Meet #1
October 2004

Intermediate Mathematics League of Eastern Massachusetts

Average team score: 126.9
Average meet for the season: 95.9

Meet #1
October 2004

Category 1
Mystery
Meet #1, October 2004

1. Diophantus Middle School beat Pythagoras Middle School in a basketball game by 6 points. The total number of points scored between the two teams was 82 points. How many points did Pythagoras Middle School score?

2. Pick a natural number. _____

Multiply your number by 15. _____

Add 25 to the result. _____

Divide by 5. _____

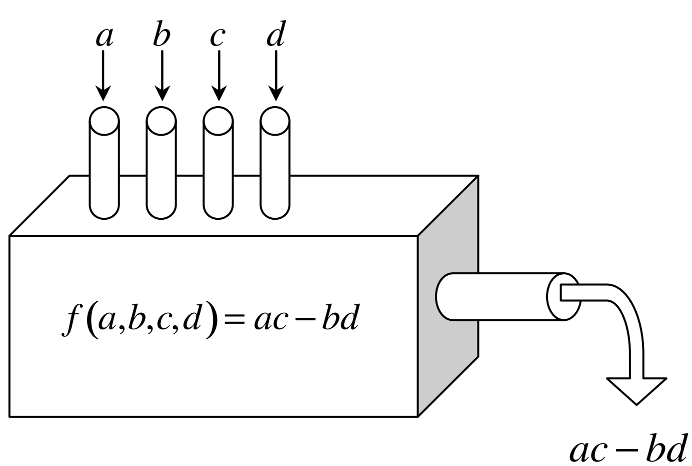
Subtract three times your original number. _____

What is the final result?

(Note: A natural number is the same as a counting number.)

3. A certain function machine takes four inputs and finds the difference between the product of the first and third numbers and the product of the second and last numbers, as suggested by the picture below. Find the value of $f(6,9,8,7)$.

Answers
1. ________________
2. ________________
3. ________________



Solutions to Category 1 Average team got 26.31 points, or 2.2 questions correct
Mystery
Meet #1, October 2004

Answers

1. 38

2. 5

3. -15

1. One way to solve this problem is to "subtract to a tie." Imagine that Diophantus Middle School did not earn those 6 points that won the game. Then the two schools would have tied with a total of 82 – 6 = 76 points, which is 38 points each. Pythagoras Middle School scored exactly **38** points, and Diophantus Middle School actually scored 38 + 6 = 44 points.

2. We will use the unspecified natural number "*n*" to show that the final result is always the same regardless of the starting number.

Pick a natural number. n

Multiply your number by 15. $15n$

Add 25 to the result. $15n + 25$

Divide by 5. $\frac{15n+25}{5} = \frac{15n}{5} + \frac{25}{5} = 3n + 5$

Subtract three times your original number.

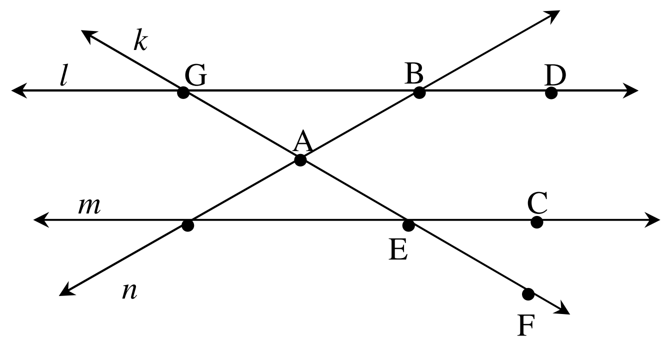
$$3n + 5 - 3n = 5$$

The unspecified natural number *n* disappears completely, and the final result is always **5**.

