

IMLEM Meet #1
October, 2014

Intermediate
Mathematics League
of
Eastern Massachusetts



Category 1
Mystery
Meet #1 - October, 2014



1) **Bill created a story intended to be kept secret. He shared it with six of his friends. Each of his friends shared that secret with nine other friends. How many people now know this secret?**

2) **What is the value of C in this sequence? 66 34 18 C 6 ...**

3) **There are 400 people in the cafeteria. If there are 40 more adults than children and 40 more adult men than adult women, then how many adult men are in the cafeteria?**

<u>Answers</u>	
1)	_____
2)	_____
3)	_____

**Solutions to Category 1
Mystery
Meet #1 - October, 2014**

1) **Bill + 6 friends + (9)(6) more friends**
= 1 + 6 + 54
= 61

- 2) **Each term is one more than half the previous term.**
For example, 34 is one more than half of 66.
So, C is one more than half of 18, so C = 10. To
check, 6 should be one more than half of 10 . . . and
it is!

- 3) **Many students may opt for using the strategy of "guess and check"**
and should be able to solve rather quickly. For those with some
algebraic expertise:

Let **X = the number of adult women**
 X + 40 = the number of adult men
 2X + 40 = the total number of adults
 2X + 40 - 40 = the number of children

Then (the number of children) + (the number of adults) = 400
(2X + 40 - 40) + (2X + 40) = 400
4X + 40 = 400
4X = 360
X = 90

So, there are 90 women, 130 men, and 180 children, for a total of
400 people.

Answers

1) 61

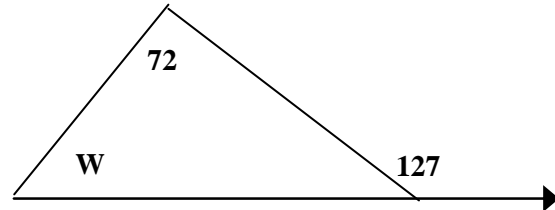
2) 10

3) 130

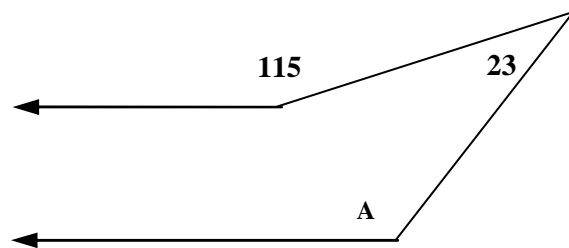
Category 2
Geometry
Meet #1 - October, 2014

For all problems below, angles are marked in degrees as shown.
Figures are not necessarily drawn to scale.

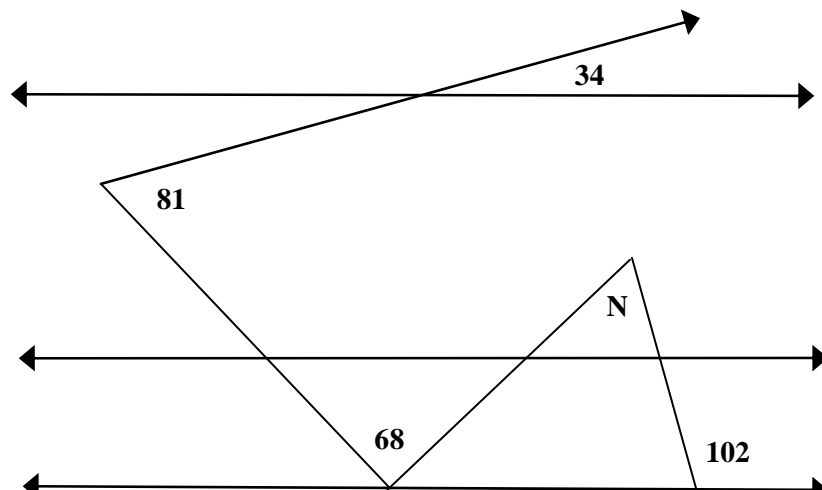
- 1) In the figure at the right, how many degrees are in the complement of angle W ?



- 2) The two horizontal lines are parallel. How many degrees are in angle A if it is less than 180 degrees ?



