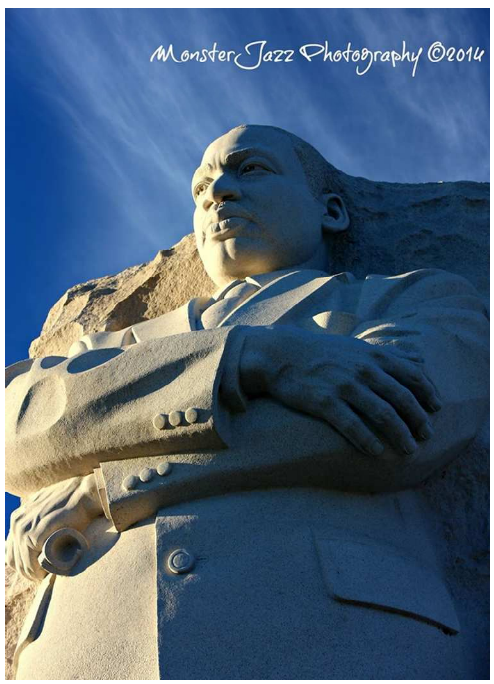
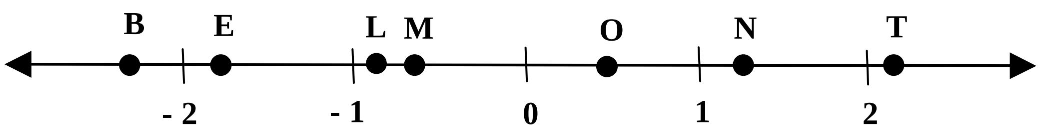
IMLEM Meet #3
January, 2015

Intermediate Mathematics League of Eastern Massachusetts



Category 1
Mystery
Meet #3 - January, 2015

1) Which one of the lettered points listed above the number line could represent the result when the coordinate of point O (not zero) is divided by the coordinate of point L ?



2) How many 3-digit positive numbers are multiples of both 8 and 12 ?

3) A Mersenne Prime is a prime number written in the form 2^P-1 where P is a prime number. Find the sum of all the Mersenne Primes that lie between 5 and 100.

Answers

1) ______

2) ______

3) ______

Solutions to Category 1
Mystery
Meet #3 - January, 2015

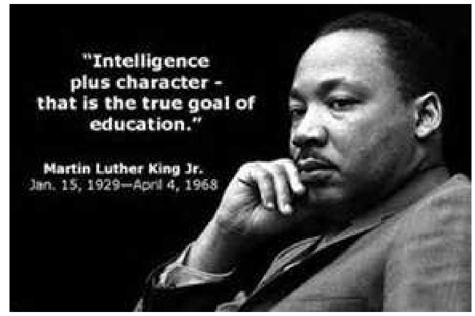
Answers
1) M
2) 37
3) 38

1) The only reasonable choice is point M, as point O (approximately 1/2) divided by point L (approximately - 7/8) is about - 4/7.

2) Multiples of 8 and 12 are multiples of their LCM, or 24. The largest multiple of 24 that is less than 1000 is (24)(41), or 984. The salient information about this is that there are 41 multiples of 24 that are less than 1000. Four of them are less than 100, namely 24, 48, 72, and 96. All the rest contain three digits. So, 41 - 4 = 37.

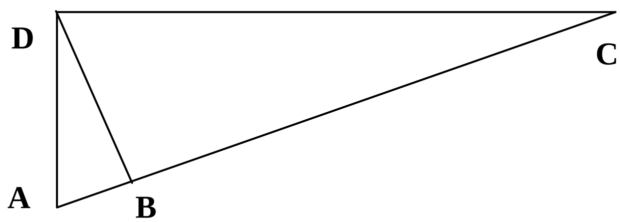
3) The most common error in solving this problem is when students choose values of P that are not prime! The only values of P that yield a prime number outcomes are when P = 3 (yielding a value of 7) and when P = 5 (yielding a value of 31). So, the sum is 31 + 7 = 38.