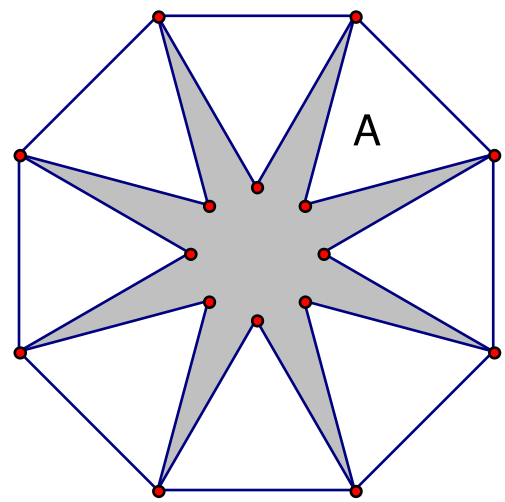
3. The function rule is $f(a,b,c,d) = ac - bd$ and we need to find $f(6,9,8,7)$. We replace *a* with 6, *b* with 9, *c* with 8, and *d* with 7 and compute $6 \cdot 8 - 9 \cdot 7 = 48 - 63 = \mathbf{-15}$.

Category 2
Geometry
Meet #1, October 2004

1. Line *l* and line *m* are parallel. The measure of angle DBA is 150°, and the measure of angle CEF is 30°. How many degrees are in the measure of angle GAB?



2. The eight-pointed star in the figure at right was created by placing equilateral triangles, such as A, along the inside edges of a regular octagon. How many degrees are in the angle measure of a point on the star?



3. If the supplement of angle *x* is five times the complement of angle *x*, how many degrees are in the measure of angle *x*? Give your answer to the nearest tenth of a degree.

Answers
1. ________________
2. ________________
3. ________________

Solutions to Category 2 Average team got 18.58 points, or 1.55 questions correct
Geometry
Meet #1, October 2004

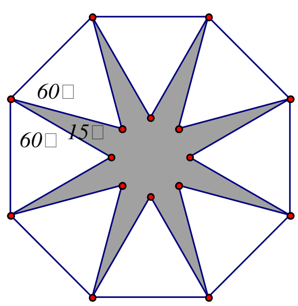
Answers

1. 120

2. 15

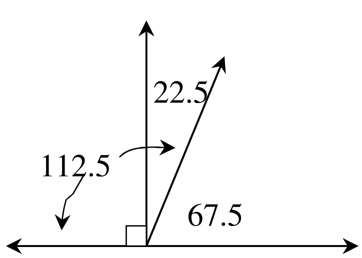
3. 67.5

1. Angle GBA is supplementary to angle DBA, whose measure is 150°, so the measure of angle GBA is 180° – 150° = 30°. Angle BGA and angle CEF are corresponding angles, so the measure of angle BGA must be equal to that of CEF, which is also 30°. Triangle GAB must have a total of 180°, so the measure of angle GAB is 180° – 2 × 30° = **120**°.



2. The interior angle of the regular octagon can be found in several ways. One way is to subdivide the octagon into six triangles, each of which has an angle sum of 180 degrees. The total interior angle is thus 6 × 180 = 1080 degrees. In a regular octagon, this total is shared equally among the eight interior angles, so each of them has an angle of 1080 ÷ 8 = 135 degrees. Each equilateral triangle has three 60 degree angles. The vertices of two triangles meet at each vertex of the octagon, occupying 2 × 60 = 120 degrees of that angle. The angle measure of a point on the star is the rest of the interior angle, or 135 – 120 = **15** degrees.

3. The supplement of angle x is 180 – x, and the complement of angle x is 90 – x. Translating the statement to algebra, we get $180 - x = 5(90 - x)$.
Solving for x, we get



$$180 - x = 450 - 5x$$
$$180 - x + 5x = 450 - 5x + 5x$$
$$180 + 4x = 450$$
$$180 - 180 + 4x = 450 - 180$$
$$4x = 270$$
$$\frac{4x}{4} = \frac{270}{4}$$
$$x = \mathbf{67.5}$$

Category 3
Number Theory
Meet #1, October 2004

1. How many pairs of primes have a sum of 24?

2. If the five-digit number $5N82N$ is divisible by 18, what is the value of N?

3. How many of the positive factors of 660 are odd?

Answers

1. ________________
2. ________________
3. ________________

Solutions to Category 3 Number Theory Meet #1, October 2004

Average team got 14.73 points, or 1.2 questions correct
The top team score was 32 out of 36 points.

