

Intermediate  
Mathematics League  
of  
Eastern Massachusetts

Category 1 – Mystery

1. In recent elections, 52% of the voters voted for candidate *A*, and 48% voted for candidate *B*. If a total of 24,000 voters participated, then how many more voters voted for candidate *A*?
2. A worm is born 20 millimeters long and grows 10% longer every week. How many millimeters long will it be after eight weeks?  
*Give you answer to two decimal places, round to the nearest hundredth.*
3. Every house in a new neighborhood has a water pipe with a diameter of 1 inch as its main water source. If there are 400 houses in the neighborhood, what should be the diameter of the pipe that supplies the whole neighborhood?  
*(So the main pipe should be able to carry as much water as 400 small pipes, and we assume the water flows at the same speed in all pipes).*

Answers	
1.	_____
2.	_____mm
3.	_____inches

Solutions to Category 1 – MysteryAnswers

1. Candidate A got 4% more of the total number of votes, or

$$4\% \cdot 24,000 = 960 \text{ more votes.}$$

1. 960  
2. 42.87  
3. 20

2. Since each week it grows 10% longer, after eight weeks the worm will be:

$$20 * 1.1^8 = 20 * 2.14358881 = 42.8717762 \cong 42.87 \text{ mm long.}$$

3. The water-carrying capacity is proportional to the cross-section of the pipe (the area). The area of a circle is proportional to the square of the diameter (or radius), so if we multiply the area by 400, it is equivalent to increasing the radius/diameter by  $\sqrt{400} = 20$ , so the big pipe's diameter should be 20 inches.

Category 2 – Geometry

**Use  $\pi = 3.14$  wherever necessary throughout the category.**

1. The area of a circle is 314 square inches.

How many inches are there in its circumference?

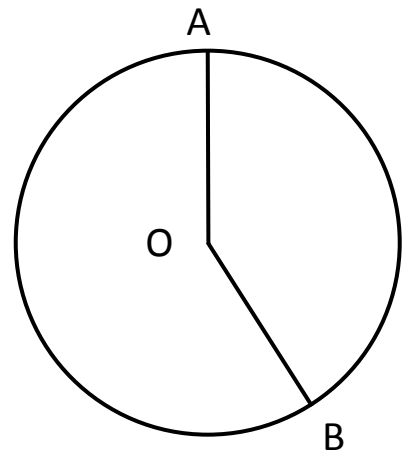
*Give an exact answer with no rounding.*

2. The radius of the circle shown is 60 centimeters.

The radius  $\overline{OA}$  points to 12 o'clock, and the radius

$\overline{OB}$  points to 5 o'clock.

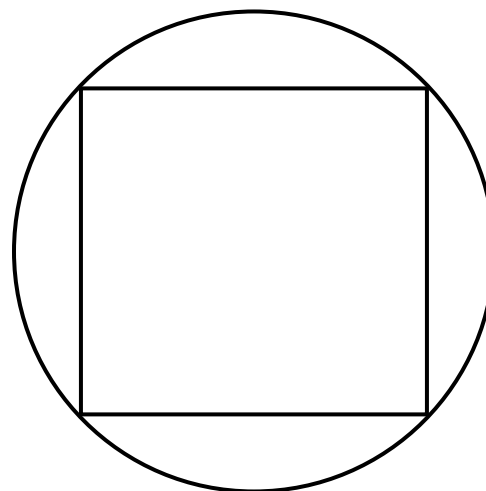
How many centimeters are there in the arc  $\widehat{AB}$ ?



3. A square is inscribed inside a circle.

What percentage of the square's perimeter is the circle's circumference?

*Round your answer to the nearest whole percent.*



Answers	
1.	_____ inches
2.	_____ cm
3.	_____ %

Solutions to Category 2 – GeometryAnswers

1. The area of a circle is  $A = \pi r^2$  so in our case

the radius is  $r = \sqrt{\frac{A}{\pi}} = \sqrt{100} = 10$  inches.

The circumference is  $2 \cdot \pi \cdot r = 2 \cdot 3.14 \cdot 10 = 62.8$  inches.

