

IMLEM Meet #2
November, 2014

Intermediate Mathematics League of Eastern Massachusetts



Category 1
Mystery
Meet #2 - November, 2014

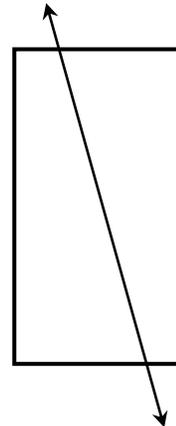


1) A recipe for Thanksgiving cookies contains cinnamon, cloves, and nutmeg in the ratio of 3:4:2. Priscilla made a large batch for the Plimoth crowd that called for 24 teaspoons of cloves. How many teaspoons of cinnamon should she use ?

- 2) $6 \## 4 = 210$
 $9 \## 2 = 711$
 $8 \## 5 = 313$
 $5 \## 2 = 37$
 $7 \## 6 = 113$
 $9 \## 8 = 117$
 $10 \## 6 = 416$
 $15 \## 3 = 1218$

What is the value of $23 \## 8$?

3) A rectangle can be separated into two separate regions by using one straight line, as in the example below. What is the largest number of regions into which a rectangle can be separated by using exactly three straight lines ?



<u>Answers</u>	
1)	_____
2)	_____
3)	_____

**Solutions to Category 1
Mystery
Meet #2 - November, 2014**

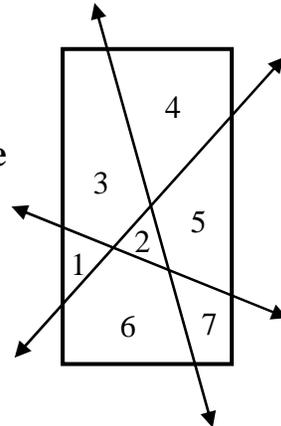
- 1) Cinnamon : cloves : nutmeg = 3 : 4 : 2.
= C : 24 : N

The scale factor is 6, comparing 24 tsp of cinnamon in the whole batch to 4 in the original ratio. So, $(3)(6) = 18$.

- 2) Analyzing the pattern yields the following observation: The difference $X - Y$ in $X \#\# Y$ gives the first digit(s) of the answer, while the sum $X + Y$ gives the final digit(s) of the answer. So, $23 \#\# 8 = (23 - 8)$ for the first digits and $(23 + 8)$ for the final digits = 1531.

<u>Answers</u>	
1)	18
2)	1531
3)	7

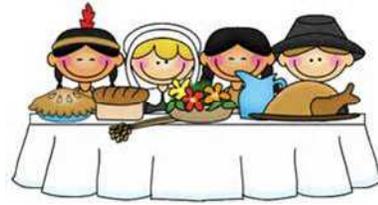
- 3) The diagram to the right is an example of how the three lines may intersect in order to produce the maximum number of sections in the rectangle. Consider . . .



- If the three lines do not intersect, then four regions are formed.
- If the three lines intersect at a point, then six regions are formed.
- If the three lines intersect at two points, then six regions are formed.
- If the three lines intersect at three points, then seven regions are formed, hence the maximum number.

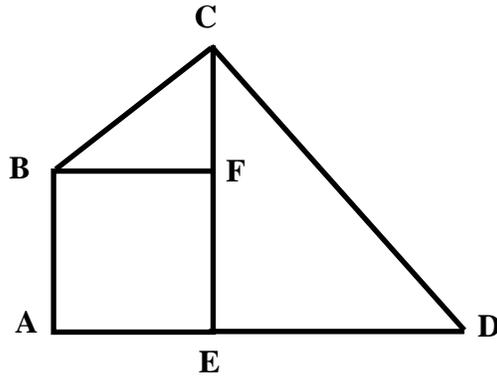
Some arrangements of intersection points may result in fewer regions if the intersection points lie on the sides of the rectangle, but not resulting in more regions.

Category 2
Geometry
Meet #2 - November, 2014

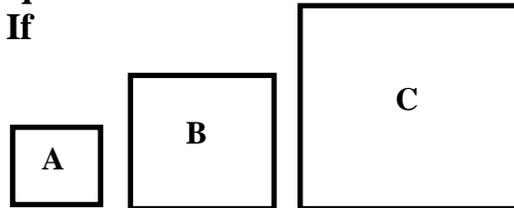


Figures are not necessarily drawn to scale.

