

IMLEM Meet #4
February, 2014

**Intermediate
Mathematics League
of
Eastern Massachusetts**

50th anniversary edition

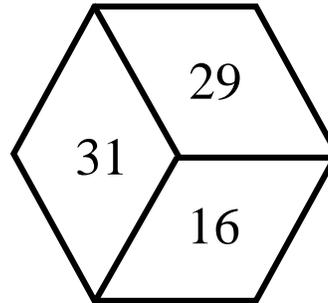
**Category 1
Mystery**

Meet #4 - February, 2014

Calculator Meet

50th anniversary edition

- 1) This cube has a number on each of its six faces. If the sum of the numbers on each pair of opposite faces is 78, then what is the sum of the three faces not shown?



- 2) The Shrewsbury Spocks soccer squad has won 17 games and lost 12 games. What is the least number of additional games that the Spocks must play so that they will have won 75% of all their games?
- 3) Mike takes 112 minutes to write 16 postcards. Sully writes four times as fast as Mike. How many postcards can Sully write in 14 hours?

ANSWERS

- 1) _____
2) _____
3) _____

Category 1 - Solutions

Mystery

Meet #4 - February, 2014

<u>ANSWERS</u>
1) 158
2) 19
3) 480

- 1) The number opposite 31 is 47 because $31 + 47 = 78$. The number opposite 29 is 49 because $29 + 49 = 78$. The number opposite 16 is 62 because $16 + 62 = 78$. The sum of the numbers on the faces not shown is $47 + 49 + 62 = 158$.

- 2) To minimize the number of extra games played, consider that the team must win every game played until 75% of all the games is won. Through trial and error with a calculator, students may try adding the same number to both the numerator and denominator of $17 / 29$ (the current fraction of games that the Spocks have won). The correct number is 19, as $(17 + 19) / (29 + 19) = 36 / 48 = 0.75 = 75\%$.

An algebraic approach would yield a faster result. Let X = the least number of games to be won. Then

$$\begin{aligned}\frac{17+X}{29+X} &= 0.75 \\ 17+X &= 0.75(29+X) \\ 17+X &= 21.75+0.75X \\ 0.25X &= 4.75 \\ X &= 19\end{aligned}$$

Therefore, the Spocks must play a minimum of 19 games in order to have won 75% of all its games.

- 3) Divide 112 by 16 to find that it takes 7 minutes for Mike to write one postcard. Sully writes four times as fast, so it takes Sully $7 / 4$, or 1.75 minutes to write one postcard. Convert 14 hours to minutes: $(14)(60) = 840$ minutes. Divide 840 by 1.75 to find that Sully can write 480 postcards.

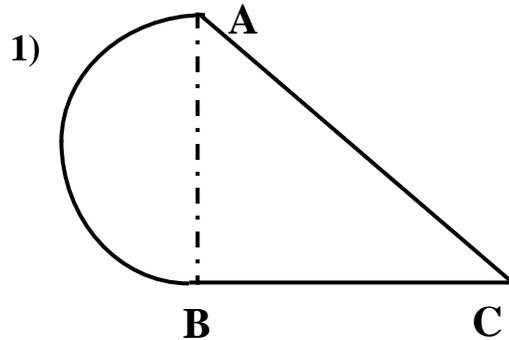
Category 2

50th anniversary edition

Geometry

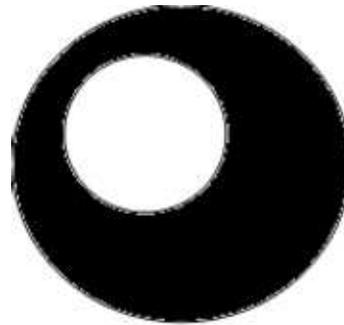
Meet #4 - February, 2014

Calculator Meet



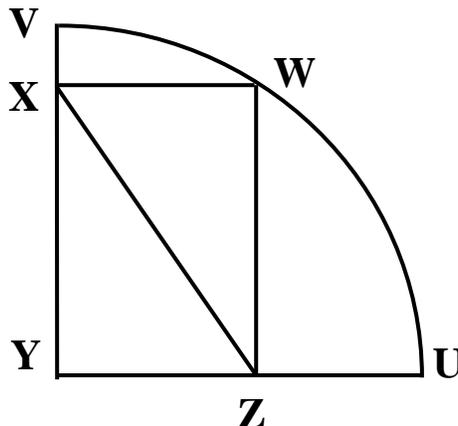
Angle ABC is a right angle. AB is the diameter of a semi-circle. How many square cm are in the area of the entire figure if $AC = 51$ cm and $BC = 45$ cm? Use $\pi \approx 3.1$.

