

IMLEM Meet #4
February, 2015

Intermediate Mathematics League of Eastern Massachusetts

This is a calculator meet!



Category 1
Mystery
Meet #4 - February, 2015
Calculator meet



"Be courteous to all, but intimate with a few, and let those few be well tried before you give them your confidence." . . . George Washington, born February 22, 1732.

- 1) All 728 students at the William Diamond Middle School lined up in a straight line along Massachusetts Avenue to participate in a relay race for charity, standing 25 yards apart. What was the distance, in yards, between the first student and the last student in this line? (For the purpose of this problem, treat the students as points on a line.)

- 2) On a number line, how many whole numbers lie between the numbers $\frac{117}{7}$ and 25π ?

- 3) How many degrees are in the smaller angle formed by the hour and minute hands of a 12-hour clock at precisely 2:48 P.M.? Keep in mind that the hands of a clock rotate at different speeds - neither hand remains stationary as time passes.

Answers

- 1) _____
- 2) _____
- 3) _____

Rosa Parks, born February 4, 1913, became known as the "first lady of civil rights" when she was arrested for refusing to give up her seat on a bus in Alabama in 1955. This act instigated the legendary Montgomery Bus Boycott that helped end racial segregation in public transportation across the United States.

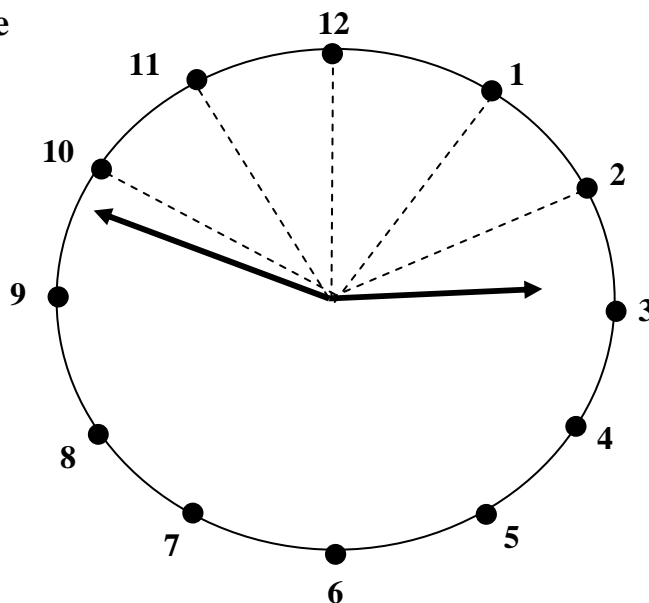


**Solutions to Category 1
Mystery
Meet #4 - February, 2015**

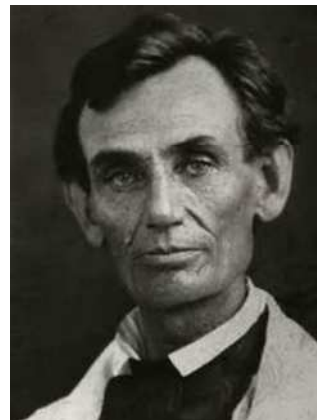
- 1) If there are 728 students, then there are 727 25-yard intervals between them. So, $(727)(25) = 18,175$.
- 2) $\frac{117}{7}$ is about 16.7. 25π is about 78.5. The number of whole numbers between them is the number of whole numbers from 17 through 78, inclusive, $= (78 - 17) + 1 = 62$.

<u>Answers</u>	
1)	18,175
2)	62
3)	156

- 3) Each hourly interval is $\frac{1}{12}$ of 360 degrees, or 30 degrees. At 2:48, the minute hand points to the 48 while the hour hand rotated $\frac{48}{60}$ of the way from pointing to the 2 at 2:00 to pointing to the 3 at 3:00 so that the angle between 2:00 and the hour hand is $\frac{48}{60}$ of 30, or 24 degrees. The angle between the minute hand and the dotted radius pointing to the 10 is $\frac{2}{5}$ of 30, or 12 degrees. So, the smallest angle formed by the hands of the clock is $(4)(30) + 12 + 24$, or 156 degrees.