Answers

1. 3

2. 6

3. 8

1. There are three (**3**) pairs of primes that have a sum of 24. They are: 19 + 5, 17 + 7, and 13 + 11.

2. For the five-digit number 5*N*82*N* to be divisible by 18, it must pass the divisibility tests for both 2 and 9. Since there is an *N* in the units place (ones place), we know that *N* has to be even. To be divisible by 9, the sum of the digits must be a multiple of 9. Currently the sum of the digits is $5 + N + 8 + 2 + N = 15 + 2N$. For $N = 0$, we get $15 + 2 \times 0 = 15$, which is not a multiple of 9. For $N = 2$, we get $15 + 2 \times 2 = 19$. For $N = 4$, we get $15 + 2 \times 4 = 23$. For $N = 6$, we get $15 + 2 \times 6 = 27$, which is a multiple of 9. This tells us that the five-digit number 56826 is divisible by 18. ($56826 \div 18 = 3157$) Note that this is the only *single-digit value* of *N* that will make $15 + 2N$ equal a multiple of 9. Thus, *N* = **6**.

3. The factor pairs of 660 are listed below. The eight (**8**) odd factors are in bold.

1 × 660
2 × 330
3 × 220
4 × **165**
5 × 132
6 × 110
10 × 66
11 × 60
12 × **55**
15 × 44
20 × **33**
22 × 30

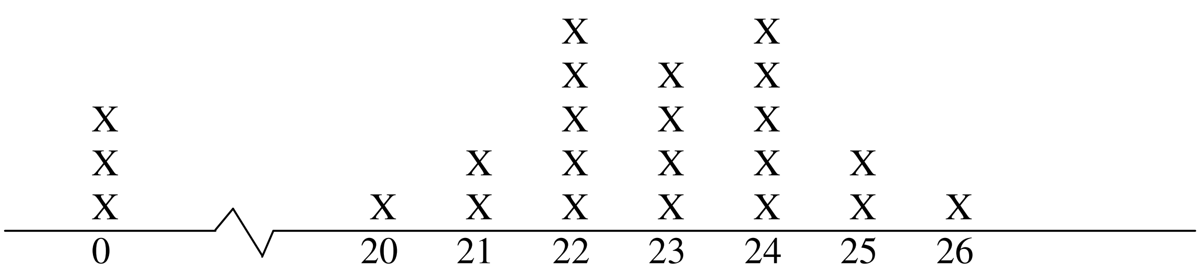
Category 4
Arithmetic
Meet #1, October 2004

1. Evaluate the following expression.

$$\frac{100-48\div 2\times 3}{2}$$

2. Leslie scored 98, 88, 92, 86, 88, and 98 on the first six math quizzes. Larson scored 89, 95, 78, 80, 96, and 95 on the first six math quizzes. What is the positive difference between the median of Leslie's quiz scores and the median of Larson's quiz scores.

3. A teacher gave her students little packages of m&m's. Each X in the line plot below represents the number of m&m's in a student's package. As you can see, there were not enough packages for everyone. To make it fair, the people who had more than the average number of m&m's gave some to those people who had less than the average, until everyone had the same number of m&m's. If Roger is the student who got the package with 26 m&m's, how many m&m's did Roger have to give away?



Answers
1. ________________
2. ________________
3. ________________

Solutions to Category 4 Average team got 24.81 points, or 2.0 questions correct
Arithmetic
Meet #1, October 2004

Answers

1. 14

2. 2

3. 6

1. Using the order of operations, we get:

$$\frac{100-48\div 2\times 3}{2}=\frac{100-24\times 3}{2}=\frac{100-72}{2}=\frac{28}{2}=\mathbf{14}$$

2. To find the median of a set of data, we must first arrange the data from least to greatest. Leslie's quiz scores are 86, 88, 88, 92, 98, 98 and Larson's quiz scores are 78, 80, 89, 95, 95, 96. The median of Leslie's scores is half way between 88 and 92, which is 90. The median of Larson's scores is half way between 89 and 95, which is 92. The positive difference between these medians is **2**.

