

IMLEM Meet #1
October, 2015

Intermediate Mathematics League of Eastern Massachusetts



Category 1

Mystery

Meet #1 - October, 2015



- 1) **C and T are two whole numbers whose sum is 20 and whose product is 36. C is greater than T. What is the value of C - T ?**

- 2) **Bugs peels carrots at the rate of 4 carrots per minute. Elmer joins bugs after 7 minutes and peels at the rate of 6 per minute. By the time they finish peeling the bag of 88 carrots, how many will Elmer have peeled?**

- 3) **A whole number has the following properties:**
 - * **Its digits are only twos and threes.**
 - * **It contains at least one two and one three.**
 - * **It is divisible by both two and three.****What is the smallest number it could be ?**

Answers

1) _____

2) _____

3) _____



**Solutions to Category 1
Mystery
Meet #1 - October, 2015**

1) **Guessing and checking, especially for the younger students, can produce viable data quickly. $C = 18$ and $T = 2$. Then $C - T = 18 - 2 = 16$.**

2) **A non-algebraic approach might be to set up a chart:**

Minute #:	1	2	3	4	5	6	7	8	9	10	11	12	13
Bugs:	4	8	12	16	20	24	28	32	36	40	44	48	52
Elmer:							6	12	18	24	30	36	
Total:	4	8	12	16	20	24	28	38	48	58	68	78	88

<u>Answers</u>	
1)	16
2)	36
3)	2232

An algebraic approach may reduce time and stress for students who have this skill:

Let $T = \#$ of minutes that Elmer peels carrots
 $6T =$ the total $\#$ of carrots that Elmer peels
 $T + 7 = \#$ of minutes that Bugs peels carrots
 $4(T + 7) =$ the total $\#$ of carrots that Bugs peels

$$\begin{aligned}
 6T + 4(T + 7) &= 88 \\
 6T + 4T + 28 &= 88 \\
 10T + 28 &= 88 \\
 10T &= 60 \\
 T &= 6
 \end{aligned}$$

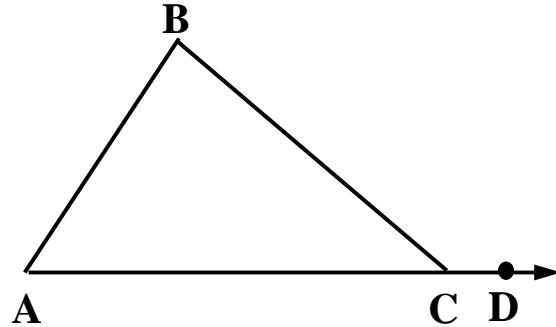
So, Elmer peeled (6)(6), or 36 carrots.

3) **For this number to be divisible by both 2 and 3, it must be even and the sum of its digits must be divisible by 3. Since we are seeking the smallest possible value, start small, then progress to larger and larger values. Utilizing only even numbers, and those that have at least one two and one three, the number 32 fails, as do 232, 322, and 332. We need to go to four digits. The number 2232 works.**

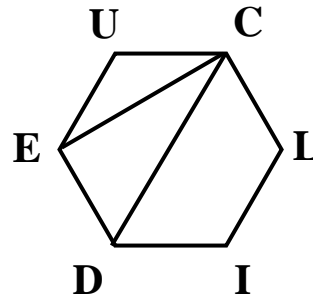
Category 2
Geometry
Meet #1 - October, 2015

For all problems below, angles are marked in degrees as shown. *Figures are not necessarily drawn to scale.*