Category 2
Geometry
Meet #3 - January, 2015

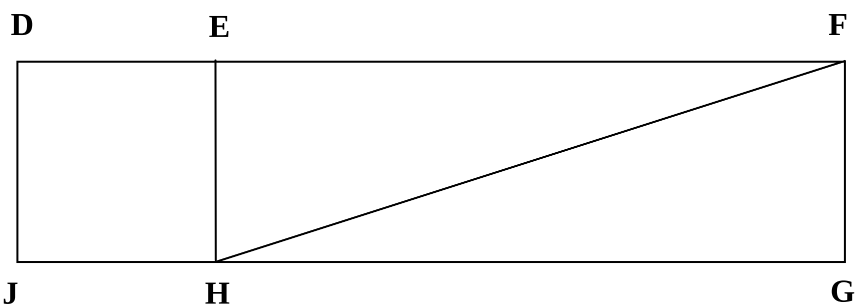


Figures are not necessarily drawn to scale.

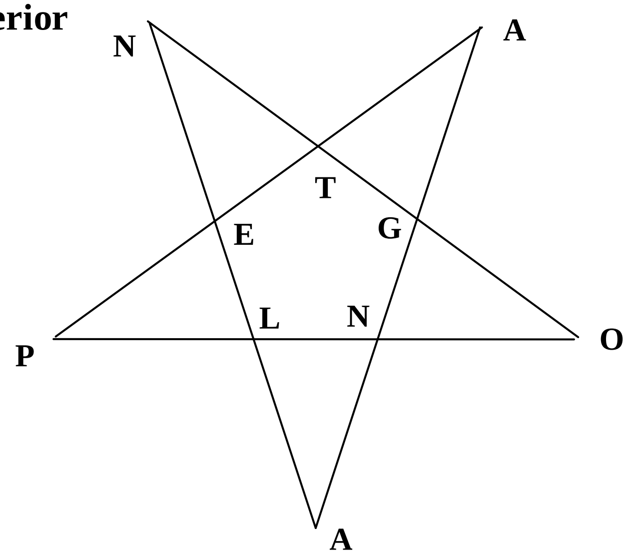
1) Angle ADC is a right angle. AB = 4 cm and BC = 9 cm. DB is perpendicular to AC. How many cm long is DB ?



2) DEHJ is a square with an area of 64 square meters. Diagonal HF = 17 meters. How many square meters are in rectangle DFGJ ?



3) Polygon PENTAGONAL is a pentagram (star) consisting of a regular pentagon with five isosceles triangles attached at its five edges. How many degrees are in one of the exterior angles (for example, angle PEN) ?



Answers

1) ______

2) ______

3) ______

Solutions to Category 2
Geometry
Meet #3 - January, 2015

Answers
1) 6
2) 184
3) 108

1) A student who knows the Pythagorean Theorem should also know that, at its foundation, is the notion of similar triangles. In this diagram are three similar triangles. Using triangles DAB and DBC, we can say that ratios of corresponding sides are proportional: $\frac{AB}{DB}=\frac{DB}{BC}$. So, $\frac{4}{DB}=\frac{DB}{9}$ and cross products are equal, so $(DB)^2=36$ and DB = 6. A few students may recognize this diagram as representing this theorem: "The altitude to the hypotenuse of a right triangle is the geometric mean (or mean proportional) to the two segments of the hypotenuse into which it is divided."

2) One side of square DEHJ is 8 meters because its area is 64 square meters. For one of the right triangles of rectangle EFGH, using the Pythagorean Theorem, $(EH)^2+(EF)^2=(HF)^2$. So, $(8)^2+(EF)^2=(17)^2$, and $64+(EF)^2=289$, so $(EF)^2=225$, and EF = 15. So, rectangle DFGJ now measures 8 by (15 + 8), or 8 by 23, so its area is (8)(23), or 184 square meters.