3. The total number of m&m's is $3 \times 0 + 1 \times 20 + 2 \times 21 + 5 \times 22 + 4 \times 23 + 5 \times 24 + 2 \times 25 + 1 \times 26 = 0 + 20 + 42 + 110 + 92 + 120 + 50 + 26 = 460$. There are 23 X's on the line plot, which means there are 23 students in the class. Everyone should get $460 \div 23 = 20$ m&m's each. This means Roger will have to give away 26 – 20 = **6** m&m's.

Category 5
Algebra
Meet #1, October 2004

1. Find the value of C so that the equation below is an identity.

$$13(2x+5)+7x+C=4(9x-8)-3x$$

2. Evaluate the expression below for $x=\frac{5}{8}$ and $y=\frac{7}{12}$.

$$(18x-9y)-(4x+6y)$$

3. In a certain triathlon, the competitors run five times as far as they swim and they bicycle four times as far as they run. If the total distance traveled in the race is 52 miles, how many miles do the competitors run?

Answers
1. ________________
2. ________________
3. ________________

Solutions to Category 5 Average team got 21.23 points, or 1.8 questions correct
Algebra
Meet #1, October 2004

Answers

1. -97

2. 0

3. 10

1. First we distribute and combine like terms on both sides of the equation as follows:

$$13(2x+5)+7x+C=4(9x-8)-3x$$
$$26x+65+7x+C=36x-32-3x$$
$$33x+65+C=33x-32$$

Notice that we have $33x$ on both sides of the equation. We can leave it or we can subtract it from both sides. The point is that this equation will an identity (true for all values of x) if we find the value of C that makes 65 + C equal to –32. Solving for C, we get:

$$65+C=-32$$
$$C=\mathbf{-97}$$

2. Substituting the values given into the equation, we get

$$\begin{aligned}(18x-9y)-(4x+6y) &= 18x-9y-4x-6y \\ &= 14x-15y \\ &= 14\cdot\frac{5}{8}-15\cdot\frac{7}{12} \\ &= 7\cdot\frac{5}{4}-5\cdot\frac{7}{4} \\ &= \frac{35}{4}-\frac{35}{4} \\ &= \mathbf{0}\end{aligned}$$

3. Let x be the distance in miles that the competitors swim. If they run five times as far as they swim, then they run $5x$ miles. If they bicycle four times as far as they run, then they bicycle $4(5x) = 20x$ miles. The total distance the competitors run, swim, and bicycle is 52 miles, which gives us the equation: $5x + x + 20x = 52$. Combining like terms, we get $26x = 52$, so $x = 2$. This means that the competitors swim 2 miles, so they must run $2 \times 5 =$ **10** miles.

Category 6
Team Questions
Meet #1, October 2004

1. If $6120 = 2^a \cdot 3^b \cdot 5^c \cdot 7^d \cdot 11^e \cdot 13^f \cdot 17^g \cdot 19^h$, then find the value of $(a+1)(b+1)(c+1)(d+1)(e+1)(f+1)(g+1)(h+1)$.

2. If $3w+6=2x+5$ and $4x+10=15y-8$ and $30y-16=4z+8$ and $w = 5$, then find the value of z.

3. Find the mean (average) of the values obtained when the numbers 2, 3, and 4 are substituted in all possible ways for *a*, *b*, and *c* in the expression ab^c. You must use each number once. Round your answer to the nearest whole number.

4. If $A \clubsuit B$ means $\frac{2A+3B}{5}$, find the value of $9\clubsuit(14\clubsuit4)$. Express your answer as a decimal to the nearest tenth.

5. The Primesons had three sons, born on the same day 6 years apart and 6 years apart. How many times were all three sons a prime age before the eldest was 50 years old? (Reminder: One is not prime, so we do not count the time when the sons were ages 1, 7, and 13 years old.)