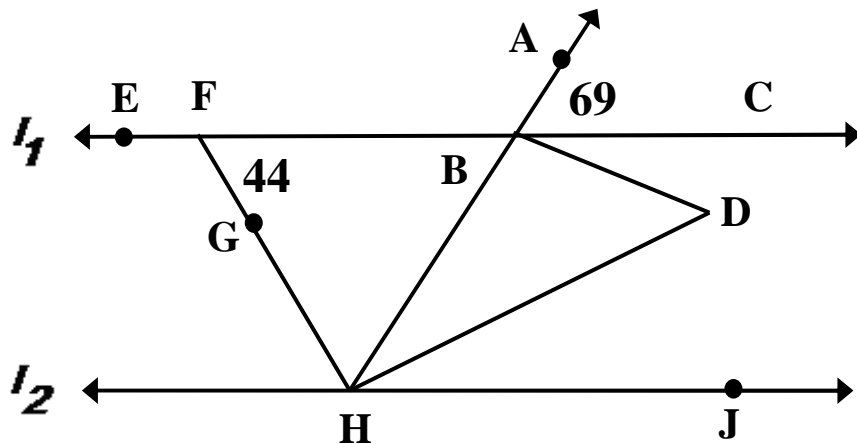
- 1) Angle B measures 112 degrees. Angle A has the same measure as angle BCA. How many degrees are in the measure of angle BCD if it is less than 180 degrees ?



- 2) Regular hexagon EUCLID has diagonals EC and CD. How many degrees are in the measure of angle ECD ?



- 3) In the diagram below, l_1 is parallel to l_2 . Angle ABC is complementary to angle CBD. Angle EFG is supplementary to angle DHJ. How many degrees are in the measure of angle D ?



<u>Answers</u>	
1)	_____
2)	_____
3)	_____

**Solutions to Category 2
Geometry
Meet #1 - October, 2015**

- 1) Since the base angles of the triangle are equal, angle $BCA = \text{half of } (180 - 112) = 34$ degrees. The exterior angle $BCD = 180 - 34$, or 146 degrees.
- 2) Each angle of the regular hexagon has a measure of 120 degrees. The diagonal DC bisects angle C , making angle $DCL = 60$ degrees. Triangle EUC is isosceles, with $UE = UC$, so the base angles UEC and $UCE = \text{half of } (180 - 120)$, or 30 degrees. Therefore, the measure of the required angle, ECD , is $120 - (30 + 60)$, or 30 degrees.
- 3) Since angle ABC is complementary to angle CBD , then angle CBD measures $(90 - 69)$, or 21 degrees. Angle HBD is 90 degrees, being adjacent to the collinear 90 -degree angle ABD . Angle BHJ corresponds to angle ABC and $= 69$ degrees. Angle DHJ is the supplement of angle EFG and $= 44$ degrees. Therefore, angle BHD measures $(69 - 44)$, or 25 degrees. Now two angles of triangle are known, so the required angle $D = 180 - (90 + 25)$, or 65 degrees.

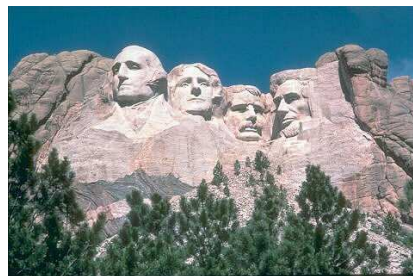
Answers

1) 146

2) 30

3) 65

Category 3
Number Theory
Meet #1 - October, 2015



- 1) Find the sum of all composite numbers between 47 and 59.

- 2) If A , B , and C are prime numbers, and $A > B > C$, then how many different factors does the product ABC have?

- 3) A two-digit whole number is divisible by E and its units digit (the number in the ones place) is also E . The two digits are different. What is the largest possible value of E ?

Answers

1) _____

2) _____

3) _____

October 31, 1941 - Mount Rushmore National Memorial was completed after 14 years of work. The memorial contains 60-foot-tall sculptures of the heads of Presidents George Washington, Thomas Jefferson, Abraham Lincoln and Theodore Roosevelt - representing America's founding, political philosophy, preservation, and expansion and conservation.