- 2) The circumference of the larger circle is 46π . The circumference of the smaller circle is 14π . What fractional part of the larger circle is shaded? Express your answer as a common fraction (lowest terms).



- 3) Arc VWU is a quarter-circle with point Y at the center. $XZ = 10$ cm. The sum of the length and width of rectangle $WXYZ$ is 13 cm. How many centimeters are in the perimeter of the figure bounded by the points $VWUZ XV$? Use $\pi \approx 3.14$.

<u>ANSWERS</u>	
1) _____	sq. cm
2) _____	
3) _____	cm



Solutions to Category 2
Geometry
Meet #4 - February, 2014

<u>Answers</u>
1) 763.2
2) $\frac{480}{529}$
3) 32.7

- 1) Use the Pythagorean Theorem to find the length of diameter AB:

$$\begin{aligned} (AB)^2 + 45^2 &= 51^2 \\ (AB)^2 + 2025 &= 2601 \\ (AB)^2 &= 576 \\ AB &= \sqrt{576} \\ AB &= 24 \end{aligned}$$

So, the radius of AB is half of 24, or 12.

The total area of the entire figure =

(area of semi-circle) + (area of triangle)

$$\begin{aligned} &= \frac{1}{2}\pi R^2 + \frac{1}{2}bh \\ &= \frac{1}{2}(3.1) \cdot (12^2) + \frac{1}{2}(45) \cdot (24) \\ &= 223.2 + 540 \\ &= 763.2 \end{aligned}$$

- 2) Using the formula for the circumference of a circle, $C=2\pi r$, we find that the radius of the smaller circle is 7 and the radius of the larger circle is 23. Using the formula for the area of a circle, πr^2 , we find that the area of the smaller circle is 49π and the area of the larger circle is 529π . By subtracting the area of the smaller circle from the area of the larger circle, we get the area of the shaded region = 480π .

Therefore, the fractional part of the larger circle that is shaded is

$$\frac{480\pi}{529\pi} = \frac{480}{529}$$

- 3) The key that unlocks this puzzle is the notion that the diagonals of a rectangle are congruent, so that $XZ = YW = 10 \text{ cm} =$ the radius of the circle $= YU = VY$.

The length of the arc VWU, the quarter-circle, is $0.25(2)(3.14)(10)$, or 15.7 cm.

$$VX + XY = \text{radius} = 10 \quad \text{and} \quad YZ + ZU = \text{radius} = 10.$$

$$VX + XY + YZ + ZU = 10 + 10 = 20$$

$$VX + (\text{length of rectangle} + \text{width of rectangle}) + ZU = 20$$

$$VX + (13) + ZU = 20, \text{ therefore } VX + ZU = 7. \text{ (Tricky, huh!!)}$$

$$\text{So, perimeter of VWUZ XV} = 7 + 10 + 15.7 = 32.7 \text{ cm.}$$

Category 3
Number Theory

Calculator meet

Meet #4 - February, 2014

50th anniversary edition

1) A fictional street - Velevis Avenue in Topsfield - has houses numbered in this arithmetic sequence: 1, 7, 13, 19, 25, and so on. What is the number on the 43rd house on Velevis Avenue?

2) As a retirement gift, Mrs. Mosca gave Mr. Mosca a clock. Instead of the customary 12 hours, the clock had 7 days, as shown. The day hand points to Saturday. If the Moscas take a 100-day vacation starting on Sunday, to what day will the hand point when they return if they return on the 100th day at about the same time of day that they left?



2) Richie Lottabux, a local millionaire, decided to make good on his New Year's resolution by giving away a total of at least one million dollars to needy individuals during the month of February. He gave away the following amounts: \$1 on the 1st, \$2 on the 2nd, \$4 on the 3rd, \$8 on the 4th, and \$16 on the 5th. By the 5th of February, Richie had given away a total of \$31. On which date had he given away an amount that brought the month's total over \$1,000,000 for the first time?

Answers

1) _____

2) _____

3) Feb. _____

Solutions to Category 3
Number Theory
Meet #4 - February, 2014

<u>Answers</u>	
1)	253
2)	Monday
3)	20

- 1) # of term: 1 2 3 4 5 N 43
value of term: 1 7 13 19 25 $6N - 5$ 253
- 2) 100 divided by 7 = 14 with remainder 2. So, the Moscas went on vacation for 14 weeks and two days, thus returning on a Monday.

3)	<u># of day</u>	<u># of dollars</u>	<u>total so far</u>
	1	1	1
	2	2	3
	3	4	7
	4	8	15
	5	16	31
	6	32	63

Students may continue using their calculators until they arrive at the 20th day's total exceeding \$1,000,000 for the first time.