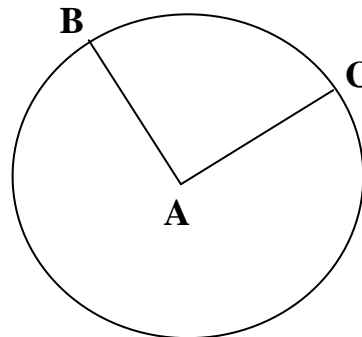


Category 2
Geometry
Meet #4 - February, 2015
Calculator meet



***"Whenever I hear anyone arguing for slavery, I feel a strong impulse to see it tried on him personally."
... Abraham Lincoln, born February 12, 1809.***

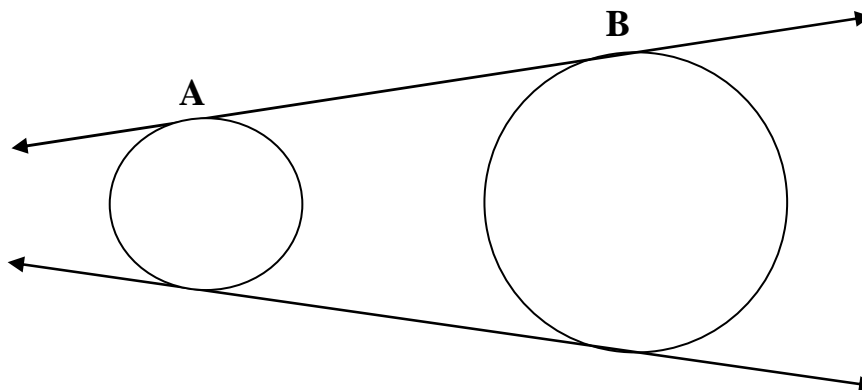
- 1) How many inches longer is the circumference of a circle of radius = 20 inches than a circle of radius = 10 inches? Use $\pi \approx 3.14$.
- 2) Given circle A with diameter = 42 feet. What is the perimeter, in feet, of sector ABC that includes radii AC and AB and arc BC? Angle A is a right angle. Use $\pi \approx 3.14$.



- 3) Two lines are tangent, externally, to two circles with diameters of 46 cm and 74 cm. AB = 48 cm. Points A and B are points of tangency. How many cm apart are the centers of the two circles? (Figure not to scale)

Answers

- 1) _____
- 2) _____
- 3) _____



**Solutions to Category 2
Geometry
Meet #4 - February, 2015**

1) Find the difference between the circumferences of the two circles = $2(\pi)(R) - 2(\pi)(r)$
 $= 2(3.14)(20) - 2(3.14)(10)$
 $= 125.6 - 62.8 = 62.8$

2) Arc BC = $\frac{1}{4}$ (circumference of circle)
 $= (0.25)(2)(3.14)(21) = 32.97$.
 So, the perimeter of the sector = arc + 2 radii
 $= 32.97 + (2)(21) = 32.97 + 42 = 74.97$ feet.

<u>Answers</u>	
1)	62.8
2)	74.97
3)	50

3) The radius drawn to a tangent from the center is perpendicular to the tangent.

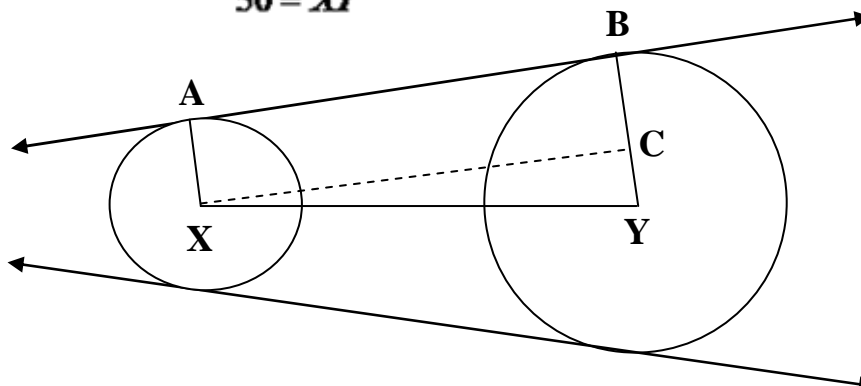
- * Let XA and YB be the two radii.
- * Draw XY to form a trapezoid with XA parallel to YB.
- * Draw a segment from X, parallel to AB, to a point C on YB.
- * We now have a rectangle and a right triangle.
- * $BY - BC = 37 - 23 = 14 = YC$.
- * $AB = XC = 48$.
- * For right triangle XCY, use the Pythagorean Theorem to find the length of XY, the distance between the centers of the two circles:

$$14^2 + 48^2 = (XY)^2$$

$$196 + 2304 = (XY)^2$$

$$2500 = (XY)^2$$

$$50 = XY$$



Category 3
Number Theory
Meet #4 - February, 2015
Calculator meet



- 1) If today is February 12 and Sabatino is exactly 633 months old, in what month was Sabatino born?

