Meet #1
October 2008

Intermediate Mathematics League of Eastern Massachusetts

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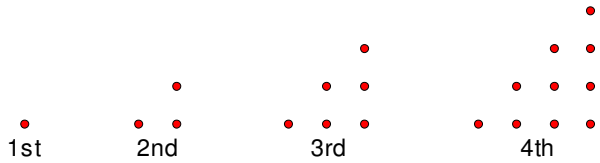
Category 1
Mystery
Meet #1, October 2008

1. Sally was reading the last Harry Potter book for the 7^{th} time. It took her 80 minutes to read the first 40 pages. Reading at that pace, how many minutes did it take her to read the entire 759 page book?

2. The first four terms in a pattern of dots are shown below. How many dots would there be in the 10^{th} term of the pattern?



3. Mike is going to cover the rectangular floor of his kitchen with square tiles that are 8 inches on each side. The floor is 8 feet by 14 feet. How many tiles will he need to cover the whole kitchen floor?

Answers

1. _____
2. _____
3. _____

Solutions to Category 1
Mystery
Meet #1, October 2008

Answers

1. 1518

2. 55

3. 252

1. Reading 40 pages in 80 minutes means she takes 2 minutes for each page. Since there are 759 pages, it will take 759 x 2 = **1518** minutes.

2. Each term adds the counting numbers up to the number of the term. The 4^{th} term has 1 + 2 + 3 + 4 dots. The 10^{th} term would have 1+2+3+4+5+6+7+8+9+10 = **55** dots. Adding the consecutive counting numbers starting with one gives you the triangular numbers, which can also be found by the formula : $T_n = \frac{n(n+1)}{2} \rightarrow T_{10} = \frac{10(10+1)}{2} = \frac{10(11)}{2} = \frac{110}{2} = \mathbf{55.}$

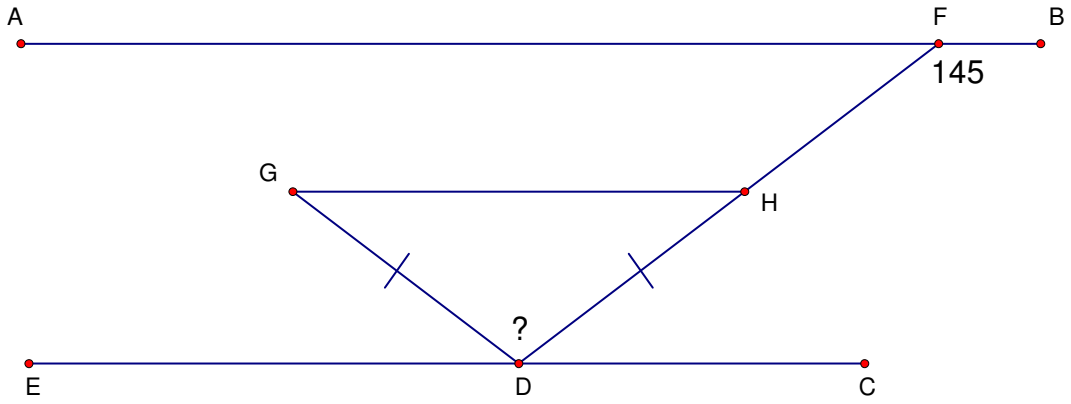
3. Three 8 inch tiles would have a length of 24 inches or 2 feet. So every 2 feet across is equivalent to 3 tiles. The 8 ft across the floor would take 12 tiles and the 14 ft across would take 21 tiles. The floor is therefore 12 tiles by 21 tiles, which is a total of 12 x 21 = **252** tiles.

An alternate solution would be to change all lengths to inches so 8 ft by 14 ft would be 96 inches by 168 inches with an area of 16128 square inches. Each tile has an area of 64 square inches. Mike would need 16128 ÷ 64 = **252** tiles.