**Solutions to Category 3
Number Theory
Meet #1 - October, 2015**

- 1) **Composite numbers, unlike prime numbers, have at least three factors.**

$$48 + 49 + 50 + 51 + 52 + 54 + 55 + 56 + 57 + 58 = 530.$$

- 2) **Select any three prime numbers - A, B, and C - multiple them, and the product will have eight factors:
1, A, B, C, AB, AC, BC, and ABC.**

- 3) **The two-digit numbers where the two digits are different and the number is divisible by its units digit are:**

12, 15, 21, 24, 25, 31, 32, 35, 36, 41, 42, 45, 48, 51, 52, 61, 62, 63, 64, 65, 71, 72, 75, 81, 82, 84, 85, 91, 93, 95, and 96.

Therefore, the largest possible value of E comes compliments of the number 48 and is therefore 8.

Answers

1) 530

2) 8

3) 8

Category 4
Arithmetic
Meet #1 - October, 2015



- 1) Using the standard order of operations, find the value of

$$1+2^3[4+5(6-7(8-9))]$$

- 2) The arithmetic mean (average) of four different whole numbers is 65. The largest of the numbers is 80. What is the smallest possible value of any of the four numbers?
- 3) The average of a set of seven different positive integers is 26. The median is 15. What is the largest possible value of any of the seven integers?

Answers

1) _____

2) _____

3) _____



**In October, 1959, The
"Twilight Zone"
first premiered on television.**

Solutions to Category 4
Arithmetic
Meet #1 - October, 2015

1) $1+2^3[4+5(6-7(8-9))]$
 $= 1+8[4+5(6-7(-1))]$
 $= 1+8[4+5(13)]$
 $= 1+8[4+65]$
 $= 1+8[4+5(6-(-7))]$
 $= 1+8[69]$
 $= 1+552$
 $= 553$

2) $80 + 79 + 78 + X = 4(65)$
 $237 + X = 260$
 $X = 23$

3) The smallest three numbers must be below 15 and the largest three must be greater than 15. If the average of the seven numbers is 26, then their sum is $7(26)$, or 182. To get the largest of the seven numbers, let the three numbers below 15 be the smallest possible different values, namely 1, 2, and 3. Let the first two numbers that are greater than 15 be 16 and 17. The sum of the first six numbers is $1 + 2 + 3 + 15 + 16 + 17 = 54$. Therefore, the seventh and final number is $182 - 54$, or 128.

Answers

1) 553

2) 23

3) 128

Category 5

Algebra

Meet #1 - October, 2015

1) Find the value of $5x^2 - 2x^3$ if $X = -4$.

2) For what value of N is $2N - 6[3N - 2(7N - 4)] - 5N = 456$?

3) If Jean were ten years older, she would be three years older than she would have been if she were four times as old as she was 38 years ago. How many years old will Jean be in 27 years?

Answers

1) _____

2) _____

3) _____



Henry Ford introduced the Model T on October 2, 1908.

**Solutions to Category 5
Algebra
Meet #1 - October, 2015**

$$\begin{aligned} 1) \quad & 5x^2 - 2x^3 \\ & = 5(-4)^2 - 2(-4)^3 \\ & = 5(16) - 2(-64) \\ & = 80 + 128 \\ & = 208 \end{aligned}$$

$$\begin{aligned} 2) \quad & 2N - 6[3N - 2(7N - 4)] - 5N = 456 \\ & 2N - 6[3N - 14N + 8] - 5N = 456 \\ & 2N - 6[-11N + 8] - 5N = 456 \\ & 2N + 66N - 48 - 5N = 456 \\ & 63N - 48 = 456 \\ & 63N = 504 \\ & N = 8 \end{aligned}$$

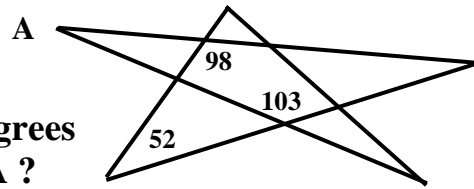
3) Let X = Jean's age now, in years
 $X + 10$ = Jean's age 10 years from now, in years
 $X - 38$ = Jean's age 38 years ago, in years

$$\begin{aligned} X + 10 &= 4(X - 38) + 3 \\ X + 10 &= 4X - 152 + 3 \\ X + 10 &= 4X - 149 \\ -3X &= -159 \\ X &= 53 \\ X + 27 &= 80 \end{aligned}$$

So, Jean is now 53 years old. In 27 years, she will be $53 + 27$, or 80 years old.

