IMLEM Meet #1
October, 2016

Intermediate Mathematics League of Eastern Massachusetts

Category 1
Mystery
Meet #1 - October, 2016

1) **I have $289 but I need a total of $2016. I sold some of my things on eBay and received $327. How much more money do I need? Express your answer as a whole number of dollars. (Note: For example, if the answer were three dollars, then it would be written as the whole number 3 and not as 3.00 as it would under other circumstances.)**

2) **I read three times as many pages every day as I read on the day before. If I read the first two pages on Wednesday and the next six pages on Thursday, then on what day of the week will I read the 700th page of <u>Harry Potter and the Order of Phoenix</u>?**

3) **Five straight lines are painted onto a rectangular soccer field, separating members of a soccer team into their own spaces, one player per space. What is the greatest number of players that can be on this soccer team?**

<u>Answers</u>

1) _____

2) _____

3) _____

Solutions to Category 1
Mystery
Meet #1 - October, 2016

Answers
1) 1400
2) Monday
3) 16

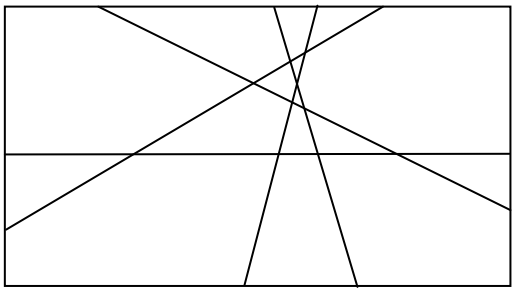
1) Subtract the total assets from the needed amount:
2016 - (289 + 327) = 2016 - 616 = 1400.

2)

Day:	Wed	Thu	Fri	Sat	Sun	Mon
pages read:	2	6	18	54	162	486
total read:	2	8	26	80	242	728

By Sunday, only 242 pages have been read. However, on Monday, there were enough pages read (486) to have reached page 700, since the total amassed by Monday is 728 pages.

3) Making a drawing can be effective, first by drawing two intersecting lines, followed by a third line that does not pass through the point of intersection of the first two lines. Each subsequent line should be drawn to maximize the number of spaces into which the plane (or rectangle, in this case) is divided. An example is shown below.



Another strategy is to count the number of spaces into which the rectangle is divided, keeping track as each new line is added to the scene, while looking for a pattern in the numbers:

Number of lines:	1	2	3	4	5
Number of spaces:	2	4	7	11	16

Another line of reasoning (pun intended) is that

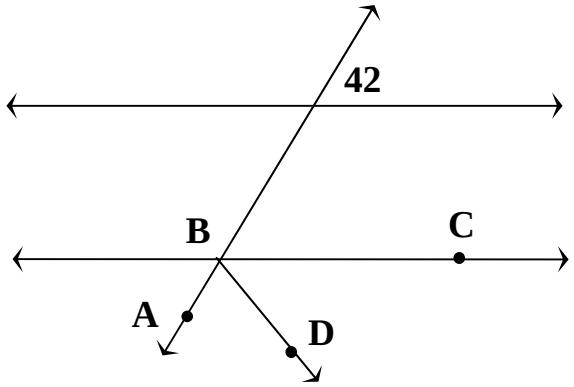
- **when you draw one line in the rectangle, it can cross zero other lines,**
- **when you draw the second line in the rectangle, it can cross, at most, one other line,**
- **when you draw the third, it can cross, at most, two other lines,**
- **when you draw the fourth, it can cross, at most, three other lines,**
- **when you draw the fifth, it can cross, at most, four other lines.**

Assume you make the maximum number of intersections each time you draw.

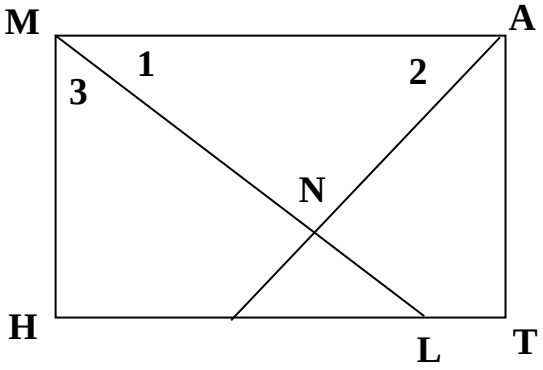
Category 2
Geometry
Meet #1 - October, 2016

For all problems below, angles are marked in degrees as shown. ***Figures are not necessarily drawn to scale.***

