

IMLEM
1999-2000



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calculator today!**

Category 1 - Mystery
Meet #5 - March, 2000

- 1) **Jessica's hair is tied at her neckline with an elastic band. Below the band is the rest of her hair which consists of three parts: a French braid, a straight section, and a regular braid. The French braid is three times as long as the straight section, and the straight section is half as long as the regular braid. If the regular braid is 6 inches long, then how many inches long is the hair below the elastic band ?**
- 2) **In a loaf of multi-grain bread, $\frac{1}{4}$ of the grains are dark rye, $\frac{1}{5}$ are light rye, 10% are oat, $\frac{5}{18}$ are cracked wheat, and the rest are whole wheat. If there are 19,980 grains in the loaf, then how many grains are whole wheat ?**
- 3) **$N \cdot 10^Y$ is the product $2^{23} \cdot 5^{19}$ written in scientific notation, where $1 \leq N < 10$, and Y is an integer. What is the value of $N + Y$? Express your answer as a decimal.**

ANSWERS

- 1) _____ inches
- 2) _____ grains
- 3) _____

SOLUTIONS - Meet #5 - Category 1

ANSWERS

CATEGORY 1 MYSTERY

1) 18

2) 3441

3) 21.6

1) Regular braid = 6 inches.

Straight section = $\frac{1}{2}$ of the regular braid
= $\frac{1}{2}$ (6)
= 3 inches

French braid = 3 times the straight section
= 3 (3)
= 9 inches.

Total length = $6 + 3 + 9 = 18$ inches.

2) The fraction of the loaf, represented by whole wheat, is

$$\begin{aligned} & 1 - \left(\frac{1}{4} + \frac{1}{5} + 10\% + \frac{5}{18} \right) \\ &= 1 - \left(\frac{45}{180} + \frac{36}{180} + \frac{18}{180} + \frac{50}{180} \right) \\ &= 1 - \left(\frac{45 + 36 + 18 + 50}{180} \right) \\ &= 1 - \left(\frac{149}{180} \right) \\ &= \frac{31}{180} \end{aligned}$$

$\frac{31}{180}$ of 19,980 = 3441.

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SOLUTIONS - Meet #5 - Category 1

Category 1, continued . . .

$$\begin{aligned} 3) \quad & 2^{23} \cdot 5^{19} \\ &= 2^4 \cdot 2^{19} \cdot 5^{19} \\ &= 2^4 \cdot (2^{19} \cdot 5^{19}) \\ &= 2^4 \cdot (2 \cdot 5)^{19} \\ &= 2^4 \cdot (10)^{19} \\ &= 16 \cdot (10)^{19} \\ &= [(1.6) \cdot 10^1] \cdot (10)^{19} \\ &= (1.6) \cdot [10^1 \cdot (10)^{19}] \\ &= (1.6) \cdot [10^{20}] \end{aligned}$$

So, $N = 1.6$, and $Y = 20$.

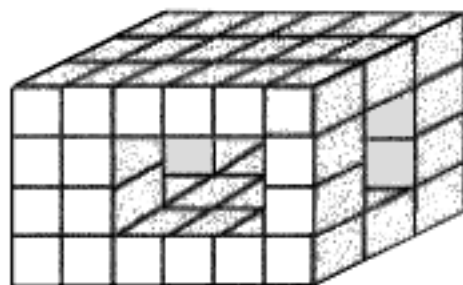
$$\begin{aligned} \text{Therefore,} \quad & N + Y \\ &= 1.6 + 20 \\ &= \mathbf{21.6} \end{aligned}$$



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Category 2 - Geometry
Meet #5 - March, 2000

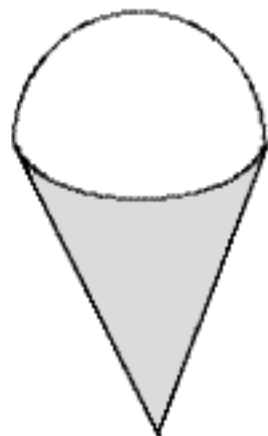
- 1) **How many of the smallest cubes are required to build the figure below ? (Note: If you see a "hole" then assume that it passes through completely.)**



- 2) **A pyramid and a rectangular solid have equal volumes. Both shapes have equal bases. The altitude (height) of the pyramid is 72 feet. How many feet are in the altitude of the rectangular solid ?**
- 3) **An ice cream cone holds a scoop of vanilla ice cream in the shape of a sphere, as pictured below, so that half of the scoop is inside the cone, and half is outside. The ice cream is allowed to melt. When it is completely melted, it fills the cone. If the cone has an altitude of 7 inches, then how many inches are in the diameter of the original ice cream sphere ? Express your answer as an improper fraction in lowest terms.**

ANSWERS

- 1) _____ cubes
2) _____ centimeters
3) _____ inches



SOLUTIONS - Meet #5 - Category 2

ANSWERS

CATEGORY 2 GEOMETRY

1) **48**

2) **24**

3) $\frac{7}{2}$

1) If the figure were solid, it would contain $4 \times 6 \times 3$ of the smallest cubes, or 72. There are 24 cubes missing, so the total is $72 - 24$, or **48**.