3) Each interior angle of the regular pentagon measures (3)(180)/5, or 108 degrees. Any one of the exterior angles of the pentagram is vertical to one of these interior 108 degree angles and, therefore, is equal to 108 degrees.

Category 3
Number Theory
Meet #3 - January, 2015

1) Express the binary number 101001001 (base 2) as a base 10 numeral.

2) Compute. Express your answer in scientific notation:

$$\frac{200\times10^7}{0.04\times10^6}\div\frac{0.0005\times10^{-3}}{80\times10^0}$$

3) Convert the base 3 numeral 202110 to a base 5 numeral.

More than 517,000,000 of the famed Elvis Presley postage stamp have been sold since the stamp was first issued on January 8, 1993, more than any other single stamp ever issued by the United Sates Postal Service (USPS) since Benjamin Franklin became our first Postmaster General in 1775.

Answers

1) ______

2) ______

3) ______

Solutions to Category 3
Number Theory
Meet #3 - January, 2015

Answers
1) 329
2) 8×10^{12}
3) 4202

1) From right to left, 101001001 (base 2) =
1(1) + 0(2) + 0(4)+ 1(8) + 0(16) + 0(32) + 1(64) + 0(128) + 1(256)
= 1 + 0 + 0 + 8 + 0 + 0 + 64 + 0 + 256
= 329

2) $$\frac{200\cdot 10^{7}}{0.04\cdot 10^{6}} \div \frac{0.0005\cdot 10^{-3}}{80\cdot 10^{0}} = \frac{200\cdot 10^{7}}{0.04\cdot 10^{6}} \cdot \frac{80\cdot 10^{0}}{0.0005\cdot 10^{-3}}$$

$$= \frac{16000\cdot 10^{7}}{0.000020\cdot 10^{3}} = \frac{16\cdot 10^{3}\cdot 10^{7}}{2\cdot 10^{-5}\cdot 10^{3}} = \frac{16\cdot 10^{3+7}}{2\cdot 10^{-5+3}} = \frac{16\cdot 10^{10}}{2\cdot 10^{-2}} = 8\cdot 10^{10-(-2)} = 8\cdot 10^{12}$$

3) First convert the base 3 numeral to a base 10 numeral and then to a base 5 numeral:
202110 (base 3) . . . (from right to left):
= 0(1) + 1(3) + 1(9) + 2(27) + 0(81) + 2(243)
= 0 + 3 + 9 + 54 + 0 + 486
= 552 (base 10)
To convert 552 (base 10) to base 5, divide 552 by subsequent powers of five until there is a remainder less than 5:
552 / 125 = 4 with remainder 52.
52 / 25 = 2 with remainder 2.
= 4(125) + 2(25) + 0(5) + 2(1)
= 4202 (base 5)

Category 4
Arithmetic
Meet #3 - January, 2015

1) Find the value of $8^0 + 7^1 + 6^2 + 5^3 + 4^4$

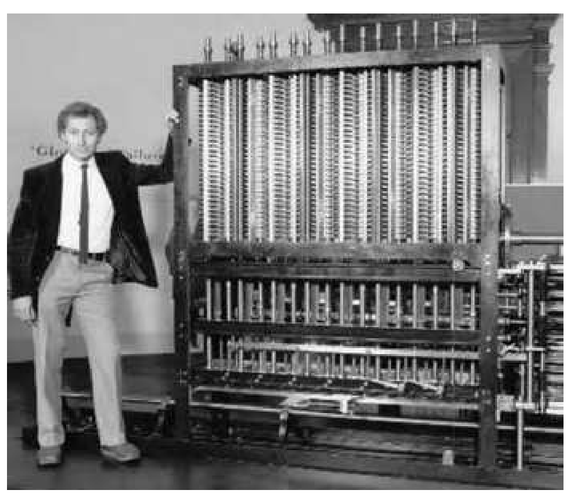
2) Evaluate $\left(\left(\sqrt[3]{\sqrt{64}}\right)^{-1}\right)^{-4}$

3) $A^{\frac{B}{C}}$ **means** $\sqrt[C]{A^B}$ **or, equivalently,** $\left(\sqrt[C]{A}\right)^B$

Find the value of $10{,}000^{\frac{3}{4}} - 64^{\frac{2}{3}} \cdot 32^{\frac{3}{5}}$

On this date, January 8, Charles Babbage patented the first computer in the year 1889.