Answers
1. ______________ $= A$
2. ______________ $= B$
3. ______________ $= C$
4. ______________ $= D$
5. ______________ $= E$
6. __________________

6. Using the values the team obtained in questions 1 through 5, evaluate the following expression:

$$\sqrt{DE^2 - \left(\sqrt{3A} + \frac{C}{2B}\right)}$$

Solutions to Category 6 Average team got 21.23 points, or 3.5 questions correct
Team Questions
Meet #1, October 2004

Answers

1. 48

2. 19

3. 76

4. 8.4

5. 5

6. 14

1. The usual way of writing the prime factorization of 6120 is $2^3 \cdot 3^2 \cdot 5 \cdot 17$. Using all the bases, we can write the equivalent: $2^3 \cdot 3^2 \cdot 5^1 \cdot 7^0 \cdot 11^0 \cdot 13^0 \cdot 17^1 \cdot 19^0$.
The product that we are looking for is thus:
$(a+1)(b+1)(c+1)(d+1)(e+1)(f+1)(g+1)(h+1)$
$= (3+1)(2+1)(1+1)(0+1)(0+1)(0+1)(1+1)(0+1)$
$= 4 \cdot 3 \cdot 2 \cdot 1 \cdot 1 \cdot 1 \cdot 2 \cdot 1 = \mathbf{48}$, which, incidently, is the number of factors of 6120

2. We could substitute the given value of *w* into the first equation and then work our way through each equation to the value of *z*. On the other hand, we can by-pass the values of *x* and *y* completely if we notice something about the equations. If we quadruple the first equation, we get $12w + 24 = 8x + 20$. If we double the second equation, we get $8x + 20 = 30y - 16$. The third equation is $30y - 16 = 4z + 8$. Now we can equate them all as follows: $12w + 24 = 8x + 20 = 30y - 16 = 4z + 8$. Since we are given the value *w* and we seek the value of *z*, we need only look at $12w + 24 = 4z + 8$. Substituting 5 for *w*, we get:

$$12 \cdot 5 + 24 = 4z + 8$$
$$60 + 24 = 4z + 8$$
$$84 = 4z + 8$$
$$76 = 4z$$
$$z = \frac{76}{4} = \mathbf{19}$$

3. The list at right shows the six values obtained when the numbers 2, 3, and 4 are substituted in all possible ways for *a*, *b*, and *c* in the expression ab^c.
The sum of these values is 454, so their mean (average) is 454 ÷ 6 = 75.666... Rounding to the nearest whole number, we get **76**.

$$2\cdot 3^4 = 2\cdot 81 = 162$$
$$2\cdot 4^3 = 2\cdot 64 = 128$$
$$3\cdot 2^4 = 3\cdot 16 = 48$$
$$3\cdot 4^2 = 3\cdot 16 = 48$$
$$4\cdot 2^3 = 4\cdot 8 = 32$$
$$4\cdot 3^2 = 4\cdot 9 = 36$$

4. To find the value of 9♣(14♣4), we must first calculate the value of (14♣4), which is $\frac{2\cdot 14+3\cdot 4}{5}=\frac{28+12}{5}=\frac{40}{5}=8$. Now we can calculate 9♣8, which is $\frac{2\cdot 9+3\cdot 8}{5}=\frac{18+24}{5}=\frac{42}{5}=\mathbf{8.4}$.

5. There are five (**5**) sets of three primes that are 6 and 6 between them. They are: 5, 11, 17; 7, 13, 19; 11, 17, 23; 17, 23, 29; and 31, 37, 43.

6. Substituting the correct values for *A* through *E* into the expression, we get:

$$\begin{aligned}\sqrt{DE^2-\left(\sqrt{3A}+\frac{C}{2B}\right)} &= \sqrt{8.4\cdot 5^2-\left(\sqrt{3\cdot 48}+\frac{76}{2\cdot 19}\right)}\\ &= \sqrt{8.4\cdot 25-\left(\sqrt{144}+\frac{76}{38}\right)}\\ &= \sqrt{210-(12+2)}\\ &= \sqrt{210-14}\\ &= \sqrt{196}\\ &= \mathbf{14}\end{aligned}$$