- 1) The area of square ABFE is 25 square feet. The area of triangle BFC is 10 square feet. The measure of angle D in triangle CDE is 45 degrees. How many feet are in the length of segment AD ?



- 2) The perimeter of square A is $\frac{2}{3}$ of the perimeter of square B, and the perimeter of square B is $\frac{2}{3}$ of the perimeter of square C. If one side of square A is 4 centimeters, then how many square centimeters are in the area of square C ?



- 3) Triangle XYZ has a perimeter of 10 inches. The lengths of its sides are all whole numbers. If n is the length of side YZ, then what is the difference between the largest and smallest possible values of n ?

<u>Answers</u>	
1)	_____
2)	_____
3)	_____

**Solutions to Category 2
Geometry
Meet #2 - November, 2014**

- 1) Start with the square. If the area is 25, then one side is the square root of 25, or 5. Then the base of the triangle, BF, is also 5. If the area of triangle BFC is 10, then its altitude, CF, is 4. That makes $CE = 5 + 4 = 9$. Because angle D is 45 degrees, right triangle CED is isosceles. So, ED is also 9. Finally, $AD = AE + ED = 5 + 9 = 14$.

Answers

1) 14

2) 81

3) 2

- 2) Since one side of A is 4, so its perimeter is $4(4)$, or 16. Then this 16 is $\frac{2}{3}$ of the perimeter of B, so the perimeter of B is $16(\frac{3}{2})$, or 24. Then 24 is $\frac{2}{3}$ of the perimeter of C, so the perimeter of C is $24(\frac{3}{2})$, or 36. So, one side of C is $\frac{36}{4}$, or 9. The area of C is $9(9)$, or 81.
- 3) This problem utilizes the notion that the sum of any two sides of a triangle must be larger than the third side. Since the perimeter is a fixed value of 10, this chart shows the possibilities for the lengths of the sides (and the ones that fail):

<u>side 1</u>	<u>side 2</u>	<u>side 3</u>	<u>possible?</u>
1	1	8	no
1	2	7	no
1	3	6	no
1	4	5	no
2	2	6	no
2	3	5	no
2	4	4	yes
3	3	4	yes

The first six options fail because the sum of the first two sides does not exceed the third side. The bottom two succeed because the sum of any two of the sides does exceed the third side.

So, the smallest possible value of n is 2 and the largest possible value is 4, and their difference is 2.

Category 3
Number Theory
Meet #2 - November, 2014



- 1) Simplify (reduce to lowest terms) the fraction $\frac{363}{594}$
if $363=3 \times 11^2$ and $594=2 \times 3^3 \times 11$.

- 2) Consider these factorizations:

$$C=2 \times 3^2 \times N \quad \text{and} \quad D=2^2 \times 5 \times N^2$$

If all of the lettered values are positive integers (whole numbers)
and $D = 1620$, then what is the value of C ?

- 3) Jacqueline watches the TV sitcom "Big Bang Theory" every 12 days. She watches "Cosmos" every 18 days and "Scorpion" every 28 days. If she watched all three shows on a Tuesday, on what day of the week will she next watch all three shows ? Assume that all of the shows are aired every day of the week.

Answers

1) _____

2) _____

3) _____

**Solutions to Category 3
Number Theory
Meet #2 - November, 2014**

Answers

1) $\frac{363}{594} = \frac{3 \times 11^2}{2 \times 3^3 \times 11} = \frac{11}{2 \times 3^2} = \frac{11}{18}$

1) $\frac{11}{18}$

2) 162

3) Tuesday

2) If $D=1620$, then $1620=2^2 \times 5 \times N^2$ and

$81=N^2$ and $N=9$. Then substituting 9 for N in the first equation,

$C=2 \times 3^2 \times 9$ and $C=162$.

3) First, find the LCM of 12, 18, and 28. Then divide the answer by 7 to see how many 7-day cycles there are. The remainder will be added onto Tuesday to determine the day of the week that all three shows will next be watched.

$12=2^2 \times 3$ and $18=2 \times 3^2$ and $28=2^2 \times 7$.

The LCM is the product of all the different factors, with each raised to the highest power that it appears in any of the factorizations. So,

the LCM is $2^2 \times 3^2 \times 7$ or 252.

Now 252 divided by 7 is 36 with a remainder of 0, so all three shows will next be watched on a Tuesday !