- 2) Below are seven terms of a geometric sequence. What is the value of $A + B + C$? Express your answer as a decimal.

8 12 18 A B C 91.125

- 3) $\sum_{K=1}^n K^2 = \frac{(n)(n+1)(2n+1)}{6}$ is the formula for adding consecutive

square numbers in the following series: $1^2 + 2^2 + 3^2 + \dots + n^2$.

For example, $\sum_{K=1}^4 K^2 = 1^2 + 2^2 + 3^2 + 4^2 = 1 + 4 + 9 + 16 = 30$

We can achieve the same result by substituting 4 for n into the

formula: $\frac{(n)(n+1)(2n+1)}{6} = \frac{(4)(4+1)(2(4)+1)}{6} = \frac{(4)(5)(9)}{6} = \frac{180}{6} = 30$.

**** Important - this formula works for series that begin with $K = 1$.**

Find the value of the following sum: $64 + 81 + 100 + 121 + \dots + 3249$.

Answers

- 1) _____
- 2) _____
- 3) _____

Bill Russell, shown above with his coach, Red Auerbach, was born on Feb 12, 1934 in West Monroe, Louisiana. Bill led the Boston Celtics professional basketball team to 11 world championships between 1956 and 1969. He was the guest of honor and keynote speaker at the 2005 Mathcounts National Competition held in Detroit, Michigan and was proud to sit and chat with the Massachusetts team, coached by Mr. Findell.

Solutions to Category 3
Number Theory
Meet #4 - February, 2015

Answers

- 1) Divide 633 by 12 . . . that will produce a quotient representing the number of complete years, as well as a remainder that represents the number of months prior to Sabatino's birthday month.

$633 / 12 = 52$ with remainder 9. Nine months prior to February (or, more easily, three months after February), is May.

- 1) May
2) 128.25
3) 63,225

- 2) The common ratio linking two consecutive terms is $12 / 8 = 1.5$. So,

$$18 \times 1.5 = 27 = A$$

$$27 \times 1.5 = 40.5 = B$$

$$40.5 \times 1.5 = 60.75 = C$$

Checking: $60.75 \times 1.5 = 91.125$, the final term.

So, $A + B + C = 27 + 40.5 + 60.75 = 128.25$ (must be expressed in decimal form).

- 3) To find the required sum, subtract the sum of the squares of the integers from 1 through 7, inclusive, from the sum of the squares of the integers from 1 through 57, inclusive.

$$\left(\sum_{k=1}^{57} k^2 \right) - \left(\sum_{k=1}^7 k^2 \right) = \left(\frac{(57)(57+1)(2 \cdot 57+1)}{6} \right) - \left(\frac{(7)(7+1)(2 \cdot 7+1)}{6} \right)$$

$$= \left(\frac{(57)(58)(115)}{6} \right) - \left(\frac{(7)(8)(15)}{6} \right) = \left(\frac{380,190}{6} \right) - \left(\frac{840}{6} \right)$$

$$= 63,365 - 140 = 63,225.$$

Category 4
Arithmetic
Meet #4 - February, 2015
Calculator meet

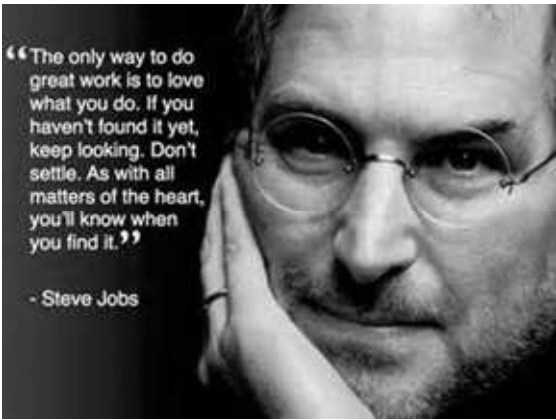


- 1) The "Rule of 72" is often used to approximate the number of years, Y , it would take to double the value of an investment when given an annual rate of R percent, as follows: $(R)(Y) = 72$. For example, if \$50 were invested at an annual rate 6%, then it would take $72 / 6$, or 12 years, for it to double in value to \$100. If that same \$50 had been invested at 4% instead, then how many more years would it have taken for the \$50 to double in value?
- 2) Gronk took his 74 teammates out for a steak dinner. The average price for each dinner was \$76.80. A 5% tax was then added onto the cost of the meal. He left the wait staff a generous 35% tip, based on the price of the meal only. What was the total amount, including meal, tax, and tip, that Gronk paid so that he and his teammates could have dinner?
- 3) Early in the race, Jen and Berry were running at the same speed. Berry then increased her speed by 20% while Jen increased her speed by only half as much. Berry is now running at 60 spans per second. How many spans per second is Jen running now?

