

IMLEM Meet #2
November, 2013

Intermediate
Mathematics League
of
Eastern Massachusetts

Category 1 - Solutions

Mystery

Meet #2 - November, 2013

ANSWERS

1) 235

2) 12.5

3) 12

- 1) This problem can be solved by scaling:
 $3 + 5 = 8$ total raisins in the ratio.
 $376 \div 8 = 47$. Scaling 5 raisins by a factor of 47 yields $5(47) = 235$.
Solving the proportion $5/8 = X/376$ is also effective, as is simply finding $5/8$ of 376.

- 2) Walking: $3:09 - 2:45 = 24$ minutes of walking; At 0.2 mile every six minutes, James walks for four 6-minute intervals for a total of $4(0.2) = 0.8$ mile. Bicycling: $3:54 - 3:09 = 45$ minutes. He rides for four and a half 10-minute intervals for a total of $(4.5)(2.6) = 11.7$ miles. In all, he travelled $0.8 + 11.7 = 12.5$ miles.

- 3) Add the numbers in this sequence until the total is at least \$650, keeping track of the number of days.

<u>Day #</u>	<u>\$</u>	<u>Total \$</u>
1	2	2
2	5	7
3	10	17
4	17	34
5	26	60
6	37	97
7	50	147
8	65	212
9	82	294
10	101	395
11	122	517
12	145	662

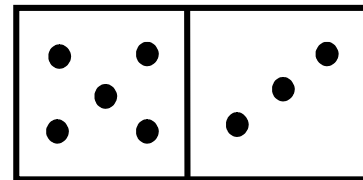
Note: For the student who would like to find a formula to solve this problem, looking at the \$ column, each value is one more than the square of the Day #. This discovery could possibly shorten the amount of calculation needed.

On the 12th day, Jennifer had enough money in her piggy bank to buy the \$650 Smahtie Phone.

Category 2
Geometry
Meet #2 - November, 2013

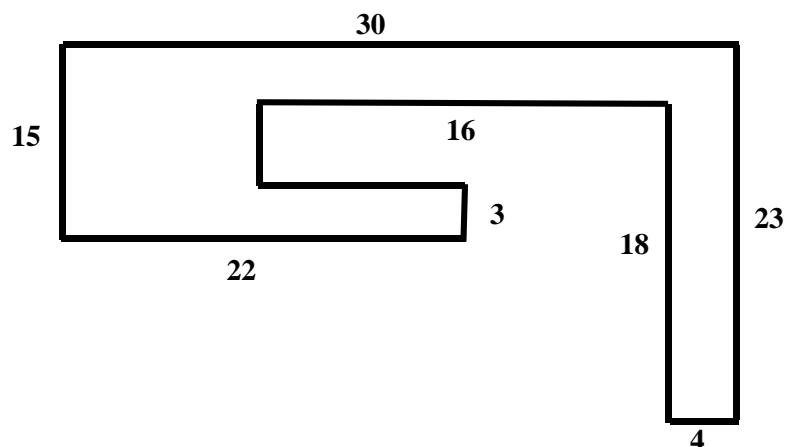
1) A pentadecagon is a polygon with 15 sides. ("penta" means 5, while "deca" means 10.) In a certain pentadecagon, eleven of the sides have the same length while each of the remaining sides measures 16 cm. If the perimeter of the pentadecagon is 207 cm, then how many cm long is one of the shorter sides?

2) The figure to the right is an example of a Domino - a rectangle consisting of two congruent squares that share a side. A giant model of a Domino measures 6 inches by 12 inches. How many Dominoes are used to completely fill the bottom of a square pan whose area is 81 square feet? (12 inches = 1 foot)



3) Find the perimeter of the figure below. All angles are right angles. Measurements are not to scale.

