There are 10 ways for the guitars to be distributed.