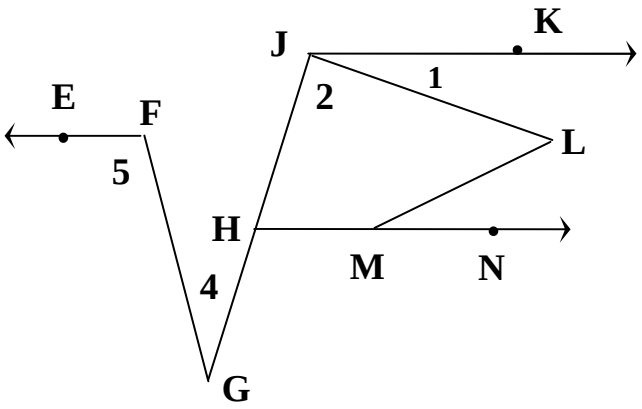
1) Two horizontal lines are parallel. Ray BD bisects angle ABC. How many degrees are in the measure of acute angle CBD?



2) Angle 1 is the complement of angle 2. MATH is a rectangle. Angle 3 measures 57 degrees. How many degrees are in the measure of angle NLT ?



3) Ray FE is parallel to ray JK and to ray HM. Angle 2 is twice the measure of angle 1. How many degrees are in the measure of acute angle L if angle LMN = 41 degrees, angle 4 = 38 degrees, and angle 5 = 116 degrees?



Answers

1) _____

2) _____

3) _____

Solutions to Category 2
Geometry
Meet #1 - October, 2016

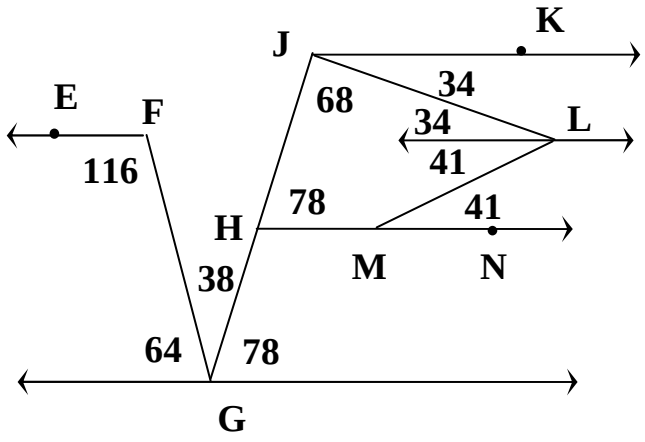
Answers
1) 69
2) 147
3) 75

1) Angle ABC is the supplement of the 42-degree angle and equals 138 degrees. Bisecting angle ABC yields a pair of 69-degree angles.

2) By definition, each vertex of the rectangle measures 90 degrees. Angles 3 and 1 are complementary, as are angles 1 and 2 (given), so angles 3 and 2 are equal. Angle 3 = 57 degrees, so does angle 2, therefore angle NAT = 33 degrees. Now, referring to quadrilateral NATL: Angle ANL + angle NAT + angle T + angle NLT = 360

$$90 + 33 + 90 + \text{angle NLT} = 360$$
$$213 + \text{angle NLT} = 360$$
$$\text{angle NLT} = 147$$

3) The key to solving this puzzle lies in the drawing of two lines, each parallel to the given parallel lines, passing through the points G and L. Using the following properties of parallel lines being cut by transversals produces these properties and allow for filling in the diagram with the given angle measures: 1) collinear angles have a sum of 180 degrees, and 2) alternate interior angles are congruent, and 3) same side interior angles are supplementary. Angle L = 34 + 41, or 75 degrees.



Category 3
Number Theory
Meet #1 - October, 2016

1) The factors of a positive whole number, N, include the numbers 1, N, and all positive numbers, W, such that N divided by W is a positive whole number. How many factors of 36 are also multiples of 4 ?

2) There are three whole numbers, A, B, and C, that are between 120 and 130 that are each the product of exactly two different prime numbers. What is the sum A + B + C ?

3)  **is a positive integer greater than 1.**

 **is the square of an integer.**

 **is the cube of an integer.**

What is the smallest possible number of different positive factors (divisors) that  **can have ?**

Answers

1) _____

2) _____

3) _____

Solutions to Category 3
Number Theory
Meet #1 - October, 2016

1) The factors of 36 are: 1, 2, 3, 4, 6, 9, 12, 18, 36. Those factors that are multiples of four are: 4, 12, and 36. So, there are three.

2)

121 = 11 x 11	**No - the factors must be different.**
122 = 2 x 61	**Yes - both 2 and 61 are prime.**
123 = 3 x 41	**Yes - both 3 and 41 are prime.**
124 = 2 x 2 x 31	**No - too many factors.**
125 = 5 x 5 x 5	**No - too many factors.**
126 = 2 x 3 x 3 x 7	**No - too many factors.**
127 = prime	**No - prime**
128 = 2 x 2 x 2 x 2 x 2 x 2 x 2	**No - too many factors.**
129 = 3 x 43	**Yes - both 3 and 43 are prime.**

Therefore, A + B + C = 122 + 123 + 129 = 374.