<u>ANSWERS</u>
1) _____ cm
2) _____
3) _____



Solutions to Category 2
Geometry
Meet #2 - November, 2013

<u>Answers</u>
1) 13
2) 162
3) 150

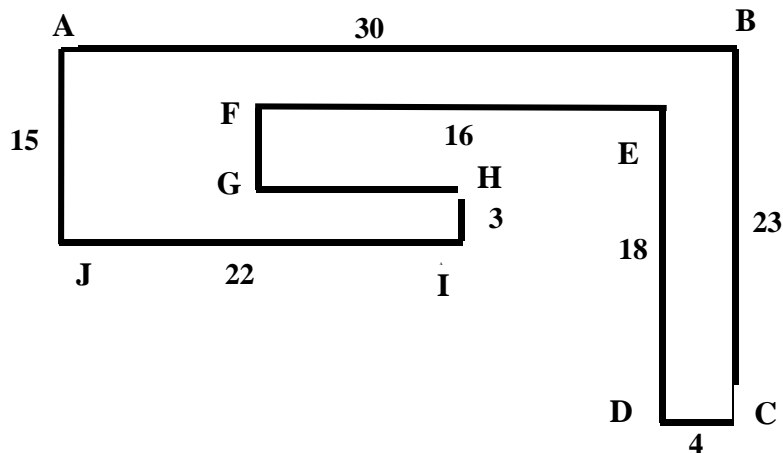
- 1) $207 - (4 \times 16) = 207 - 64 = 143$ (the total length of the 11 congruent sides. $143/11 = 13$ (the length of each of the shorter sides).
- 2) Each Domino contains 6×12 , or 72 square inches (or half of a square foot). A square pan of 81 square feet contains 162 of these model Dominoes. Also consider that two Dominoes combine to make one square foot.

Another approach is to convert the 81 square feet to square inches, where 1 square foot = 144 (12 x 12) square inches. $81 \times 144 = 11,664$ square inches. Dividing 11,664 by 72 yields 162 Dominoes. Of course, this technique involves much more arithmetic.

- 3) The width of the narrow horizontal bar at the top of the figure can be found by subtracting 18 from 23, (DE from BC) which is 5. The sum of this 5 and the 3 (HI) is the difference between the 15 (AJ) and the unlabelled vertical segment (FG), so the length of that vertical segment (FG) is 7. The adjacent horizontal unlabelled segment (HG) when added to the width of the leftmost large space ($30 - 20 = 10$) totals to 22 (IJ), so the length of that horizontal unlabelled segment (GH) is $22 - 10$, or 12. (Lengths are not to scale.)

So, the perimeter:

$$AJ + AB + BC + DC + DE + FE + FG + GH + HI + IJ = 15 + 30 + 23 + 4 + 18 + 16 + 7 + 12 + 3 + 22 = 150.$$



Category 3
Number Theory
Meet #2 - November, 2013

1) What is the GCF (greatest common factor) of F and G if $F \times G = 210$? F and G are positive integers.

2) If $2^A \times 3^B \times 5^C = 16^3 \times 27^2 \times 25^4$ then find the value of $A + B + C$.

3) Megan sees her friend, Barbara, at her favorite restaurant, Pasta ya Prime, every 12 days. She sees Jake there every 18 days and Chara every 40 days. They all met there on April 1. In what month will they meet next?

<u>Answers</u>	
1)	_____
2)	_____
3)	_____

Solutions to Category 3
Number Theory
Meet #2 - November, 2013

Answers

1) 1

2) 26

3) March

1) Since $210 = 2 \times 3 \times 5 \times 7$, and all of these factors are relatively prime (their GCF = 1), then any combination of products that yields two factors of 210 will be also relatively prime.

For example, let $F = 6$ and $G = 35$ (GCF = 1)

another: let $F = 2$ and $G = 105$ (GCF = 1)

another: let $F = 10$ and $G = 21$ (GCF = 1)

2) The bases on the right side of the equation are all powers of the bases on the left side:

$$16^3 = (2^4)^3 = 2^{12}$$

$$27^2 = (3^3)^2 = 3^6$$

$$25^4 = (5^2)^4 = 5^8$$

So, $A + B + C = 12 + 6 + 8 = 26$

3) The LCM (lowest common multiple, often called the least common multiple) of 12 and 18 and 40 can be found either by listing multiples until the first common multiple is found or by using prime factorization.

Listing: 12 24 36 48 60 ...

18 36 54 72 90 ...

40 80 120 160 ...

Eventually (and painstakingly), all lists lead to the number 360.

Prime factorization: $12 = 2^2 \times 3$, $18 = 2 \times 3^2$, and $40 = 2^3 \times 5$.