2) Since the two figures have equal bases, the rectangular solid only needs to be one-third of the height of the pyramid in order to have a volume equal to that of the pyramid. One-third of 72 is **24**.

Another way to show this is as follows:

Let H = the height of the rectangular solid.

- * Use the formulas for the volumes of the two figures,
- * set the two formulas equal to each other,
- * substitute known values,
- * then solve for H :

volume of rect. solid = volume of pyramid

$$LWH = \frac{1}{3}(\text{area of base})(\text{height})$$

$$LWH = \frac{1}{3}(LW)(\text{height})$$

$$LWH = \frac{1}{3}(LW)(72)$$

Dividing both sides by the product LW yields the following statement:

$$H = \frac{1}{3}(72)$$

$$H = \mathbf{24.}$$

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SOLUTIONS - Meet #5 - Category 2

Category 2, continued . . .

- 3) This solution is similar to #2:

volume of sphere = volume of cone

$$\frac{4}{3} \cdot \pi R^3 = \frac{1}{3} \cdot \pi R^2 H$$

Since the radius of the sphere is the same as the radius of the circular rim of the cone, we can divide both sides by $\frac{1}{3} \cdot \pi R^2$, giving the following equation:

$$4R = H$$

Substituting 7 for the height of the cone:

$$4R = 7$$

$$4R + 4 = 7 + 4$$

$$R = 1.75$$

If the radius of the sphere is 1.75 inches, then its diameter is twice 1.75, or **3.5** inches.

A more interesting outcome is found in the step above which says that $4R = H$! In other words, the height of the cone is the same as four times the radius of its circular base, or twice its diameter! Knowing that, the diameter of the circular base (same as the diameter of the sphere) is half of 7, or $\frac{7}{2}$.

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Category 3 - Number Theory

Meet #5 - March, 2000

- 1) **Ten students in a class of 23 at the Wellesley Middle School like Pepsi, while 18 like Coke. What is the greatest (most) possible number of students who may not like either Coke or Pepsi ?**

- 2) **Set A = {all composite numbers between 29 and 43}**
Set B = {all whole numbers between 25 and 44 which are divisible by 3}
Set C = {all 2-digit whole numbers, the sum of whose digits is less than 7}

Find the sum of all numbers in the set $(A \cap B) \cap C$.

- 3) **There are 360 students at the Brown Middle School in Newton.**
 - * **50% like to watch TV.**
 - * **$\frac{2}{5}$ like to go to the movies.**
 - * **60% like to listen to the radio.**
 - * **$\frac{1}{4}$ like to watch TV and go to the movies.**
 - * **20% like to go to the movies and listen to the radio.**
 - * **$\frac{2}{9}$ like to watch TV and listen to the radio.**
 - * **10% like to do all three activities.**

How many students at the Brown Middle School do not like any of the three activities ?

ANSWERS

1) _____

2) _____

3) _____

SOLUTIONS - Meet #5 - Category 3

ANSWERS

CATEGORY 3 NUMBER THEORY

- 1) 5
- 2) 105
- 3) 26
- 1) The greatest number who may not like either drink is 23 - 18, or 5, accounting for the fact that of all of the ten students who like Pepsi, it does not matter how many of them may not like Coke.
- 2) Set A = {30, 32, 33, 34, 35, 36, 38, 39, 40, 41, 42}
Set B = {27, 30, 33, 36, 39, 42}
Set C = {10, 11, 12, 13, 14, 15, 20, 21, 22, 23, 24, 30, 31, 32, 33, 40, 41, 42, 50, 51, 60}

$$\begin{aligned} (A \cap B) \cap C &= \{(30, 33, 39, 42)\} \cap \{\text{Set C}\} \\ &= \{30, 33, 42\} \end{aligned}$$