- 3) All three horizontal lines are parallel. How many degrees are in the supplement of angle N ?



<u>Answers</u>	
1)	_____
2)	_____
3)	_____

Solutions to Category 2
Geometry
Meet #1 - October, 2014

- 1) The supplement of the 127-degree angle is $(180 - 127)$, or 53 degrees and lies in the interior of the triangle. The remaining angle, W is $(180 - (72 + 53))$, or 55 degrees, as the sum of the angles of a triangle is 180 degrees. The question, however, asks for the *complement* of angle W. So, $90 - 55 = 35$ degrees.

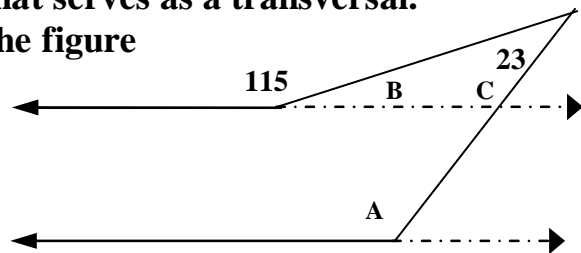
<u>Answers</u>	
1)	35
2)	92
3)	143

- 2) Strategy: Extend the parallel lines to the right, creating angles with the far-right slanted line segment that serves as a transversal.

The triangle in the upper-right of the figure has three angles: B, C, and 23.

Angle B is the supplement of 115 and measures 65 degrees. Angle C = $180 - (65 + 23)$, or 92 degrees.

Angle A is equal to angle C (corresponding angles). Therefore, angle A = 92 degrees.



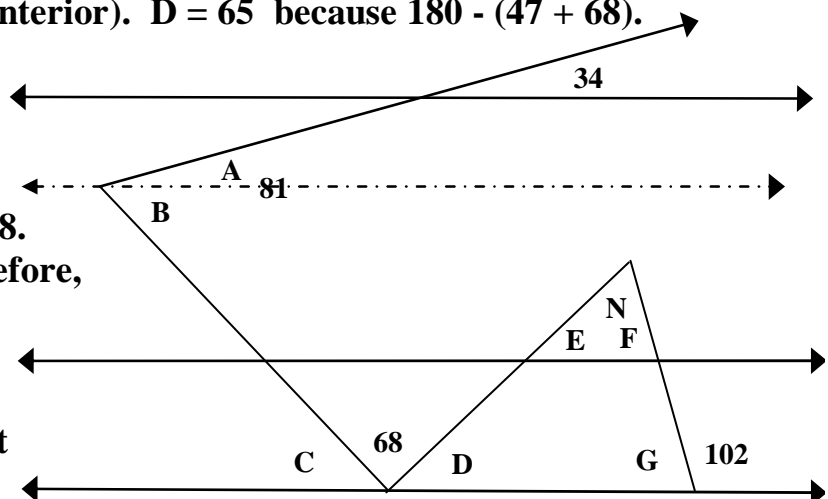
- 3) Strategy: draw a line through the vertex of the 81 degree angle that is parallel to the three horizontal parallel lines. The 81 degree angle is now subdivided into two angles: A = 34 (corresponding angles are equal) and B = 47 ($81 - 34 = 47$).

B = C = 47 (alternate interior). D = 65 because $180 - (47 + 68)$.

E corresponds to D = 65. G is the supplement of 102, so G = 78.

F corresponds to G = 78. $N + E + F = 180$. Therefore, $N = 180 - (65 + 78) = 180 - 143 = 37$.


Finally, the supplement of N is $(180 - 37) = 143$.



Category 3
Number Theory
Meet #1 - October, 2014



1) What is the sum of all the prime numbers between 40 and 60 ?

2)  has an odd number of factors.

 is divisible by 3.

 > 20 .

 < 70 .

What is the value of  ?

3) How many of the three-digit numbers that can be made using the digits 1, 3, and 5 are not multiples of three ? Each digit can be used more than once.

Answers


1) _____

2) _____

3) _____

**Solutions to Category 3
Number Theory
Meet #1 - October, 2014**

1) $41 + 43 + 47 + 53 + 59 = 243$

2)  is a perfect square if it has an odd number

of factors. If that square is between 20 and 70, then it could be any of these: 25, 36, 49, or 64.