The LCM of (12, 18, 40) = $2^3 \times 3^2 \times 5 = 360$

360 days is almost a full year which, when added to April 1, brings us to late March.

Category 4
Arithmetic
Meet #2 - November, 2013

1) The harmonic mean of two numbers is the reciprocal of the arithmetic mean (or average) of their reciprocals. It is one of the three Pythagorean means and is a better way to calculate certain types of averages, like the average of rates. Using this concept, find the harmonic mean of 40 mph and 60 mph.

2) At the vintage Filene's Basement in Downtown Boston, the following pricing policy was in effect in 1965:

- new products: full price
- products at least 7 days old: 25% off the original price
- products at least 14 days old: 50% off the original price
- products at least 21 days old: 75% off the original price
- products at least 28 days old: donated to charity

If Martha bought a 9-day-old dress that was originally priced at \$ 8.92 and a 25-day-old pair of shoes that was originally priced at \$ 9.48, then how much money did she pay in all?

3) Simplify: $\frac{\frac{3}{4} + 2.625}{7.\bar{6} - \frac{5}{3}}$ Express your answer as a common fraction.

<p><u>ANSWERS</u></p> <p>1) _____ mph</p> <p>2) \$ _____</p> <p>3) _____</p>

Solutions to Category 4
Arithmetic
Meet #2 - November, 2013

<u>Answers</u>
1) 48
2) 9.06
3) $\frac{9}{16}$

$$\begin{aligned}
 1) \quad \frac{1}{\frac{1}{2}\left(\frac{1}{40} + \frac{1}{60}\right)} &= \frac{1}{\frac{1}{2}\left(\frac{60}{2400} + \frac{40}{2400}\right)} = \frac{1}{\frac{1}{2}\left(\frac{100}{2400}\right)} \\
 &= \frac{1}{\frac{1}{48}} = 48
 \end{aligned}$$

- 2) dress: $0.75 (8.92) = \$ 6.69$
 shoes: $0.25 (9.48) = \$ 2.37$
 total: $\$ 9.06$

$$3) \quad \frac{\frac{3}{4} + 2.625}{7.\bar{6} - \frac{5}{3}} = \frac{0.75 + 2.625}{7\frac{2}{3} - \frac{5}{3}} = \frac{\frac{27}{8}}{\frac{14}{3} - \frac{5}{3}} = \frac{27}{8} \times \frac{1}{6} = \frac{27}{48} = \frac{9}{16}$$

Category 5

Algebra

Meet #2 - November, 2013

1) Sam has three more eggs than Sham but twice as many eggs as Faro.
If there are 62 eggs in all, then how many eggs does Sham have?

2) If $X + 3Y - 4A = 17$ and $X + 7Y - 4A = 45$, then what is the value of $3X - 12A$?

3) The formula that converts a temperature in Celsius (C) degrees to Fahrenheit (F) degrees is

$$F = \frac{9}{5}C + 32$$

A roasted turkey is sufficiently cooked when a thermometer inserted into the thickest part of the thigh registers 180 degrees Fahrenheit. Priscilla only has a Celsius thermometer. How many minutes longer must she roast the turkey if its current temperature is 70 degrees Celsius and every five minutes in the oven produces a rise in temperature of one degree Celsius? Round your answer to the nearest minute.

ANSWERS

1) _____

2) _____

3) _____

Solutions to Category 5
Algebra
Meet #2 - November, 2013

Answers

1) 23

2) -12

3) 61

- 1) Let $X =$ the number of Faro's eggs
 $2X - 3 =$ the number of Sham's eggs
 $2X =$ the number of Sam's eggs

$$X + (2X - 3) + 2X = 62$$

$$5X - 3 = 62$$

$$5X = 65$$

$$X = 13 \text{ (Faro)}$$

$$2X - 3 = 23 \text{ (Sham)}$$

- 2) Compare the two equations and find their difference:

$$X + 3Y - 4A = 17$$

$$X + 7Y - 4A = 45$$

Difference: $4Y = 28$

$$Y = 7$$

Substituting 7 for Y into the first equation: $X + 3(7) - 4A = 17$

$$X + 21 - 4A = 17$$

$$X - 4A = -4$$

Multiply both sides by 3:

$$3X - 12A = -12$$

Done!!