Category 2
Geometry
Meet #1, October 2008

1. How many degrees are in the supplement of a 72° angle?

2. In the figure below angle BFH = 145°. The line segments $\overline{AB}$, $\overline{GH}$, and $\overline{EC}$ are all parallel. Triangle GDH is an isosceles triangle with GD = HD. How many degrees are in the measure of angle GDH?



3. The complement of one angle is equal to twice the supplement of a different angle. If the two angles are supplementary to each other, then how many degrees are in the measure of the smaller angle?

Answers

1. _____
2. _____
3. _____

Solutions to Category 2
Geometry
Meet #1, October 2008

Answers

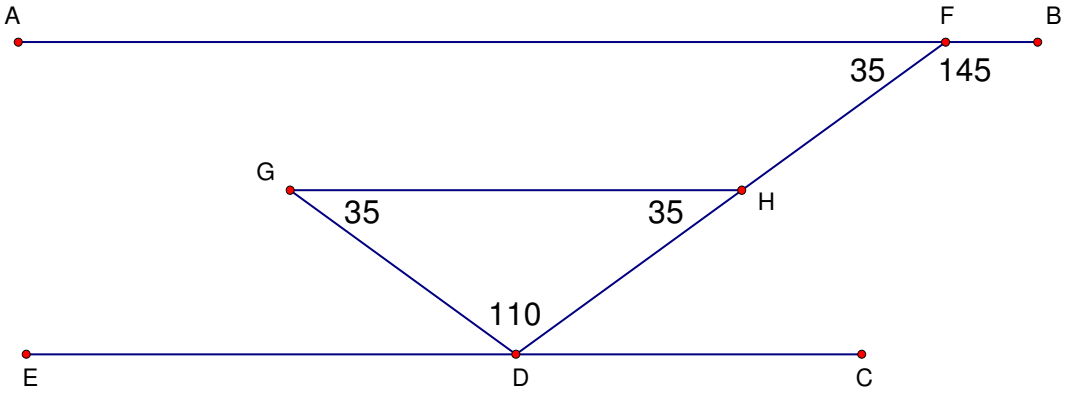
1. 108

2. 110

3. 30

1. Supplementary angles have a sum of 180°, so the supplement of a 72° angle is 180 – 72 = **108.**

2. Angle AFH is the supplement of angle BFH so its measure is 35°. Angle GHD is corresponding to angle AFH so it is also 35°. Since the triangle is isosceles, angle DGH is also 35°. Since the three angles of the triangle must have a sum of 180°, angle GDH has a measure of **110°**.



3. From the first statement we know :

$90 - x = 2(180 - y)$

$90 - x = 360 - 2y$

$2y - x = 270$

Since the two angles are supplementary we also know that $x + y = 180$.

Adding the two equations together gives us :

$(2y - x) + (x + y) = 270 + 180$

$3y = 450$

$y = 150$ and $x = 30$ So the smaller angle is 30°

Category 3
Number Theory
Meet #1, October 2008

1. The number 210 is divisible by how many prime numbers?

2. The seven digit number 2A5A756 is divisible by 3. What is the sum of the possible values of A?

3. The prime numbers 23 and 29 are the smallest two primes that are 6 apart and have no primes between them. What is the sum of the smallest two primes that are 8 apart and have no other primes between them?

Answers

1. _____
2. _____
3. _____

Solutions to Category 3
Number Theory
Meet #1, October 2008

Answers

1. 4

2. 12

3. 186

1. 210 is divisible by:
1, 2, 3, 5, 6, 7, 10, 14, 15, 21,20, 35, 42, 70, 105, and 210.
Of those only 2, 3, 5, and 7 are prime numbers, so there are **4.**

2. In order for 2A5A756 to be divisible by 3, the sum of the digits must be divisible by 3. The sum of the digits is 2A + 25 and that needs to be a multiple of 3. The smallest possible value of 2A + 25 is 2(0)+25 = 25 and the largest possible value is 2(9) + 25 = 43. The multiples of 3 between 25 and 43 are 27, 30, 33, 36, 39, and 42. We can solve the equation 2A + 25 = 27 or 30 or 33 or 36 or 39 or 42 to find A. Subtracting 25 gives us 2A = 2, 5, 8, 11, 14, or 17. We then divide by 2, but ignore any decimals since A is a digit. The possible values of A then are: 1, 4, or 7 and they have a sum of **12.**