<u>Answers</u>	
1)	208
2)	8
3)	80

Category 6
Team Round
Meet #1 - October, 2015



- 1) Angle measures of this figure are given in degrees as shown. How many degrees are in angle A ?
- 2) Barnabas scored zero points on a test that awarded eight points for each correct answer but deducted five points for each incorrect or unanswered question. If there were between 30 and 50 questions in all, then how many questions were on the test?
- 3) The number 75 has the property that it is divisible by its units digit (the number in the ones place). Here, 75 is divisible by 5. How many whole numbers between 10 and 40 have this property?
- 4) If C, A, and P are consecutive integers such that $0 < C < A < P$ and the units digit of the product CP is 9, then what is the units digit of A ?
- 5) Let #n# be defined as the product of all of the factors of 2n, for every positive integer value of n. For example, #6# = $12 \times 6 \times 4 \times 3 \times 2 \times 1 = 1728$. What is the value of #20# ?
- 6) On a Cartesian (rectangular) coordinate plane, if the point A = (4, -9) is reflected across the y-axis to create the point A', and then the point A' is reflected across the line $y = x$ to create the point A'', then what is the sum of the coordinates of the point A'' ?

ANSWERS

- 1) _____
- 2) _____
- 3) _____
- 4) _____
- 5) _____
- 6) _____

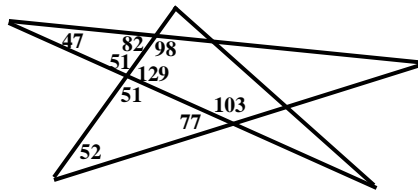
NOTE: *For the first time in over 20 years, since the inception of the Team Round, IMLEM is introducing a variety of Team Round formats. The standard one used since 1993 contained five questions, plus a sixth question that incorporated the answers to the first five. Today's round has six independent questions. A future round may have nine independent questions, or may be like the standard of the past 20 years. Enjoy this new IMLEM adventure!*

**Solutions to Category 6
Team Round
Meet #1 - October, 2015**

ANSWERS

- 1) 47
2) 39
3) 12
4) 0
5) 2,560,000
6) -13

- 1) Based on collinear angles having a sum of 180 degrees, and the sum of the angles of a triangle = 180 degrees, the resulting relevant angles are on this diagram.



- 2) There would have to be eight incorrect or unanswered questions for every five correct answers, so there could be 13 or 26 or any multiple of 13 questions on the test. Since the number is greater than 30

but less than 50, the correct number of questions is 39.

- 3) The numbers are 11, 12, 15, 21, 22, 24, 25, 31, 32, 33, 35, and 36, so there are twelve such numbers in all.

- 4) This chart shows sets of three consecutive integers - C, A, and P (the columns) - and the units digit of CP is given at the bottom of each column. For example, in the first column, the three consecutive integers are 1, 2, and 3. $CP = (1)(3) = 3$. The next set of three consecutive integers is in the next column, 2, 3, and 4. $CP = (2)(4) = 8$. The first column where $CP = 9$ is the column of numbers 9, 10, and 11 where $CP = 9$. This pattern repeats and continues forever so that the A value in every column where the units digit of C = 9 is 0.

C	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
A	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
P	3	4	5	6	7	8	9	0	11	12	13	14	15	16	17	18	19	20	21
CP	3	8	5	4	5	8	3	0	9	0	3	8	5	4	5	8	3	0	9

- 5) $\#20\# = 40 \times 20 \times 10 \times 8 \times 5 \times 4 \times 2 \times 1 = 2,560,000$.

- 6) $A' = (-4, -9)$ so $A'' = (-9, -4)$ and the sum of the coordinates of A'' is $-9 + -4$, or -13 .