3) Sixty-four is the smallest positive integer that meets all the criteria. The factors of 64 are: 1, 2, 4, 8, 16, 32, and 64, so there are seven in all.

Answers

1) 3

2) 374

3) 7

Category 4
Arithmetic
Meet #1 - October, 2016

1) Using the standard order of operations, find the value of $3^4 + 4^3$ **.**

2) Evaluate: $\left(17 - \left[9 \cdot (68-49)^2 - 4\left(2^6 - 6^2\right)^2\right]\right)^2$

3) Consider the set H = { 5, 8, 10, 3, 10, 5, 17, 10, 7 }.
Let A = the mean of set H
B = the median of set H
C = the mode of set H.
What is the mean of A, B, and C ? Round your answer to the nearest tenth.

Answers

1) _____

2) _____

3) _____

Columbus Day, celebrating the landing in the Americas of the Spanish explorer, Christopher Columbus, falls on the second Monday of October.

Solutions to Category 4
Arithmetic
Meet #1 - October, 2015

Answers
1) 145
2) 9216
3) 8.8

1) $3^4 + 4^3$

= (3)(3)(3)(3) + (4)(4)(4)

= 81 + 64

= 145

2) $\left(17-\left[9\cdot(68-49)^2-4\left(2^6-6^2\right)^2\right]\right)^2$

= $\left(17-\left[9\cdot(19)^2-4(64-36)^2\right]\right)^2$

= $\left(17-\left[9\cdot 361-4(28)^2\right]\right)^2$

= $\left(17-\left[3249-4(784)\right]\right)^2$

= $\left(17-\left[3249-3136\right]\right)^2$

= $\left(17-\left[113\right]\right)^2$

= $(-96)^2$

= 9216

3) Arranging the nine numbers in order, from smallest to largest:

H = { 3, 5, 5, 7, 8, 10, 10, 10, 17 }.

Then A = the mean of H = (3+5+5+7+8+10+10+10+10+17) / 9 = 75 / 9 = $8.\overline{3}$ **.**
B = the median of H = the middle number = 8.
C = the mode of H = the most frequent value = 10.

The mean of A, B, and C = $(8.\overline{3}$ **+ 8 + 10) / 3 =** $26.\overline{3}$ **/ 3 = 8.77 . . .** $\approx$ **8.8.**

Category 5
Algebra
Meet #1 - October, 2016

1) If $3N - 15 = 24$ and $6(2M + 7) = 18$, then what is the value of $N - M$?

2) An identity is an equation that is true for all values of the variable. What value of C makes the following equation an identity in the variable H ?

$$5(H + 2) + 11 = 3(8 + H) + 2H + C$$

3) The symbol "¢" is used in the following equation as a sign of multiplication. What value of E makes the equation true?

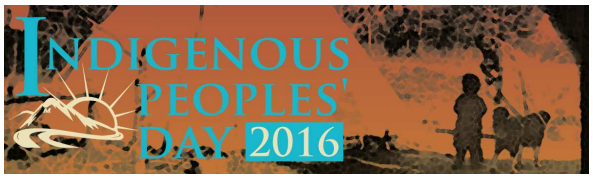
$$8 ¢ 15 ¢ 21 ¢ 100 ¢ E = 4 ¢ 30 ¢ 63 ¢ 1000$$

Answers

1) _____

2) _____

3) _____



The UN declared August 9 as Indigenous Peoples' Day," celebrating the lives of Native Peoples around the globe. In the USA, many states now celebrate this holiday concurrently with Columbus Day.

Solutions to Category 5
Algebra
Meet #1 - October, 2016

1) $3N - 15 = 24$ and $6(2M + 7) = 18$

$3N = 39$ $\quad$ $12M + 42 = 18$

$N = 13$ $\quad$ $12M = -24$

$M = -2$

Therefore, $N - M = 13 - (-2) = 15$.