3. Listing the primes we get :

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97……

Those last two are the first two that are 8 apart with no other primes between them. The sum is 89 + 97 = **186.**

Category 4
Arithmetic
Meet #1, October 2008

1. Evaluate this expression. Give your answer as a decimal rounded to the nearest hundredth.

$$\frac{1+3\times11}{10+2^2}$$

2. If *A* represents the median of the set of numbers below and *B* represents the mode of the same set, what is the mean of *A* and *B*? Express your answer as a common fraction. (A "common fraction" is written in the form $\frac{M}{N}$ where $\frac{M}{N}$ cannot be simplified. A "mixed number" is not a common fraction.)

12, 8, 5, 7, 3, 7, 2, 5, 2, 4, 10, 8, 12, 5

3. In a math class Monisha took 10 quizzes. Her scores on those quizzes were as follows : 79, 81, 84, 88, 90, 91, 93, 95, 97, and 99. The teacher graded in a weird way though. She put all 10 scores in a hat and pulled out 8 of them. She then found the mean of those 8 scores and that was Monisha's grade. Without any rounding, the teacher calculated a grade of exactly 90 for Monisha's grade in the class. List the two scores that were NOT pulled out of the hat.

Answers

1. _____
2. _____
3. _____

Solutions to Category 4
Arithmetic
Meet #1, October 2008

Answers

1. 2.43

2. $\frac{11}{2}$

3. 84 and 93
or 84, 93
or 93 and 84
or 93, 84

1. $\frac{1+3\times11}{10+2^2} = \frac{1+33}{10+4} = \frac{34}{14} = \frac{17}{7} = 2.\overline{428571} \approx 2.43$

2. Putting the list of numbers in order gives us :
2, 2, 3, 4, 5, 5, 5, 7, 7, 8, 8, 10, 12, 12
The median of this list is between the 5 and 7, so the median is 6 and the mode is 5 since it appears the most frequently. The mean of 5 and 6 is $\frac{5+6}{2} = \frac{11}{2}$. (This problem asks for a common fraction so $5\frac{1}{2}$ is not an acceptable answer.)

3. The ten quiz grades were 97, 99, 84, 88, 91, 95, 79, 81, 93 and 90 and they have a sum of 897. In order for 8 of the quiz grades to have a mean of 90 they would need to add up to 720. That means that the two scores left in the hat have a sum of 897 – 720 = 177. The ways you can have a sum of 177 from two whole numbers are listed below :

88 + 89
87 + 90
86 + 91
85 + 92
84 + 93
83 + 94
82 + 95
81 + 96
80 + 97
79 + 98

We don't need to go any further since there were no grades less than 79. Of all those possibilities, the only pair that represents scores Monisha actually received is **84 and 93** which must be the two numbers still in the hat.

Category 5
Algebra
Meet #1, October 2008

1. Evaluate the expression below if $x = 6$, $y = 1\frac{1}{2}$, and $z = 3$.

$$-2x + 8y - \frac{7}{3}z$$

2. Find the value of A that makes the equation below an identity.

$$3Ax + 2(4x - 5) - 11x = 4(2 + 3x) - 18$$

3. Solve each of the first three equations below and use the solutions to solve the final equation for x. What is the value of x?

Equation #1 $3a - 8 = 13$

Equation #2 $5b - 19 = 29 - 3b$

Equation #3 $\frac{7c-3}{5} = \frac{3c-4}{2}$

Final Equation : $ax + bx + 40 = cx$

Answers
1. _____
2. _____
3. _____

Solutions to Category 5
Algebra
Meet #1, October 2008

Answers

1. – 7

2. 5

3. 40

1. By substituting the values in for the variables we get:

$$-2x + 8y - \frac{7}{3}z =$$
$$-2(6) + 8\left(1\frac{1}{2}\right) - \frac{7}{3}(3) =$$
$$-12 + 12 - 7 =$$
$$\mathbf{-7}$$

2. For this to be an identity it must be a true statement for all values of x. Simplifying the equation will find the value of A that creates the identity.