.....

Steve Jobs, born in February of 1955, was an American entrepreneur, marketer, and inventor who was the co-founder, chairman, and CEO of Apple Inc. He is widely recognized as a pioneer of the personal computer revolution, transforming one industry after another, from computers and smart phones (iPhone) to music (iTunes) and movies (Pixar).

<u>ANSWERS</u>
1) _____
2) _____
3) _____



Solutions to Category 4
Arithmetic
Meet #4 - February, 2015

Answers

1) 6

2) 8064

3) 55

1) Using the rule of 72, $72 / 4 = 18$ years, which is 6 years more than 12.

2) $(75)(\$76.80) = \5760 , the total cost of the food only.

$(\$5760)(0.05) = \288 , the amount of tax.

$(\$5760)(0.35) = \2016 , the amount of the tip.

Meal + tax + tip = $\$5760 + \$288 + \$2016 = \8064 .

3) Working backwards, Berry's speed early in the race was $60 / 1.2 = 50$ spans per second.

Jen's speed now: $(50)(1.1) = 55$ spans per second.

Category 5
Algebra
Meet #4 - February, 2015
Calculator meet



- 1) Tom completes three passes of every five that he throws. In an average football season, Tom throws 465 passes. How many passes does he *fail* to complete?

- 2) Four burgers and three drinks cost \$20.05. Seven burgers and four drinks cost \$32.85. How much money would it cost to provide a classroom of 23 students so that each student receives two burgers and one drink?

- 3) On the Isle of Sodor, a set of parallel train tracks is 400 miles long. On one end of one track, Train Thomas leaves the station at 5:00 PM. On the opposite end of the other track, Train Diesel leaves the station at 6:30PM. If Train Thomas travels 40 mph and Train Diesel travels 60 mph, then at what time will they meet?

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Father Frank O'Gara of Whitefriars Street Church in Dublin, Ireland, tells the real story of Saint Valentine, who was a Roman priest. At that time, 269 A.D., young people were forbidden by the Church to be married. Valentine broke the

<u>ANSWERS</u>
1) _____
2) \$ _____
3) _____ P.M.

law and, eventually, was caught, imprisoned, and tortured for performing marriage ceremonies against the command of Claudius II. Valentine was sentenced to a three-part execution of a beating, stoning, and finally decapitation, all because of his stand for Christian marriage. The last words he wrote were in a note to justice Asterius' daughter: "from your Valentine."



Solutions to Category 5
Arithmetic
Meet #4 - February, 2015

<u>Answers</u>	
1)	186
2)	209.99
3)	9:54

- 1) Let P = the number of passes that Tom completes. Then write and solve a proportion.

$$\frac{\text{\# of passes completed}}{\text{\# of passes thrown}} = \frac{3}{5} = \frac{P}{465} \quad \text{Cross products are equal: } 5P = (3)(465)$$

$$5P = 1395 \quad \text{and} \quad P = 279.$$

Tom completes 279 passes, so he fails to complete $465 - 279$, or 186 passes.

- 2) Let B = the number of burgers
 D = the number of drinks.

$$4B + 3D = 20.05 \quad \text{and} \quad 7B + 4D = 32.85.$$

To eliminate D , multiply both sides of the first equation by 4 and both sides of the second equation by -3 , then add the two equations:

$$16B + 12D = 80.2 \quad \text{and} \quad -21B - 12D = -98.55$$

Adding yields $-5B = -18.35$, so $B = 3.67$ and then $D = 1.79$.

So, a burger costs \$3.67 and a drink costs \$1.79.

So that members of the class of 23 students each has two burgers and one drink, the total cost will be $(23)[(2)(3.67) + 1.79] = (23)[7.34 + 1.79]$
 $= (23)(9.13) = \$209.99$.

- 3) Let T = the number of hours that Diesel rides.
 $T + 1.5$ = the number of hours that Thomas rides.
 $60T$ = the number of miles that Diesel rides.
 $40(T + 1.5)$ = the number of miles that Thomas rides.