Answers
1) 15
2) - 3
3) 30

2) $5(H + 2) + 11 = 3(8 + H) + 2H + C$

$5H + 10 + 11 = 24 + 3H + 2H + C$

$5H + 21 = 5H + 24 + C$

$21 = 24 + C$

$-3 = C$

3) Although there is a lot of arithmetic involved here, there is a more simplified approach utilizing factorization. A number on one side of the equation may be expressed as a factorization of a number on the other side. For example, the 8 on the left side can be expressed as 2 x 4, where the number 4 is on the right side. The entire equation could be expressed as such:

$8 ¢ 15 ¢ 21 ¢ 100 ¢ E = 4 ¢ 30 ¢ 63 ¢ 1000$

$(2 ¢ 4) ¢ 15 ¢ 21 ¢ 100 ¢ E = 4 ¢ (2 ¢ 15) ¢ (3 ¢ 21) ¢ (10 ¢ 100)$

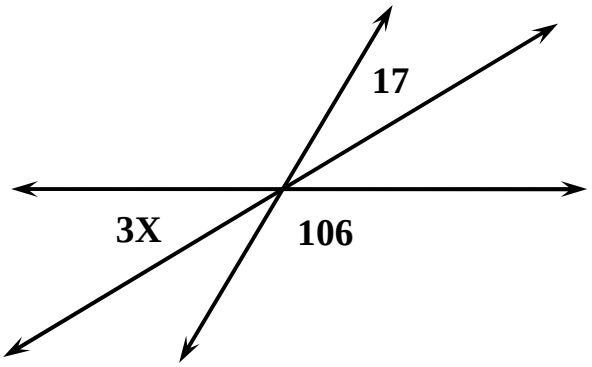
Now divide both sides by the product of the common factors of $2 ¢ 4 ¢ 15 ¢ 21 ¢ 100$, leaving us with the following equation:

$E = 3 ¢ 10$

$E = 30$

Category 6
Team Round
Meet #1 - October, 2016

1) **Three lines intersect at a common point and angle degree measures are labeled as shown. What is the value of X ?**



2) **Fifteen carpenters work for a total of 24 hours to frame a house. How many hours would it take for 20 workers to complete the same job if all workers work at the same rate?**

3) **M ♥ P means** $3P^2-4M+7$. **G ♣ A means** $-4G^3+5AG$. **What is the value of (- 6 ♣ - 10) - (8 ♥ - 9) ?**

4) **Farmer Fred's garden has 3 lettuce plants for every 7 tomato plants and 4 tomato plants for every 11 cucumber plants. If Fred has 240 lettuce plants, then how many cucumber plants does he have?**

5) **What value of N makes the following equation true?**

$$7(3N + 2) + 5(4N - 1) - 8(9N - 10) = 6(7 - 11N) - 4(2N + 8) + 337$$

6) **The average height of the 64 students at the Teen Center is 146 centimeters. The average height of the 28 boys is 128 centimeters. How many centimeters is the average height of the girls?**

ANSWERS

1) _____

2) _____

3) _____

4) _____

5) _____

6) _____

Solutions to Category 6
Team Round
Meet #1 - October, 2016

ANSWERS	
1)	19
2)	18
3)	946
4)	1540
5)	6
6)	160

1) The angle opposite (or "vertical to") the 17-degree angle also measures 17 degrees. Since the angles now marked 3X, 17, and 106 form a straight angle, then $3X + 17 + 106 = 180$ . . . $3X + 123 = 180$ . . . $3X = 57$ . . . $X = 19$.

2) Since each of the 15 carpenters works 24 hours, the total number of carpenter-hours worked is (15)(24), or 360. Divide 360 by 20 to get 18, the number of hours that each of the 20 workers must work to complete the job if 20 workers do the job.

3) (- 6 ♣ - 10) - (8 ♥ - 9)

$$= \left((-4)(-6)^3+(5)(-10)(-6)\right)-\left((3)(-9)^2-(4)(8)+7\right)$$
$$= \left((-4)(-216)+(300)\right)-\left((3)(81)-(32)+7\right)$$
$$= (864+300)-(243-32+7) \quad = \quad 1164-218 \quad = \quad 946.$$

4) Scaling should help solve this problem. Fred's 240 lettuce plants is 80 times more than is stated in the first ratio mentioned. His actual number of tomato plants is (80)(7), or 560. The next ratio mentions 4 tomato plants. Again scaling, (4)(140) is 560, thus enabling us to scale for the actual number of cucumber plants: (140)(11) is 1540.

5) $7(3N + 2) + 5(4N - 1) - 8(9N - 10) = 6(7 - 11N) - 4(2N + 8) + 337$

$21N + 14 + 20N - 5 - 72N + 80 = 42 - 66N - 8N - 32 + 337$

$- 31N + 89 = - 74N + 347$

$43N = 258$

$N = 6$

6) The total height of all 64 students is (64)(146), or 9344 cm. The total of the boys' heights is (28)(128), or 3584 cm. The 36 girls have a total height of 9344 - 3584, or 5760 cm. So, the average height of the girls is 5760 / 36, or 160 cm.