$3Ax + 2(4x - 5) - 11x = 4(2 + 3x) - 18$
$3Ax + 8x - 10 - 11x = 8 + 12x - 18$
$3Ax - 3x - 10 = 12x - 10$
$3Ax = 15x$
$3A = 15$
$A = 5$

3. Solve Equations 1 through 3 first :

Equation #1 $3a - 8 = 13 \rightarrow 3a = 21 \rightarrow a = 7$

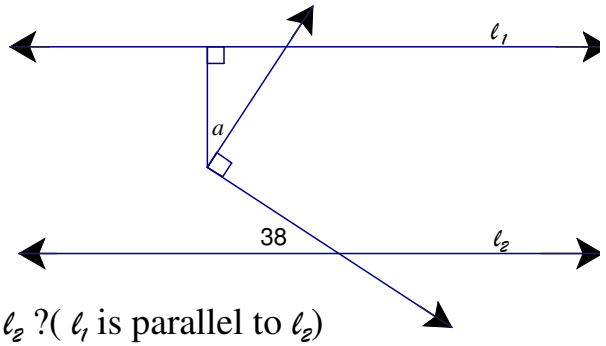
Equation #2 $5b - 19 = 29 - 3b \rightarrow 8b = 48 \rightarrow b = 6$

Equation #3 $\frac{7c-3}{5} = \frac{3c-4}{2} \rightarrow 14c - 6 = 15c - 20 \rightarrow 14 = c$

Now substitute the values of a, b, and c into the equation below

Final Equation :
$ax + bx + 40 = cx$
$7x + 6x + 40 =$14x
$13x + 40 = 14x$
$40 = x$

Category 6
Team Questions
Meet #1, October 2008



1. What is the measure of angle *a* in the diagram to the right given that $\ell_1 \parallel \ell_2$?(ℓ_1 is parallel to ℓ_2)

2. In the equation below, what is the value of A + B + C + D if A, B, C, and D are all positive integers? (HINT : If the two sides of an equation are not equal to zero, then their reciprocals are equal as well.)

$$\frac{1}{A+\frac{1}{B+\frac{1}{C+\frac{2}{D}}}} = \frac{13}{70}$$

3. Mr. Marker bought a package of 4 dry erase markers for his white board. The package had a red marker, a blue marker, a green marker, and a black marker and each marker's cap was the same color as the marker. His students played a trick on him and switched around all the markers' caps so that none of the markers had the correct colored cap on them. In how many ways could the students have switched the caps so that none of them were the right color?

4. The numbers 11, 14, 15, 16, 18, 19, 22, 23, and 26 are to be placed in a 3 by 3 magic square such that the middle row, middle column, and two diagonals must have the same sum while the outer rows and columns do not. Which of the numbers must be placed in the middle square?

5. How many whole numbers from 1 to 100 inclusive are divisible by 3 different prime numbers? (Note: 12 = 2x2x3 but is only divisible by 2 <u>different</u> prime numbers)

Answers

1. _____ = *A*
2. _____ = *B*
3. _____ = *C*
4. _____ = *D*
5. _____= *E*
6. _____

6. Using the values the team obtained in questions 1 through 5, evaluate the expression below.

$$\sqrt[3]{\frac{C(A-E)}{B-1} \times E} + \sqrt{C+D}$$

Solutions to Category 6 Team Questions
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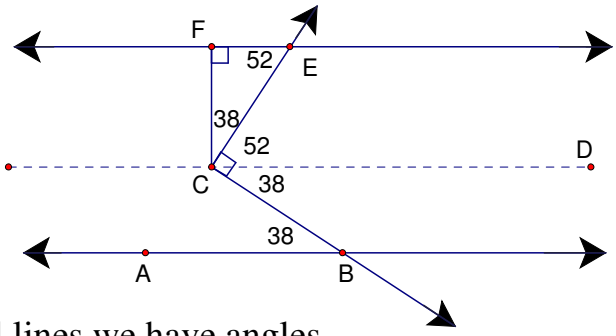