An astutely observant student may notice that the total is always one less than a power of two indicated on that day. For example, on the 6th day, the total donated was \$63, or $2^6 - 1$. The total donated by the 19th day was \$524,287, or $2^{19} - 1$ and the total donated by the 20th was \$1,048,576, or $2^{20} - 1$.

**Category 4
Arithmetic**

Calculator Meet

Meet #4 - February, 2014

50th anniversary edition

- 1) Victor bought a new silver dollar in 2009 for \$32.00. It has since increased in value by 30%. What is its current value?

- 2) Li-Mei folds a rectangular sheet of normal-sized paper in half, lengthwise, and then again in half but in the opposite direction. It is said that no normal-sized sheet of paper can be folded in the manner more than six times. Assume that Li-Mei is able to make six folds. The surface area of the resulting rectangle is what percent of the surface area of the original paper? Round your answer to the nearest tenth of a percent.

- 3) The formula for compound interest is $A = P(1+r)^t$ provided that
P = the principal, or amount invested,
r = the annual (yearly) rate at which the principal is invested,
t = the amount of time, in years, that the money is invested, and
A = the total amount of money, including the principal and the interest.
Saul found his great-grandfather's bank book in a box in the attic. His savings account on February 14, 1928 had a balance of \$7.12. Assuming an average rate of 5% over the course of time, how many dollars is the savings account worth on February 14, 2014? Round your answer to the nearest dollar.

ANSWERS

1) \$ _____

2) _____ %

3) \$ _____

Solutions to Category 4
Arithmetic
Meet #4 - February, 2014

Answers

1) 41.60

2) 1.6

3) 473

$$\begin{aligned} 1) \quad & 32 + 30\% (32) \\ & = 32 + 9.6 \\ & = 41.6 \end{aligned}$$

41.60 is also acceptable (standard form for dollars and cents)

$$\begin{aligned} 2) \quad & 50\% \text{ of } 1 \\ & = (0.5)(0.5)(0.5)(0.5)(0.5)(0.5)(1) \\ & = 0.015625 \\ & = 1.6 \% \text{ (rounded to the tenth of a percent)} \end{aligned}$$

Alternate solution: $\left(\frac{1}{2}\right)^6 = \frac{1}{64} = 0.015625 = 1.6 \%$

3) $A = P(1+r)^t$

$$A = 7.12(1+0.05)^{2014-1928}$$

$$A = 7.12(1.05)^{86}$$

$$A = 7.12(66.41707)$$

$$A = 472.8895$$

$$A = 473 \text{ (rounded to the nearest whole number)}$$

Category 5
Algebra

Calculator Meet

Meet #4 - February, 2014

50th anniversary edition

- 1) **A four-foot tall stick, placed vertically into the ground, casts a seven-foot long shadow onto the ground late in the afternoon. How long a shadow does the 555-foot tall Washington Monument cast at the same time of day in the same District? Round your answer to the nearest foot.**

- 2) **My favorite cantaloupe weighs $\frac{5}{6}$ of its own weight plus $\frac{5}{6}$ of a pound. How many pounds does my favorite cantaloupe weigh? (a classic from the very first IMLEM competition in 1963)**

- 3) **Rob and Liz live 390 miles apart. They each leave their homes at the same time and head in each other's direction on the same road. Rob's average speed is 30 miles per hour less than Liz' average speed. After three hours on the road, they pass each other! How many miles per hour was Liz travelling?**

ANSWERS

1) _____ feet

2) _____ pounds

3) _____ mph

Solutions to Category 5
Algebra
Meet #4 - January, 2014

Answers

1) 971

2) 5

3) 80

1) Solving a proportion should help:

$$\frac{4}{7} = \frac{555}{X} \quad \text{So, } 4X = (7)(555),$$

$$4X = 3885, \quad \text{and } X = 971.25$$

Rounded to the nearest foot, the shadow is about 971 feet long.

2) Let X = the weight of the cantaloupe. So, $\frac{5}{6}X + \frac{5}{6} = X$ and

$$\frac{5}{6} = \frac{1}{6}X \quad \text{So, multiplying both sides by 6 yields } 5 = X. \quad \text{So, my}$$

favorite cantaloupe weighs 5 pounds.