The only one divisible by 3 is 36.

3) The full list of three-digit numbers is: 111, 113, 115, 131, 133, 135, 151, 153, 155, 311, 313, 315, 331, 333, 335, 351, 353, 355, 511, 513, 515, 531, 533, 535, 551, 553, and 555. Checking that there should be twenty-seven numbers: $3 \times 3 \times 3 = 27$.

The numbers that are divisible by 3 are those whose digit-sum is a multiple of 3: 111, 135, 153, 315, 333, 351, 513, 531, and 555.

That is nine numbers. Therefore, there are $27 - 9$, or 18 numbers, that are not multiples of three.

Answers

1) 243

2) 36

3) 18

Category 4
Arithmetic
Meet #1 - October, 2014



- 1) Using the standard order of operations, find the value of

$$2[2(2+2 \times 2)]^2$$

- 2) The arithmetic mean (average) of 17, 26, 42, and 59 is equal to the arithmetic mean of 19 and N. What is the value of N?
- 3) In preparation for handing out candy at Halloween, Frank bought forty 30-cent candy bars, X 50-cent candy bars, and fifteen 75-cent candy bars. If the median cost of all the candy bars is 50 cents, and X is a whole number, then what is the smallest possible value of X?

Answers

1) _____

2) _____

3) _____

**Solutions to Category 4
Arithmetic
Meet #1 - October, 2014**

1)

$$\begin{aligned} & 2[2(2+2 \times 2)]^2 \\ = & 2[2(2+4)]^2 \\ = & 2[2(6)]^2 \\ = & 2[12]^2 \\ = & 2[144] \\ = & 288 \end{aligned}$$

Answers

1) 288

2) 53

3) 26

- 2) The average of 17, 26, 42, and 59 is their sum divided by 4. The sum is 144. Dividing 144 by 4 yields 36. Then 36 is the average of 19 and N. 19 is 17 less than 36, so N is 17 more than 36. So, N = 53.
- 3) Since there are 40 of the least expensive candy bars, and only 15 of the most expensive, then we have to compensate by having at least 26 of the mid-priced candy bars in order for the 50-cent bar to be the median price.

Category 5
Algebra
Meet #1 - October, 2014



- 1) If $3W + 7 = 34$ and $5A - 9 = -39$, then what is the value of $-2WA$?
- 2) The letters of the alphabet, $\{A, B, C, \dots, Z\}$, correspond to the consecutive integers $\{-8, -7, -6, \dots, 17\}$. For example, $A = -8$, $B = -7$, $N = 5$, and so on. The number, zero, is included among the integers. Find the sum of the numbers corresponding to the letters in the following word:

HALLOWEEN

- 3) Find the value of N if

$$2(N + 1) + 3(4N - 5) - 6(7N + 8) - 9(10N - 11) = 12(13 - 14N) + 32$$

<u>Answers</u>	
1)	_____
2)	_____
3)	_____

**Solutions to Category 5
Algebra
Meet #1 - October, 2014**

$$\begin{array}{ll}
 1) \quad 3W + 7 = 34 & 5A - 9 = -39 \\
 \quad \quad 3W = 27 & \quad \quad 5A = -30 \\
 \quad \quad \quad W = 9 & \quad \quad \quad A = -6
 \end{array}$$

$$\text{So, } -2WA = (-2)(9)(-6) = 108$$

Answers

1) 108

2) 14

3) 3

$$\begin{array}{cccccccccccccccccccc}
 2) \quad A & B & C & D & E & F & G & H & I & J & K & L & M & N & O & P & Q & R & S & T \\
 -8 & -7 & -6 & -5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11
 \end{array}$$

$$\begin{array}{cccccc}
 U & V & W & X & Y & Z \\
 12 & 13 & 14 & 15 & 16 & 17
 \end{array}$$

$$\begin{aligned}
 \text{HALLOWEEN} &= (-1) + (-8) + (3) + (3) + (6) + (14) + (-4) + (-4) + (5) \\
 &= 14
 \end{aligned}$$

$$\begin{aligned}
 3) \quad 2(N + 1) + 3(4N - 5) - 6(7N + 8) - 9(10N - 11) &= 12(13 - 14N) + 32 \\
 2N + 2 + 12N - 15 - 42N - 48 - 90N + 99 &= 156 - 168N + 32 \\
 - 118N + 38 &= 188 - 168N \\
 50N &= 150 \\
 N &= 3
 \end{aligned}$$