Their total distance = 400 miles, so $60T + 40(T + 1.5) = 400$

$$60T + 40T + 60 = 400$$

$$100T + 60 = 400$$

$$100T = 340$$

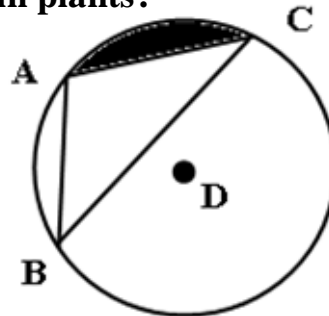
$$T = 3.4$$

So, 3.4 hours after 6:30 P.M. is 9:54 P.M.

Category 6
Team Round
Meet #4 - February, 2015

- 1) Twenty-four coins fell out of Dagwood's pocket. Twice as many landed tails up as landed heads up. How many coins landed tails up?
- 2) What is the minimum (smallest) possible product when three different integers are selected from this set? { - 9, - 6, - 4, 0, 4, 5, 7 }
- 3) Farmer Fred planted 288 vegetable plants in his garden: 72 tomato, 56 cucumber, 23 herbs, and 29 pepper. One-third of the remaining plants were squash. The rest were pumpkin plants. Fred represented the plants in a pie (circle) chart. How many degrees should be in the central angle of the sector representing pumpkin plants?

- 4) In the circle D at the right, angle ABC measures 45 degrees. If the diameter of the circle is 52 cm, then how many square cm are in the shaded area? Use $\pi \approx 3.1$. Round your final answer to the nearest tenth.



- 5) $Y = A\left(1 + \frac{R}{N}\right)^{NT}$ represents the value, Y, of an amount, A, that grows

at an annual interest rate, R, for T years. N is the number of times per year that interest is compounded. Owen buys a 50" diagonal TV and charges it to his credit card. He can only afford to pay a total of no more than \$700. The annual (yearly) interest rate on his card is 12.49%, compounded monthly. If Owen needs to pay off this debt in seven months, then what is the greatest sticker price amount he can afford? Round your answer to the appropriate whole dollar amount. Remember his total spending limit!

- 6) Using the answers from questions #1-5, evaluate the following expression:

$$H\sqrt{10(C - B + D) + E + 1} \text{ where } H = \sqrt{A}$$

ANSWERS

- 1) _____ = A
- 2) _____ = B
- 3) _____ = C
- 4) _____ = D
- 5) _____ = E
- 6) _____

**Solutions to Category 6
Team Round
Meet #4 - February, 2015**

ANSWERS

- 1) 16 = A
2) - 315 = B
3) 90 = C
4) 185.9 = D
5) 651 = E
6) 9

- 1) $2X + X = 24 \dots 3X = 24 \dots X = 8 \dots 2X = 16$.
- 2) The product of the three negative integers is - 216, while $(- 9)(5)(7)$ yields - 315.
- 3) Pumpkin + squash: $288 - (72 + 56 + 23 + 29) = 288 - 180 = 108$. Pumpkin = $\frac{2}{3}(108) = 72$. $72 / 288 = 0.25 =$ the part of the circle that represents pumpkin = 90 degrees.
- 4) If angle B, an inscribed angle, measures 45 degrees, then central angle ADC measures 90 degrees, thus making arc AC $\frac{1}{4}$ of the circle's circumference. If the diameter of the

circle is 52 cm, then the radius is half of 52, or 26 cm. The area of sector ACD is $\frac{1}{4}$ of the area of the circle, or $(\frac{1}{4})(3.1)(26)(26)$, or 523.9 square cm. The shaded area = (sector area) - (area of triangle ADC = $523.9 - [(\frac{1}{2})(26)(26)] = 523.9 - 338 = 185.9$.

- 5) The most challenging part of this problem is deciding on the appropriate rounding. Substitute $Y = 700$, $R = 0.1249$, $T = 7/12$, and $N = 12$.

$$Y = A\left(1 + \frac{R}{N}\right)^{NT} \dots\dots 700 = A\left(1 + \frac{0.1249}{12}\right)^{(12)\left[\frac{7}{12}\right]}$$

$$\dots\dots 700 = A(1.0104083)^7 \dots\dots 700 = A(1.075172965)$$

$A = 651.058037$. Rounding down to \$651 is most appropriate, as Owen's spending limit of \$700 is honored. Rounding up to \$652 would exceed the spending limit.

- 6) $\sqrt[4]{10(C - B + D) + E + 1}$ where $H = \sqrt{A}$
 $= \sqrt[4]{10(90 - (-315) + 185.9) + 651 + 1}$ where $H = \sqrt{16}$
 $= \sqrt[4]{6561} = 9$.