3) Rate \times Time = Distance

Let S = Rob's rate of speed

then $S + 30$ = Liz' rate of speed

Since Rob and Liz travel toward one another, then by the time they pass each other, they will have travelled a total distance of 390 miles while taking the same amount of time to do so (3 hours).

$$\text{Rob's distance} + \text{Liz' distance} = 390$$

$$(\text{Rob's rate})(\text{Rob's time}) + (\text{Liz' rate})(\text{Liz' time}) = 390$$

$$(S)(3) + (S + 30)(3) = 390$$

$$3S + 3S + 90 = 390$$

$$6S + 90 = 390$$

$$6S = 300$$

$$S = 50$$

$$S + 30 = 80$$

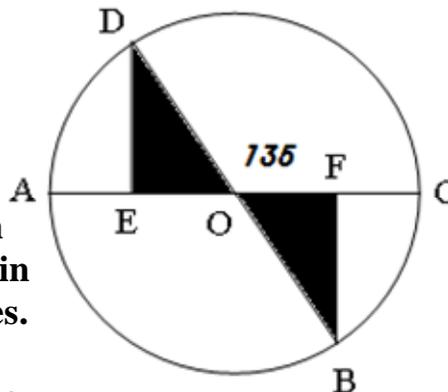
Therefore, Liz' rate of speed was 80 miles per hour.

Category 6

50th anniversary edition

Team Round

Meet #4 - February, 2014



1) In the circle at the right, O is the center, DE and BF are each perpendicular to AC, AC and DB are diameters, each with length 52 feet. How many square feet are in the shaded area? Angle DOC = 135 degrees.

2) Neptunia captured some sea creatures at the beach and noticed that a) the ratio of sand dollars to periwinkles was 3:8, and b) the ratio of sand dollars to fiddler crabs was 4:7. If there were 60 more periwinkles than sand dollars, then how many fiddler crabs were there?

3) Find the value of E in this geometric sequence:

7 A B C 1792 E F 114,688

4) Lowah Costa, a store manager, marked down all prices by 60% in preparation for a special sale event. The following day, Lowah was informed that the sale would occur a month later. By what per cent did she need to mark up the sale price in order to restore the original price of each item?

5) Route 495 encircles Washington, D.C. There are 37 places to enter or exit this highway (called exits) numbered from 1 through 37,

inclusive. Sheldon entered the highway at Exit #23. Let's say that the exits are two miles apart along the road. Sheldon got distracted and passed 714 exits, finally taking the 715th exit. How many miles apart are the two places where Sheldon entered and exited Route 495?

ANSWERS

1) _____ = A

2) _____ = B

3) _____ = C

4) _____ = D

5) _____ = E

6) _____

6) Using the answers from questions #1-5 evaluate the following expression:

$$\sqrt{\sqrt{(D - \sqrt{B + 1} + 2) + A - E} - 2}$$

**Solutions to Category 6
Team Round
Meet #4 - February, 2013**

ANSWERS

1) $338 = A$

2) $63 = B$

3) $7168 = C$

4) $150 = D$

5) $24 = E$

6) 18

1) Each angle adjacent to angle AOC measures 45 degrees, making the remaining acute angles also 45 degrees. If the diameter is 52, then the radius is 26. So, the total shaded area is $(2)(0.5)(26/\sqrt{2})(26/\sqrt{2}) = 338$

2) Let $X =$ the number of sand dollars
So, $X + 60 =$ the number of periwinkles

$$\frac{X}{X+60} = \frac{3}{8} \quad \text{so} \quad 3(X+60) = 8X$$

$$3X + 180 = 8X$$

$$180 = 5X$$

$$36 = X$$

Let $F =$ the number of fiddler crabs

$$\frac{X}{F} = \frac{4}{7} \quad \text{so} \quad \frac{36}{F} = \frac{4}{7} \quad \text{and} \quad 4F = (7)(36)$$

$$\text{and } 4F = 252 \quad \text{so} \quad F = 63.$$

3) Let $N =$ the common multiplier so that $7(N)(N)(N)(N) = 1792$, so $7N^4 = 1792 \dots N^4 = 256 \dots N = 4$. So, $1792(4) = E$ and $E = 7168$.

4) Let $X =$ the original price of any item, so the sale price = $X - (60\% \text{ of } X)$, or $0.4X$. Let $N =$ the factor by which the sale price is multiplied to attain the original price. Then $(N)(0.4X) = X$, and $N = X/(0.4X)$ so $N = 1/0.4$ or 2.5 . Therefore, the sale price must be multiplied by 250% to attain the original price, and so must be increased by 150% .

5) $715 / 37 = 19$ complete trips around Route 495, plus another 12 exits. Sheldon left the highway 12 exits beyond where he started, or $(2)(12)$ miles = 24 miles.

$$\begin{aligned} 6) \quad \sqrt{\sqrt{(D-\sqrt{B+1}+2)+A-E-2}} &= \sqrt{\sqrt{(150-\sqrt{63+1}+2)+338-24-2}} \\ \sqrt{\sqrt{(150-\sqrt{64}+2)+312}} &= \sqrt{\sqrt{(150-8+2)+312}} = \sqrt{\sqrt{144+312}} \\ &= \sqrt{12+312} = \sqrt{324} = 18. \end{aligned}$$