A shorter concluding step would be to acknowledge that the prime factorization of 252 contains the number 7 . . . thus making 252 divisible by 7 and there would be no remainder.

Category 4
Arithmetic
Meet #2 - November, 2014

- 1) Last year, the number of World War II veterans marching in parades in Massachusetts was about 720, while the number in Pennsylvania was 640. In anticipation of the 70th anniversary of the end of World War II, those numbers are expected to increase by 20% next year. How many more Massachusetts veterans than Pennsylvania veterans are expected to march in next year's parades? (Note: The image at the bottom of this page is the World War II monument in Washington, D.C. which was first open to the public in 2004.)

- 2) Connor paid \$ 28.14 for a science kit, including a 5% sales tax. How many dollars was the tax? Express your answer as a decimal.

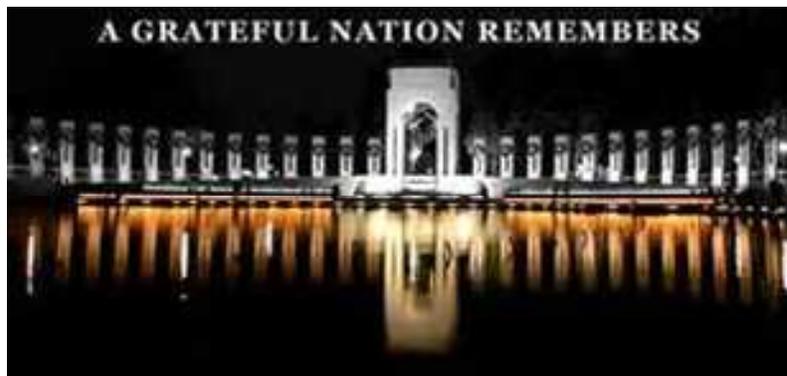
- 3) Evan spent $\frac{2}{9}$ of his year's allowance on gifts for his family and friends and $\frac{1}{5}$ on items for himself. He put the rest into his savings account at the bank. If he saved \$468, then how many dollars was his yearly allowance?

ANSWERS

1) _____

2) \$ _____

3) \$ _____



**Solutions to Category 4
Arithmetic
Meet #2 - November, 2014**

Answers

- 1) Calculate as follows:

$$\begin{aligned} & (120\% \text{ of } 720) - (120\% \text{ of } 640) \\ &= (1.2 \times 720) - (1.2 \times 640) \\ &= 864 - 768 \\ &= 96 \end{aligned}$$

1) 96

2) 1.34

3) 810

- 2) 105% of the cost of the science kit is \$28.14.

So, 28.14 divided by 1.05 gives the cost of the science kit = \$26.80.

The tax is 5% of the cost of the kit = $0.05 \times 26.80 = \$1.34$.

- 3) The fraction of Evan's allowance that is set aside for savings is

$$\begin{aligned} & 1 - (2/9 + 1/5) \\ &= 1 - (10/45 + 9/45) \\ &= 1 - (19/45) \\ &= 26/45 \end{aligned}$$

\$468 is $26/45$ of his yearly allowance, so his yearly allowance is

$$\begin{aligned} & 468 \text{ divided by } 26/45 \\ &= 468 \times 45/26 \\ &= 810. \end{aligned}$$

Category 5

Algebra

Meet #2 - November, 2014



- 1) If $2A + 6B = 26$, then what is the value of $5A + 15B - 10$?

- 2) Mutt and Jeff live 100 miles apart. Each rides a bicycle toward the other's house along a straight road connecting the two. Mutt pedals at a constant rate of 8 miles per hour (mph) while Jeff pedals at a constant rate of 12 miles per hour. If Mutt and Jeff leave their houses at the same time, then how many miles are they from Jeff's house when they meet ?

Footnote: Mutt and Jeff were characters in a long-running comic strip by the same name from 1907 to 1983, published in daily newspapers around the world. For more information:
http://en.wikipedia.org/wiki/Mutt_and_Jeff

- 3) Rick travelled from Lompoc to Visalia, averaging 60 kilometers per hour (kph). He averaged 90 kilometers per hour on the return trip home, following the same route. What was Rick's average rate of speed, in kilometers per hour, for the entire round trip ?