Answers

1. 38

2. 11

3. 9

4. 16

5. 8

6. 11

1. By drawing in a new line through the vertex of the right angle and parallel to the two existing parallel lines we have angles ABC and DCB being alternate interior and therefore both 38°. Since ∠BCD and ∠DCE are complementary ∠DCE = 52°. We can either then say that ∠FCE must also be 38° since it is complementary to ∠DCE, or say that ∠FEC = 52° since it is alternate interior to ∠DCE and then that ∠FCE =**38°** since it is complementary to ∠FEC as they are both in a right triangle.

2. Start by taking the inverse of both fractions to see that A = 5 and then remove 5 from both sides. Then take the inverse of both again.

$$A+\frac{1}{B+\frac{1}{C+\frac{2}{D}}}=\frac{70}{13}=5\frac{5}{13}\rightarrow\quad\frac{1}{B+\frac{1}{C+\frac{2}{D}}}=\frac{5}{13}\rightarrow$$

This shows us that B = 2 and then subtract 2 from both sides. Repeat the process again to find C and D.

$$B+\frac{1}{C+\frac{2}{D}}=\frac{13}{5}=2\frac{3}{5}\rightarrow\quad\frac{1}{C+\frac{2}{D}}=\frac{3}{5}\rightarrow C+\frac{2}{D}=\frac{5}{3}=1\frac{2}{3}$$

Which shows us that $C=1$ and $D=3$.

So $A+B+C+D=5+2+1+3=11$

3. The following 9 ways have all the caps on wrong

Correct Way →	ABCD	(representing the 4 colors)
Wrong Ways :	BADC	
	BCDA	
	BDAC	
	CADB	
	CDBA	
	CDAB	
	DABC	
	DCAB	
	DCBA	

4. In order for this magic square to follow the instructions there are going to be 4 groups of three numbers with the same sum, and all of them will include the same number(the number in the middle). Basically it means that the 8 numbers not in the middle need to be paired off so that the 4 pairs each have the same sum. Currently the 9 numbers have a sum of 164 which is divisible by 4. We need to remove the number that will be in the middle and then divide the numbers into 4 pairs. If we remove a number not divisible by 4 the sum of the 8 remaining numbers will not be divisible by 4 and we will not be able to break them into 4 pairs with the same sum. So we need a number which is divisible by 4 to go in the center and 16 is the only number divisible by 4 in that list. With **16** in the middle the numbers can be broken into pairs with a sum of 37 as such {(11, 26); (14, 23); (15, 22); and (18, 19)}. One such square is shown to the right.

11	**18**	**23**
15	**16**	**22**
14	**19**	**26**

5. We can make an organized list to find all such numbers :

2x3x5 = 30 and any multiple of 30 will work, so that gives us {30, 60, 90}
2x3x7 = 42 and any multiple of 42 will work, so that gives us {42, 84}
2x3x11 = 66 giving us just {66}
2x3x13 = 78 giving us just {78}
2x3x17 = 102 which is too big
2x5x7 = 70 giving us just {70}
2x5x11 = 110 which is too big
2x7x11 = 154 which is too big
3x5x7 = 105 which is too big

So the only number between 1 and 100 that have 3 different prime factors are {30, 60, 90; 42, 84; 66; 78; 70} for a total of **8** #s

6. $\sqrt[3]{\frac{C(A-E)}{B-1} \times E} + \sqrt{C+D} = \sqrt[3]{\frac{9(38-8)}{11-1} \times 8} + \sqrt{9+16} =$

$\sqrt[3]{\frac{9(30)}{10} \times 8} + \sqrt{25} = \sqrt[3]{27 \times 8} + 5 = \sqrt[3]{216} + 5 = 6 + 5 = \mathbf{11}$