$$3) F = \frac{9}{5}C + 32 \quad \dots \quad 180 = \frac{9}{5}C + 32 \quad \dots \quad 148 = \frac{9}{5}C \quad \dots \quad 148 \left(\frac{5}{9}\right) = C$$

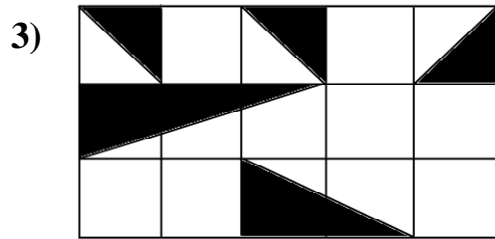
$$\dots \quad 82\frac{2}{9} = C$$

$$82\frac{2}{9} - 70 = 12\frac{2}{9} \quad \dots \quad 12\frac{2}{9}(5) = 61\frac{1}{9} \approx 61.$$

Therefore, it would take an additional **61** minutes to roast the turkey to perfection! Scrumptious!

Category 6
Team Round
Meet #2 - November, 2013

- 1) If the GCF of A and B is 12, and $AB = 4320$, then what is the LCM (lowest common multiple) of A and B ?
- 2) The lengths of the sides of a nonagon (9-sided polygon) are consecutive odd integers. If the perimeter is 189 cm, then how many cm long is the longest side ?



A rectangle is subdivided into 15 congruent squares as shown. Each square measures 4 inches on each side. How many square inches are in the unshaded area ?

- 4) When the fraction $\frac{3}{7}$ is changed to its decimal form, and N is the value of the 75th digit to the right of the decimal point, then what is the value of $N - 2$?
- 5) If the hundreds digit and the units (ones) digit of a 3-digit number are swapped, then the new number is 495 more than the original number. If the tens digit is 7, then what is the largest possible value of the original number?
- 6) Using the answers from questions #1-5, evaluate the following expression:

<u>ANSWERS</u>	
1) _____	= A
2) _____	= B
3) _____	= C
4) _____	= D
5) _____	= E
6) _____	

$$\sqrt{\sqrt{\frac{A}{D+4} + \frac{E-C-3}{B+1}}}$$

Solutions to Category 6
Team Round
Meet #2 - November, 2013

ANSWERS

- 1) 360 = A
 2) 29 = B
 3) 176 = C
 4) 6 = D
 5) 479 = E
 6) 4

1) $AB = (\text{GCF of A and B})(\text{LCM of A and B})$
 $4320 = 12 (\text{LCM of A and B})$
 $4320 / 12 = 360$

2) Guessing and checking is a reasonable approach and may yield a quick answer. Otherwise, let $X =$ the length of the shortest side, $X+2$ the next longer side, etc., until $X + 16$ is the longest. $9X + 72 = 189$
 $9X = 117$
 $X = 13$
 $X + 16 = 29$

Another strategy: $189/9 = 21$ (the middle number). Count off four odd integers smaller and four odd integers larger to create this list 13, 15, 17, 19, 21, 23, 25, 27, 29 where 29 is the longest side.

3) Each square has an area of 4×4 , or 16 square inches. The total area of the rectangle is 15×16 , or 240 square inches. Each shaded triangle contains half the area of the rectangle that frames it. Each triangle in the top row has half of 16, for a total of 24, while the one in the middle row has half of 3×16 , or 24, and the one in the bottom row has half of 2×16 , or 16. The sum of the shaded areas is $24 + 24 + 16$, or 64. The unshaded area is the difference $240 - 64$, or 176 square inches.

4) $3/7 = 0.428571428571 \dots$ There are six digits in each block, so the 75th digit, N, is 8, and so $N - 2 = 8 - 2 = 6$.

5) Three numbers meet the conditions of the original number: 176, 277, 378, 479, with the largest being 479.

6)
$$\sqrt{\sqrt{\frac{A}{D+4} + \frac{E-C-3}{B+1}}} = \sqrt{\sqrt{\frac{360}{6+4} + \frac{479-176-3}{29+1}}} = \sqrt{\sqrt{\frac{360}{10} + \frac{300}{30}}}$$

$$= \sqrt{\sqrt{36+10}} = \sqrt{6+10} = \sqrt{16} = 4$$