IMLEM Meet #1
October, 2013

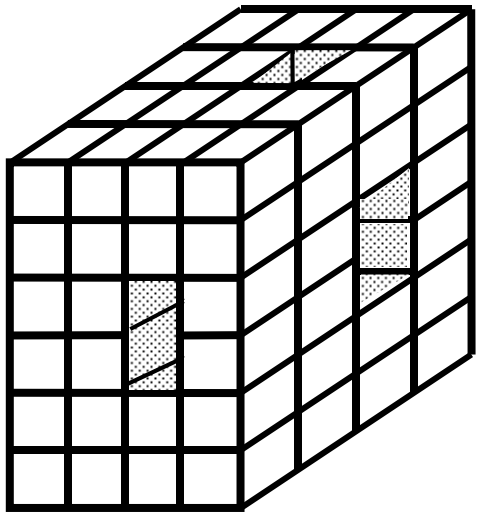
Intermediate Mathematics League of Eastern Massachusetts

Category 1
Mystery
Meet #1 - October, 2013

1) If this pattern repeats continuously, then what is the 573rd letter in the pattern?

HAMILTONHAMILTONHAMILTONHAMIL . . .

2) The figure below consists of small cubes. How many of the these cubes are there? Assume that, if there is a "hole" on any surface, then that hole goes all the way through the figure.



3) There are three whole numbers between 3000 and 4000 that are divisible by 3, 11, and 13. What is their sum?

Answers

1) _____

2) _____

3) _____

Solutions to Category 1
Mystery
Meet #1 - October, 2013

Answers
1) L
2) 78
3) 10,296

1) There are eight letters in the word HAMILTON. To find the 573rd letter, see what multiple of 8 is in its vicinity. Dividing 573 by 8 yields 71 with a remainder of 5. It is the remainder of 5 that is helpful in solving this problem. There are 71 complete clusters of the 8-letter configuration. The 5th letter after that final complete cluster is L.

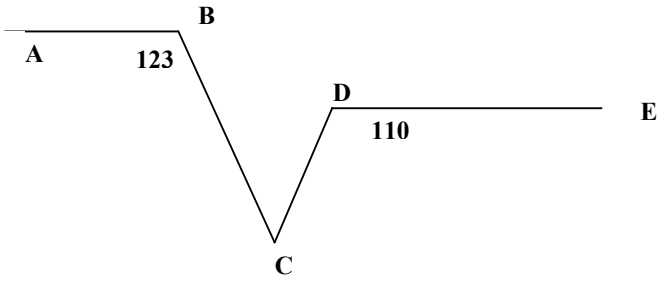
2) If the entire figure were uninterrupted by the holes, the total number of cubes would be (4)(4)(6) = 96 cubes. The holes intersect in the interior of the figure, so we have to be careful about counting the ones missing. The hole in the front (2 cubes high) goes all the way to the back, thus eliminating 8 cubes. We are now down to 88 cubes. The hole on the right side (also 2 cubes high) implies another gap of 8 cubes. However, two of them were eliminated from our first count. So, we subtract six more cubes to bring our new total to 82 cubes. The hole of a single cube on the top implies a gap of 6 cubes, but two of them were already counted. So, we subtract 4 more cubes which brings us to 78 cubes.

3) To narrow our 1000 or so choices for answers, consider that the numbers 3, 11, and 13 are relatively prime (sharing only a common factor of 1). So, their product is their LCM: (3)(11)(13) = 429. The 7th, 8th, and 9th multiples of 429 fall between 3000 and 4000. They are 3003, 3432, and 3861. Their sum is 3003 + 3432 + 3861 = 10,296.