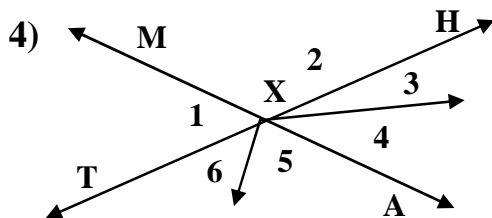
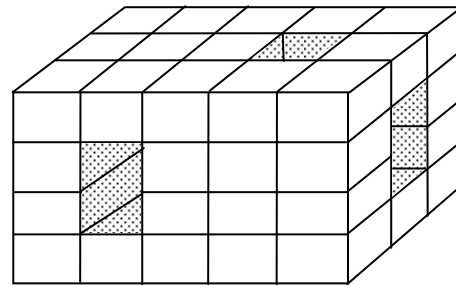
Category 6

Team

Meet #1 - October, 2014

- 1) In 1492, Columbus sailed the ocean blue. How many years later did Magellan complete the very first trip around the world in a year that is the first perfect square integer that is greater than 1492?
- 2) C is an "abundant" number if the sum of its factors (excluding C) is greater than C. What is the sum of all abundant numbers greater than 40 but less than 50 ?

- 3) How many of the smallest cubes are there in this figure? All holes extend completely through the figure.



Lines MA and TH intersect at point X. Angles 1 and 4 are complementary. Angles 2 and 6 are supplementary. Angle 3 measures 18 degrees. How many degrees are in angle 5 ?

- 5) A number is divisible by 18 if it is divisible by 9 and by 2. How many numbers between 2300 and 2600 are divisible by 18 ?

ANSWERS

- 1) _____ = A
- 2) _____ = B
- 3) _____ = C
- 4) _____ = D
- 5) _____ = E
- 6) _____

- 6) Using the answers from questions #1-5, add the number of factors of B to the number of prime factors of (D - C). Then multiply the answer by the number of composite factors of (A - E). What is the sum of all of the factors of the final answer ?

**Solutions to Category 6
Team
Meet #1 - October, 2014**

ANSWERS

1) $29 = A$

2) $90 = B$

3) $44 = C$

4) $72 = D$

5) $17 = E$

6) 96

1) The square number is 1521, which is the square of 39. $1521 - 1492 = 29$.

2) The abundant numbers between 40 and 50 are 42 and 48. Their sum is 90.

3) Any way you count the cubes, the total is 44.

$$\begin{array}{r}
 4) \#1 + \#2 + \#3 + \#4 + \#5 + \#6 = 360 \\
 \#1 + \#4 + \#2 + \#6 + \#3 + \#5 = 360 \\
 90 + 180 + \#3 + \#5 = 360 \\
 270 + \#3 + \#5 = 360 \\
 \#3 + \#5 = 90 \\
 18 + \#5 = 90 \\
 \#5 = 72
 \end{array}$$

5) The first number larger than 2300 that is divisible by 18 is 2304, because it is even and the sum of its digits is 9. The difference $2600 - 2300$, or 300, will accommodate $300 / 18$, or 16 more multiples of 18, with 12 (the remainder of that division) left over. So, there are 17 multiples of 18 between 2300 and 2600.

6) There are twelve *factors* of 90, namely: 1, 2, 3, 5, 6, 9, 10, 15, 18, 30, 45, and 90.

There are two *prime* factors of $(D - C)$, or $72 - 44$, or 28), namely: 2 and 7.

This sum is $12 + 2$, or 14.

There are three *composite* factors of $(A - E)$, or $29 - 17$, or 12, namely: 4, 6, and 12.

Multiplying the previous sum of 14 by 3 (the number of composite factors of 12) gives us 42.

The *factors* of 42 are: 1, 2, 3, 6, 7, 14, 21, and 42.

Their sum is: $1 + 2 + 3 + 6 + 7 + 14 + 21 + 42 = 96$.