2. The hours on the clock divide the central angle to 12 equal parts (each one being 30 degrees then). The arc from 12 to 5 will measure  $\frac{5}{12}$  of the whole circumference, or in our case  $\widehat{AB} = \frac{5}{12} \cdot 2 \cdot \pi \cdot r = \frac{5}{6} \cdot 3.14 \cdot 60 = 157$  cm

3. If we call the square's side  $L$ , then its perimeter is  $4 \cdot L$ .

Its diagonal is  $D = \sqrt{2} \cdot L$ , and that is the circle's diameter, so the circle's circumference equals  $\pi \cdot D = \sqrt{2} \cdot \pi \cdot L \cong 4.44 \cdot L = 111\% * (4 \cdot L)$

4. 62.8

5. 157

6. 111%

Category 3 – Number Theory

1. What is the 30th term in the series:  $\{58, 51, 44, 37, \dots\}$  ?
2. What is the smallest natural number  $N$  that solves:  $5 \cdot N \equiv 7 \pmod{11}$  ?
3. The old king divided his treasure of gold coins among his 12 children.  
The youngest got a sack of coins, and each child afterwards got 15 more coins than the one before him. Overall, 2,910 coins were given away.  
How many coins did the oldest child get?

Answers	
1.	_____
2.	_____
3.	_____ coins

Solutions to Category 3 – Number TheoryAnswers

1. We can see that the difference between consecutive terms is  $-7$ , so the 30th term is going to be:

$$58 - 7 * 29 = -145$$

1.  $-145$   
2. 8  
3. 325

2. The solution has to satisfy:  $5 \cdot N = 7 + 11 \cdot k$  for some natural number  $k$ , so looking at the series  $7 + 11 \cdot k$  for  $k = 0, 1, 2, \dots$  we have: 7, 18, 29, 40, 51, ... 40 is of course the smallest of these to be divisible by 5, for  $N = 8$ .

3. If the youngest got  $N$  coins, then the next child got  $(N + 15)$  coins, etc. and the oldest got  $(N + 11 \cdot 15)$  coins.

The sum of this series is  $(N + N + 11 \cdot 15) \cdot 6 = 2,910$

So we can solve:  $N = \frac{2910 - 6 \cdot 11 \cdot 15}{12} = \frac{1,920}{12} = 160$  coins, and the oldest received

$160 + 11 \cdot 15 = 325$  coins.

Category 4 – Arithmetic

1. Tom has 4%, or \$10, less money than Jerry.  
How much money do they have together?
  
2. 36% of all vehicles sold at a dealership are red cars.  
4% of the vehicles sold are motorcycles (the rest are cars).  
What percentage of cars sold are red?
  
3. You bought a TV on sale, which was marked '20% Off'.  
After paying the state's 6.25% sales tax (calculated on the discounted price),  
you paid a total of \$680.  
What was the TV's original list-price?  
*List price is the price advertised, which does not include tax.*

Answers	
1. \$	_____
2. _____	%
3. \$	_____



Solutions to Category 4 – Arithmetic

1. If \$10 is 4% of Jerry's amount, then he has  $\frac{\$10}{0.04} = \$250$

Tom has \$10 less, so together they have  $\$240 + \$250 = \$490$

2. If 4% of the vehicles are not cars, we are left with 96% of the total vehicles that are cars.

The *red* cars' share of these is :

$$\frac{36\%}{96\%} = 37.5\%$$

3. The discounted price would be  $\frac{\$680}{1.0625} = \$640$ , and this is 80% of the original price. So the original price is  $\frac{\$640}{80\%} = 1.25 * \$640 = \$800$

Answers

1. \$490 or  
\$490.00
2. 37.5%
3. \$800 or  
\$800.00

Category 5 – Algebra

1. John is 4 years younger than his brother Bill.  
In 4 years, John will be 90% of Bill's age.  
How old will Bill be then?
  
2. Your \$10 is exactly enough to buy either 6 cups of slush and 1 slice of pizza, or 2 cups of slush and 3 slices of pizza.  
How much will one slice of pizza and one cup of slush cost?
  
3. The sum of the series  $\{x, x + 3, x + 6, x + 9, \dots, x + 90\}$  is 558.  
What is the value of  $x$ ?

## Answers

1. \_\_\_\_\_
2. \_\_\_\_\_
3. \_\_\_\_\_

Solutions to Category 5 - AlgebraAnswers

1. If Bill is going to be  $N$  years old in 4 years, then John is going to be  $N - 4$  years old, and we know that  $\frac{N-4}{N} = 90\%$ , or in other words 4 is 10% of  $N$ , so  $N = 40$ .