Category 2
Geometry
Meet #1 - October, 2013

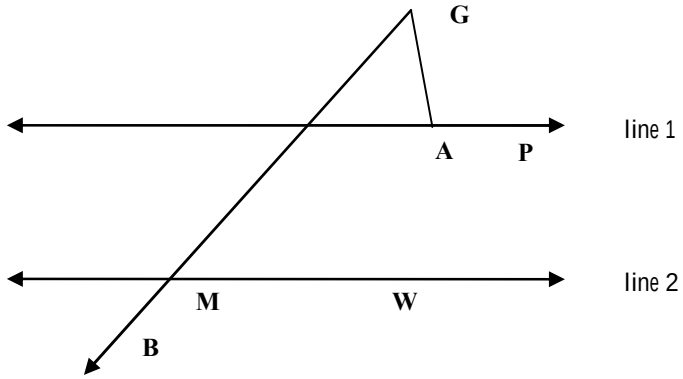
1) Two angles are "poplimentary" if the sum of their degree measures is 20. Two angles are "cuplimentary" if the product of their angle measures is 96. Angle X and angle W are both poplimentary and cuplimentary. X < W. How many degrees are in the measure of angle W ?

2)



Line segment AB is parallel to line segment DE. Degree measures of angles are as written on the diagram. What is the measure of angle C if it is less than 180 degrees?

3) Line 1 is parallel to line 2. Angle BMW measures 115 degrees. Angle BMW is supplementary to angle G. How many degrees are in the measure of angle GAP ?



Answers

1) _____

2) _____

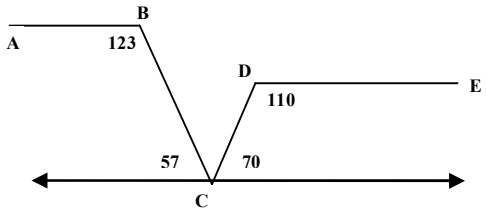
3) _____

Solutions to Category 2
Geometry
Meet #1 - October, 2013

Answers
1) 12
2) 53
3) 130

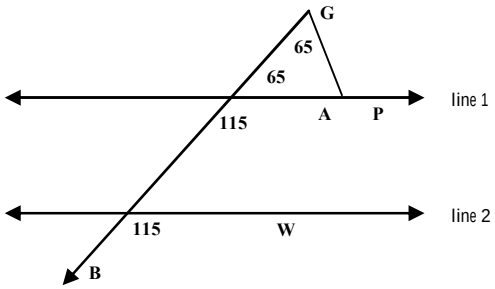
1) Algebraically, the solution boils down to $X + W = 20$ and $XW = 96$. Guessing and checking would yield a pair of numbers that solves both conditions: 8 and 12. Since the question asks for the larger solution, $W = 12$.

2) One strategy is to draw a line through C that is parallel to the other two parallel lines, thus creating pairs of angles that are either congruent or supplementary, as shown in this diagram:



The three angles at the bottom of the diagram form a straight angle, so their sum is 180 degrees. $57 + C + 70 = 180$, so $C = 53$.

3) The parallel lines again create pairs of angles that are either congruent or supplementary, as shown in the diagram below:



Since angle G is supplementary to angle BMW, it measures 65 degrees. Angle GAP measures, therefore, 130 degrees, because the degree measure of an exterior angle of a triangle is equal to the sum of the two remote interior angles.

Category 3
Number Theory
Meet #1 - October, 2013

1) What is the only whole number between 280 and 290 that is divisible by both 4 and 9 ?

2) Two of the four prime factors of 17,017 are 13 and 17. What is the larger of the other two prime factors ?

3)

☐ is a multiple of 9.

☐ > 1000.

☐ is divisible by both 5 and 7.

☐ is not divisible by 2.

☐ < 2000.

What is the value of ☐ ?

Answers

1) _____

2) _____

3) _____

Solutions to Category 3
Number Theory
Meet #1 - October, 2013

Answers
1) 288
2) 11
3) 1575

1) Since 4 and 9 are relatively prime, their product is their LCM. We need to look at multiples of 36 that lie near 280. A bit of guessing and checking yields (36)(8) = 288.

2) First, divide 17,017 by the product of 13 and 17. Then factor the resulting quotient by using divisibility rules:

(13)(17) = 221.
17,017 / 221 = 77.
77 = (7)(11).
The larger of the two factors of 77 is 11.