ANSWERS
1) _______
2) _______
3) _______



Solutions to Category 4
Arithmetic
Meet #3 - January, 2015

Answers
1) 425
2) 16
3) 872

1) $8^0 + 7^1 + 6^2 + 5^3 + 4^4 = 1 + 7 + 36 + 125 + 256 = 425$

2) $\left(\left(\sqrt[3]{\sqrt{64}}\right)^{-1}\right)^{-4} = \left(\left(\sqrt[3]{8}\right)^{-1}\right)^{-4} = \left((2)^{-1}\right)^{-4} = \left(\frac{1}{2}\right)^{-4} = 2^4 = 16$

3) $10{,}000^{\frac{3}{4}} - 64^{\frac{2}{3}} \cdot 32^{\frac{3}{5}} = \left(\sqrt[4]{10{,}000}\right)^3 - \left(\sqrt[3]{64}\right)^2 \cdot \left(\sqrt[5]{32}\right)^3$

$= 10^3 - 4^2 \times 2^3$

$= 1000 - 16 \times 8$

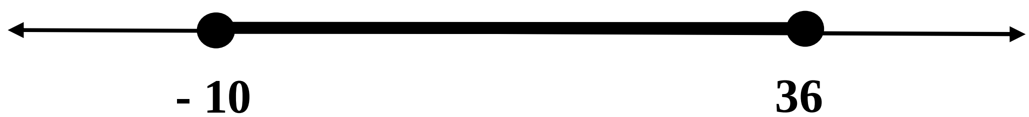
$= 1000 - 128$ **(Multiply before subtracting!)**

$= 872$

Category 5
Algebra
Meet #3 - January, 2015

1) The equation $|X-3|=5$ can be translated as, "The distance between X and 3 on the number line is exactly five units." What are the two values of X that make this equation true? You may list them in the answer box in any order.

2) The graph below is the set of all real values of W that make the following inequality true: $|W-A|\leq C$



What are the values of A and C ?

3) Find the smallest integer value of N that makes the following inequality true:

$$2(3N+7) < 3(4N-5)-9$$

On this date in history - January 8 - The U. S. Mint at Carson City, Nevada, began issuing coins in 1870. Here, the Morgan silver dollar is shown, with the "CC" mint mark shown on the reverse of the coin.

ANSWERS

1) ___ and ___

2) A = ______

C = ______

3) ______

Solutions to Category 5
Arithmetic
Meet #3 - January, 2015

Answers

1) -2 ; 8 (any order)

2) $A = 13$
$C = 23$

3) 7

1) Either $X - 3 = 5$ or $X - 3 = -5$. So, either $X = 8$ or $X = -2$.

2) Students may take a hint from the wording of #1 in order to translate this inequality as , "The distance between W and A is less than or equal to C." If the endpoints, - 10 and 36, are to be equidistant from A, then A is their midpoint, or 13. So, A = 13. The distance between A and either endpoint is 23 units. So, C = 23.

3)

$2(3N + 7) < 3(4N - 5) - 9$	original inequality
$6N + 14 < 12N - 15 - 9$	distribute
$6N - 12N < -14 - 15 - 9$	subtract 12N from both sides
$-6N < -38$	combine like terms
$N > 38/6$	divide both side by - 6 (which changes the sense of the inequality from $<$ to $>$)

The smallest integer value of N that is greater than 38/6 is 7.