Answers

1) _____

2) _____

3) _____

**Solutions to Category 5
Algebra
Meet #2 - November, 2014**

1) The expression $5A + 15B$ is 2.5 times as great as $2A + 5B$. So, $(2.5)(26) - 10 = 65 - 10 = 55$.

2) Let X = the number of hours that each man travels. Then $8X$ is Mutt's distance and $12X$ is Jeff's distance. Since they travel toward one another and meet at a point in between their houses, they travel a total of 100 miles.

$$8X + 12X = 100$$

$$20X = 100$$

$$X = 5$$

In 5 hours, Jeff travels $(12)(5)$, or 60 miles, and is therefore 60 miles from his house when they meet.

3) The average rate of speed, in kph, for the entire round trip is equal to the total distance in km divided by the total time in hours. The exact distance between Lompoc and Visalia is irrelevant. For the sake of the numbers in this problem, let's use a multiple of 60 and 90, such as 540 km, as the distance from Lompoc to Visalia. So, $540 / 60 = 9$ hours. Also, $540 / 90 = 6$ hours.

Therefore, total distance divided by total time

$$= (2)(540) / (6 + 9)$$

$$= 1080 / 15$$

$$= 72 \text{ kph.}$$

Answers

1) 55

2) 60

3) 72

Category 6

Team Round

Meet #2 - November, 2014

- 1) If $N = 4P + 26$ and P is any positive whole number, then N could be divisible by all of the following single-digit numbers except for which one? 2 3 4 5 6 7
- 2) M , A , T , and H are consecutive multiples of 7. $M < A < T < H$. What is the value of $(M - T)(H - A)$?
- 3) The lengths of two sides of a triangle are 8 and 11 units. The third side is a whole number of units. How many units are in the largest possible perimeter of this triangle?
- 4) Find the GCF (greatest common factor) of T and Y if $TY = 3024$ and the LCM (least common multiple) of T and $Y = 504$.
- 5) Avid Gardner would like to cover two spaces of his yard with mulch. One sealed bag of mulch covers 15 square feet. One yard space is a square and the other is a rectangle, both of which have the same area. The length of the rectangle is 6 feet longer than the length of the square and 10 feet longer than the width of the rectangle. How many bags of mulch must Avid purchase in order to be able to cover his garden spaces?

ANSWERS

1) _____ = A

2) _____ = B

3) _____ = C

4) _____ = D

5) _____ = E

6) _____

- 6) Using the answers from questions #1-5, find the value of

$$2D^3 - 5AB - C + E$$

**Solutions to Category 6
Team Round
Meet #2 - November, 2014**

ANSWERS

1) $4 = A$

2) $-196 = B$

3) $37 = C$

4) $6 = D$

5) $20 = E$

6) 4335

1) If P is a whole number, then $4P$ is divisible by 4, but $4P + 26$, or $4P + 6(4) + 2$ is always 2 more than a multiple of 4.

2) Since M and T have a difference of 14, and $M < T$, then $M - T = -14$. H is 14 greater than A , so $H - A = 14$ and $(M - T)(H - A) = (-14)(14) = -196$.

3) If the two given sides are "hinged," to make a straight angle, then their endpoints would be 19 units apart. Closing the hinge just a tad could accommodate a third side of 18 units. The resulting triangle would have a

maximum perimeter of $8 + 11 + 18$, or 37 units.

4) Use this fact: (the GCF of A and B)(the LCM of A and B) = AB .
So, (the GCF of A and B)(504) = 3024
Therefore, $3024 / 504 =$ the GCF of A and $B = 6$.

5) Let $X =$ the length of the side of the square.
Then $X + 6 =$ the length of the rectangle
and $X + 6 - 10 =$ the width of the rectangle.

The two areas are equal, so (area of square) = (area of rectangle)
 $(X)(X) = (X + 6)(X + 6 - 10)$
 $(X)(X) = (X)(X) - 2X - 24$
 $2X = 24$
 $X = 12$

So, each area = 144 and the total area of the two garden spaces = 288 square feet. Divide the total area by 15 to compute the number of bags of mulch that must be purchased = $288 / 15 = 19$ with a remainder of 3 sq ft. So, 20 bags must be purchased.

6) $2D^3 - 5AB - C + E \dots = 2 \cdot 6^3 - 5 \cdot 4 \cdot (-196) - 37 + 20 \dots = 2 \times 216 - (-3920) - 17 \dots$
 $= 432 + 3920 - 17 \dots = 4335$