3) We are looking for an odd multiple of (5)(7)(9) that lies between 1000 and 2000.

(5)(7)(9) = 315.
The multiples of 315 that lie between 1000 and 2000 are
4(315) = 1260,
5(315) = 1575, and
6(315) = 1890.
The only odd multiple is 1575.

Category 4
Arithmetic
Meet #1 - October, 2013

1) If a number is added to the following set, N, there would be two modes. What is that number?

N = { 11, 7, 6, 3, 7, 12, 8, 3, 4, 10, 7, 9 }

2) A number, W, is added to the set of twelve numbers in the set N from problem #1 to create the set Ω. If the mean of the numbers in set Ω is 7.6, rounded to the nearest tenth, then what is the largest possible whole-number value of W ?

3) A = 6 x 10 ÷ 2 x 3
B = $15 - 3^2 + 2^3 + 1$
C = $\frac{A}{B}$

What is the value of C ?

Answers

1) _____

2) _____

3) _____

Solutions to Category 4
Arithmetic
Meet #1 - October, 2013

Answers
1) 3
2) 12
3) 6

1) The current mode is 7, as there are more sevens in set N than any other number. If there were another 3 in the set, then there would be as many threes as sevens.

2) The total of the twelve numbers in set W is 87. If the mean is about 7.6, then the total sum of the thirteen numbers, including W, is about (13)(7.6), or 98.8. If 13 is added to set W, then the new sum would be 100. To test, divide 100 by 13. The quotient is 7.692... which does NOT round to 7.6. Therefore, try adding 12 instead of 13 to set W. That would bring the thirteen-number total to 99. To test, divide 99 by 13. The quotient is 7.615... which meets all conditions of the problem.

3) $A = 6 \times 10 \div 2 \times 3$ — **Multiply and divide from left to right.**

$= 60 \div 2 \times 3$

$= 30 \times 3$

$= 90$

$B = 15 - 3^2 + 2^3 + 1$ — **Evaluate powers first**

$= 15 - 9 + 8 + 1$ — **Now add and subtract, in order, from left to right.**

$= 6 + 8 + 1$

$= 14 + 1$

$= 15$

$C = \frac{90}{15}$

$= 6$

Category 5
Algebra
Meet #1 - October, 2013

1) If $P = -1$ and $R = \frac{1}{2}$ then evaluate $-P^2 + 4R^4 - 6PR$.

Express your answer as a decimal.

2) Simplify: $4(2X - 5) + 3(7X - 6) - 2(5X - 1)$

3) Situation #1: $10A + 1 = 7(2A - 5)$

Situation #2: $3C^2 = 75$

Situation #3: The following equation is an identity in N:

$$4N - 7 + W = -2(3N + 4) + 10N$$

Finally, find the value of X if

X = (the smaller value of C)(the larger value of C) - AW

Answers

1) _____

2) _____

3) _____

Solutions to Category 5
Algebra
Meet #1 - October, 2013

Answers
1) 2.25
2) 19X - 36
3) - 16

1) If $P = -1$ and $R = \frac{1}{2}$ then $-P^2 + 4R^4 - 6PR$

$$= -(-1)^2 + 4(\frac{1}{2})^4 - 6(-1)(\frac{1}{2})$$
$$= -(1) + 4(\frac{1}{16}) + 3$$
$$= -1 + \frac{1}{4} + 3$$
$$= 2.25$$

2) Simplify: $4(2X - 5) + 3(7X - 6) - 2(5X - 1)$

$$= 8X - 20 + 21X - 18 - 10X + 2$$
$$= 19X - 36$$

3) Situation #1: $10A + 1 = 7(2A - 5)$

$$10A + 1 = 14A - 35$$
$$36 = 4A$$
$$9 = A$$

Situation #2: $3C^2 = 75$

$$C^2 = 25$$
$$C = \pm 5$$

Situation #3: The following equation is an identity:

$$4N - 7 + W = -2(3N + 4) + 10N$$
$$4N - 7 + W = -6N - 8 + 10N$$
$$10N + W = -1 + 10N$$
$$W = -1$$

X = (the smaller value of C)(the larger value of C) - AW
= (-5)(5) - (9)(-1)
= -25 + 9
= -16