Category 6
Team Round
Meet #3 - January, 2015

"The time is always right to do the right thing"
. . . Dr. Martin Luther King, Jr.

1) Juan Nada does mathematics only in base two (binary). He ate 10110 M&Ms every day for 110 days. How many M&Ms did Juan eat in all? Every number written in this problem is in binary. Express your answer as a base ten numeral.

2) From his house, Mike biked 10 miles north, then 27 miles east, 13 miles north, 10 miles east, 8 miles south, and finally 1 mile west. How many miles is Mike from his starting point (in a straight path)?

3) How many square feet are in the area of a right triangle whose hypotenuse is 41 feet and one of whose legs is 9 feet?

4) An elephant at the Southwick Animal Farm in Mendon, MA, is E years old today. In 20 years, she will be E^2 years old. How many years old will she be 7 years from today?

5) The graph below represents all values of U that satisfy the absolute value inequality $|U - Y| < W$. What is the sum of Y + W ?

ANSWERS

1) _______ = A

2) _______ = B

3) _______ = C

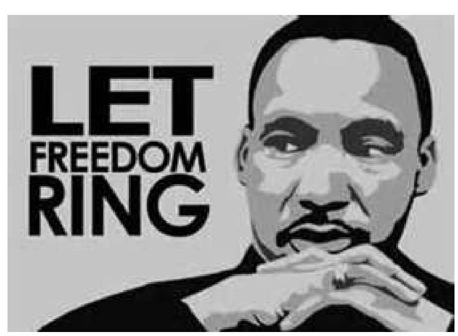
4) _______ = D

5) _______ = E

6) _______

6) Using the answers from questions #1-5, find the value of

$$\left(\sqrt[4]{\frac{AD}{E-3}(A+D)}\right)\left(\sqrt[5]{B+C+2D}\right)$$



Solutions to Category 6
Team Round
Meet #3 - January, 2015

ANSWERS	
1)	132 = A
2)	39 = B
3)	180 = C
4)	12 = D
5)	14 = E
6)	36

1) 10110 (base 2) = 0(1)+1(2)+1(4)+0(8)+1(16) = 0 + 2 + 4 + 0 + 16 = 22
110 (base 2) = 0(1)+1(2)+1(4) = 0+2+4 = 6.
(22)(6) = 132

2) Mike finishes NE of his starting point. Drop a perpendicular from this end point to the eastern extension of his starting point until they meet, forming a right triangle whose legs are 36 and 15. The Pythagorean Theorem may yield some tedious calculations, so scaling down 36 and 15 by a factor three gives values of 12 and 5. This yields a hypotenuse of 13. Then scale back up by a factor of three to get the actual distance from the starting point to the finish point = (13)(3) = 39.

3) Use the Pythagorean Theorem to find the length of the other leg.
$9^2 + L^2 = 41^2$ . . . . $81 + L^2 = 1681$ . . . . $L^2 = 1600$ . . . . $L = 40$.
Area = BH/2 = (40)(9)/2 = 180.

4) Quick guessing and checking yields E = 5, then 5 + 7 = 12.

5) The two limits, - 22 and 14, are 36 units apart. Their midpoint is - 4. So, the distance between U and the midpoint is less than 18 units. Therefore, Y = -4 and W = 18, and Y + W = -4 + 18 = 14.

6) $$\left(\sqrt[4]{\frac{AD}{E-3}(A+D)}\right)\left(\sqrt[5]{B+C+2D}\right)$$

$$= \left(\sqrt[4]{\frac{(132)(12)}{14-3}(132+12)}\right)\left(\sqrt[5]{39+180+2(12)}\right)$$

$$= \left(\sqrt[4]{\frac{(12)(11)(12)}{11}(144)}\right)\left(\sqrt[5]{243}\right) = \left(\sqrt[4]{(12)(12)(12)(12)}\right)(3) = (12)(3) = 36$$