1. 40  
2. \$3.75  
3. -27

2. Calling the prices of a cup of slush and a slice of pizza  $S, P$  respectively:

$$6 \cdot S + P = \$10$$

$$2 \cdot S + 3 \cdot P = \$10$$

From the first equation we can take  $P = \$10 - 6 \cdot S$  and plug it into the second:

$$2 \cdot S + 3 \cdot (\$10 - 6 \cdot S) = \$10 \text{ to get } \$30 - 16 \cdot S = \$10$$

and the solution is  $S = \$1.25$  from which we get  $P = \$2.50$  and so the answer is  $P + S = \$3.75$

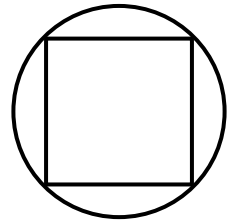
3.  $\{x, x + 3, x + 6, x + 9, \dots, x + 90\} = 31 \cdot x + (3 + 6 + \dots + 90) =$   
 $= 31 \cdot x + 93 \cdot 15 = 558$   
 $31 \cdot x = 558 - 1,395 = -837$   
 $x = -27$

Category 6

1. A square is inscribed inside a circle. What percentage of the circle's area is taken up by the square? *Round your answer to the nearest whole percent. Use  $\pi = 3.14$*
  
2. Two cars leave point *A* at the same time heading towards point *B*.  
 Car #1 drives 10% faster than car #2 and arrives at the destination 5 minutes ahead.  
 How many minutes did it take car #2 to complete the trip?
  
3. Left unchecked, the rat population on an island doubles in size every month. A rat eats 1 kg of food every month. If we start with 2 rats at the beginning of the year and let them multiply, how many kg of food will the rat population consume over one year?  
*You can assume that the rats multiply on the last day of each month, and new rats start eating the next month.*
  
4. You invest \$1,000 in a bank account that will pay you 4% interest (compounded annually), and leave the money there for 10 year. How much interest did you earn during the 10<sup>th</sup> year?  
*Round your answer to the nearest whole Dollar amount.*
  
5. A lab experiment starts at 12 o'clock and consists of 8 steps. The first step lasts one hour, and each consecutive step takes 3 times longer than the one before it.  
 What time will it be when the experiment is over?  
*Notice we don't care about AM/PM, only the hour in a 12-hour clock.*
  
6. Using the values you obtained in questions 1 through 5, evaluate the expression: *(Note A is a percent)*

$$\frac{2 * C}{(100 * A) + B + D + E}$$

Answers	
1.	_____ % = <i>A</i>
2.	_____ min = <i>B</i>
3.	_____ kg = <i>C</i>
4.	\$ _____ = <i>D</i>
5.	_____ = <i>E</i>
6.	_____

Solutions to Category 6

1. Calling the circle's radius  $R$ , its area is  $\pi \cdot R^2$ . Using Pythagoras the square's side is  $\sqrt{2} \cdot R$  and so its area is  $2 \cdot R^2$ . The ratio then is

$$\frac{2}{\pi} = \frac{2}{3.14} = 0.6369 \dots \cong 64\%$$

2. Since car #1 drives 10% faster, the trip will take car #2 10% more time than it takes car #1.

*[If you think: Car #1 will take 10% less time than car #2, you are wrong. Look back at the relationship  $\text{Time} = \frac{\text{Distance}}{\text{Speed}}$ ].*

So if the 5 minute difference equals 10% of car #1 time, then its trip took 50 minutes, and car #2 took 55 minutes.

3. During the first month, the rats consume 2 kg of food, then 4 kg the second month, 8 kg the third month and so on, eventually consuming  $2^{12}$  kg the last month. The sum of this is  $2 + 2^2 + 2^3 + \dots + 2^{12} = 2^{13} - 2 = 8,190$  kg.

*Note we start the series from 2 (not 1).*

4. After 9 years, the balance is  $\$1,000 * 1.04^9 = \$1,423.31$

After 10 years your balance is  $\$1,000 * 1.04^{10} = \$1,480.24$ , which means that in the last year you earned  $\$1,480.24 - \$1,423.31 = \$56.93 \cong \$57$  in interest.

Answers

1. 64%
2. 55
3. 8,190
4. \$57
5. 4
6. 91

5. The experiment will take  $1 + 3 + 9 + \dots + 3^7 = \frac{3^8-1}{3-1} = \frac{6560}{2} = 3,280$  hours (This is a little more than 4 months). To find out what time it'll be, we have to find the remainder when dividing by 12:  $3,280 \equiv 4 \pmod{12}$ , so it'll be 4 o'clock.

$$6. \frac{2*C}{(100*A)+B+D+E} = \frac{2*8190}{64+55+57+4} = \frac{2*8190}{180} = 91$$