Category 6
Team
Meet #1 - October, 2013

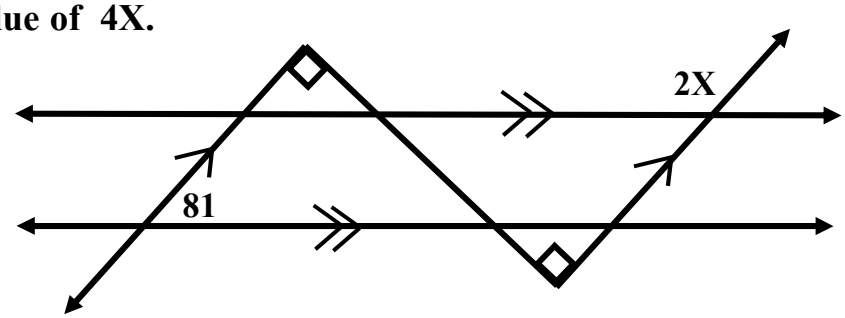
1) **Find the sum of the prime numbers between 170 and 180.**

2) **The supplement of angle M is 14 degrees more than three times its complement. How many degrees are in the complement of angle M ?**

3) **Evaluate:** $-31 + 2[\,17 - 4\,(6^2 - 2^6)^2\,]$

4) **Ethan went bowling and achieved the following scores: 143, 191, 168, 210, and 175. He bowled two more games and averaged 182 for the day. What was the mean of his final two games ? Express your answer as a decimal.**

5) **2X is the degree measure of an angle formed by two intersecting lines. Find the value of 4X.**



6) **Using the answers from questions #1-5, evaluate the following expression:**

$$\sqrt{\sqrt{2D + 0.5E - 2} - C + 6A - 6B + 106}$$

ANSWERS

1) _____ = A

2) _____ = B

3) _____ = C

4) _____ = D

5) _____ = E

6) _____

Solutions to Category 6
Team
Meet #1 - October, 2013

ANSWERS

1) 352 = A

2) 38 = B

3) - 6269 = C

4) 193.5 = D

5) 198 = E

6) 91

1) 173 + 179 = 352

2) Solving 180 - M = 3(90 - M) + 14,
M = 52, so the complement of M is 38.

3)
$$\begin{aligned} & -31 + 2[17 - 4(6^2 - 2^6)^2] \\ = \; & -31 + 2[17 - 4(36 - 64)^2] \\ = \; & -31 + 2[17 - 4(-28)^2] \\ = \; & -31 + 2[17 - 4(784)] \\ = \; & -31 + 2[17 - 3136] \\ = \; & -31 + 2[-3119] \\ = \; & -31 + (-6238) \\ = \; & -6269 \end{aligned}$$

4) The total of his first five games: 143 + 191 + 168 + 210 + 175 = 887. The total for all seven games is his average for the day (182) multiplied by 7 = 1274. The difference between his 5-game total and his 7-game total is 1274 - 887 = 387. The mean (average) of his final two games is this difference divided by 2, or 193.5 (decimal only).

5) There are m any ways to find the value of X and, subsequently, 4X. Due to the double-parallelism, all of the angles' measures can be found, either via parallel lines cut by transversals or sums of measures of triangles. Regardless of technique, 2X = the supplement of 81, or 99, so X = 49.5 and 4X = 198.

6) $\sqrt{\sqrt{2D + 0.5E - 2} - C + 6A - 6B + 106} =$

$$\sqrt{\sqrt{2(193.5) + 0.5(198) - 2} - (-6269) + 6(352) - 6(38) + 10(}$$
$$= \sqrt{\sqrt{387 + 99 - 2} + 6269 + 2112 - 228 + 106}$$
$$= \sqrt{\sqrt{484} + 8259}$$
$$= \sqrt{22 + 8259}$$
$$= \sqrt{8281}$$