The sum of 30, 33, and 42, is

$$\begin{aligned} &30 + 33 + 42 \\ &= 105. \end{aligned}$$

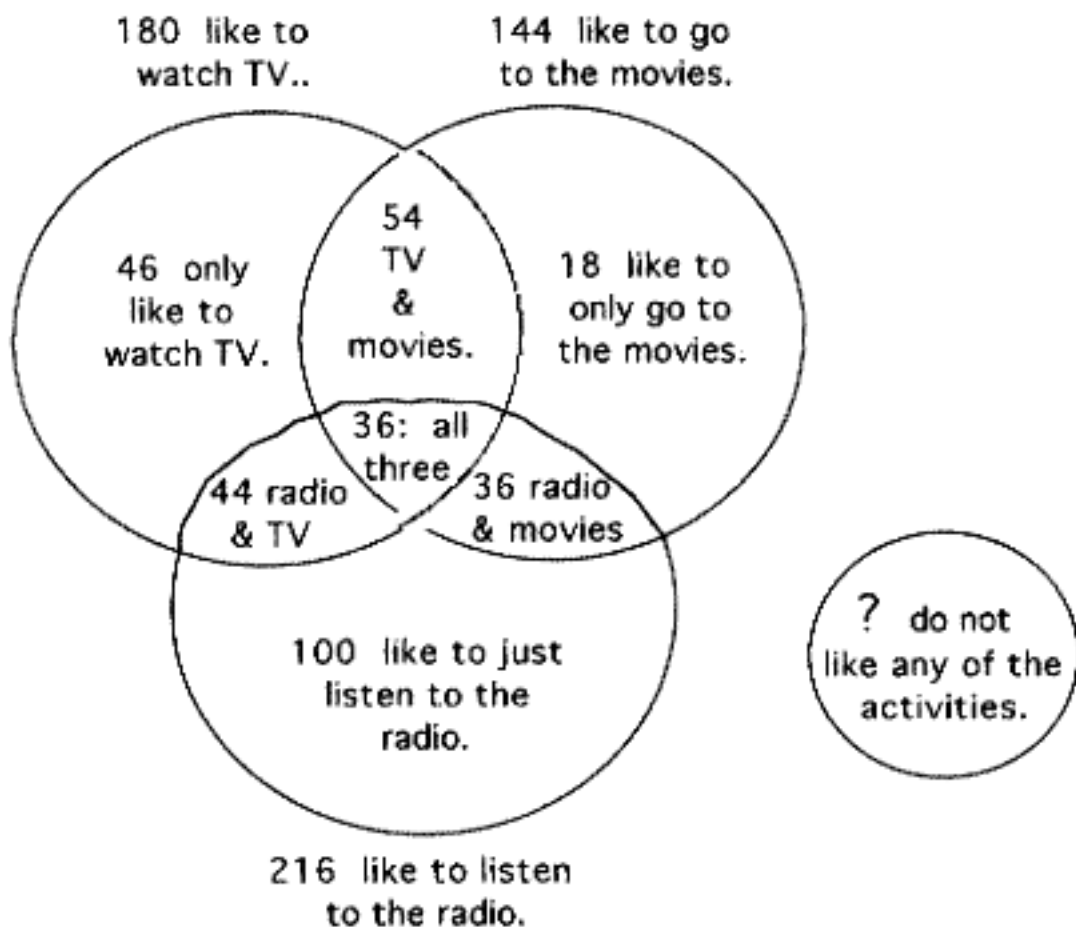
- 3)
- | | | |
|-----------------|-------------------------------|-----------|
| Watch TV | = 50% of 360 | = 180 |
| Go to movies | = $\frac{2}{5}$ of 360 | = 144 |
| Listen to radio | = 60% of 360 | = 216 |
| All three | = 10% of 360 | = 36 |
| TV & movies | = $(\frac{1}{4}$ of 360) - 36 | = 90 - 36 |
| | | = 54 |
| Movies & radio | = (20% of 360) - 36 | = 72 - 36 |
| | | = 36 |
| TV & radio | = $(\frac{2}{9}$ of 360) - 36 | = 80 - 36 |
| | | = 44 |

The Venn diagram on the next page helps to organize all of this information:

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SOLUTIONS - Meet #5 - Category 3

Category 3, continued . . .



The total number of students who like any of the activities is the sum of all the numbers inside the triple-circle part of the diagram:

$$= 46+54+18+36+44+36+100 \\ = 334.$$

Therefore, the number of students who do not like any of the activities is

$$= 360 - 334 \\ = 26.$$



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Category 4 - Arithmetic
Meet #5 - March, 2000

- 1) **There are cans of ice-cold soda in the cooler, as follows:**

creme soda	7
ginger ale	3
cola	6
root beer	5
dr. peppah	3

- Carolyn likes only creme soda and dr. peppah. If she selects a soda at random from the cooler, then what is the probability that she selects one that she likes? Express your answer as a fraction in lowest terms.**
- 2) **Two cubical dice, each with six faces numbered 1,2,3,4,5,6, are rolled. The numbers on the top faces are then multiplied. What is the probability that the product is greater than 10? Express your answer as a percent, rounded to the nearest whole percent.**
- 3) **In his pocket, Justin has four dimes, one nickel, and seven quarters. If he takes two coins from his pocket at random, then what is the probability that their total value is exactly 35¢? Express your answer as a fraction in lowest terms. (Note: A dime is worth 10¢, a quarter is worth 25¢, and a nickel is worth 5¢.)**

ANSWERS

- 1) _____
- 2) _____
- 3) _____

SOLUTIONS - Meet #5 - Category 4

ANSWERS

CATEGORY 4 ARITHMETIC

1) $\frac{5}{12}$

2) 47%

3) $\frac{14}{33}$

1)
$$\frac{\text{\# of creme soda or dr. peppah}}{\text{total \# of sodas}}$$

$$= \frac{7+3}{7+3+6+5+3}$$

$$= \frac{10}{24}$$

$$= \frac{5}{12}$$

- 2) The following chart shows the outcomes for the dice (1,2,3,4,5,6) along the left and top edges, as well as all possible products, multiplying each number of the first die with each number of the second die:

		First die					
		1	2	3	4	5	6
Second die	1	1	2	3	4	5	6
	2	2	4	6	8	10	12
	3	3	6	9	12	15	18
	4	4	8	12	16	20	24
	5	5	10	15	20	25	30
	6	6	12	18	24	30	36

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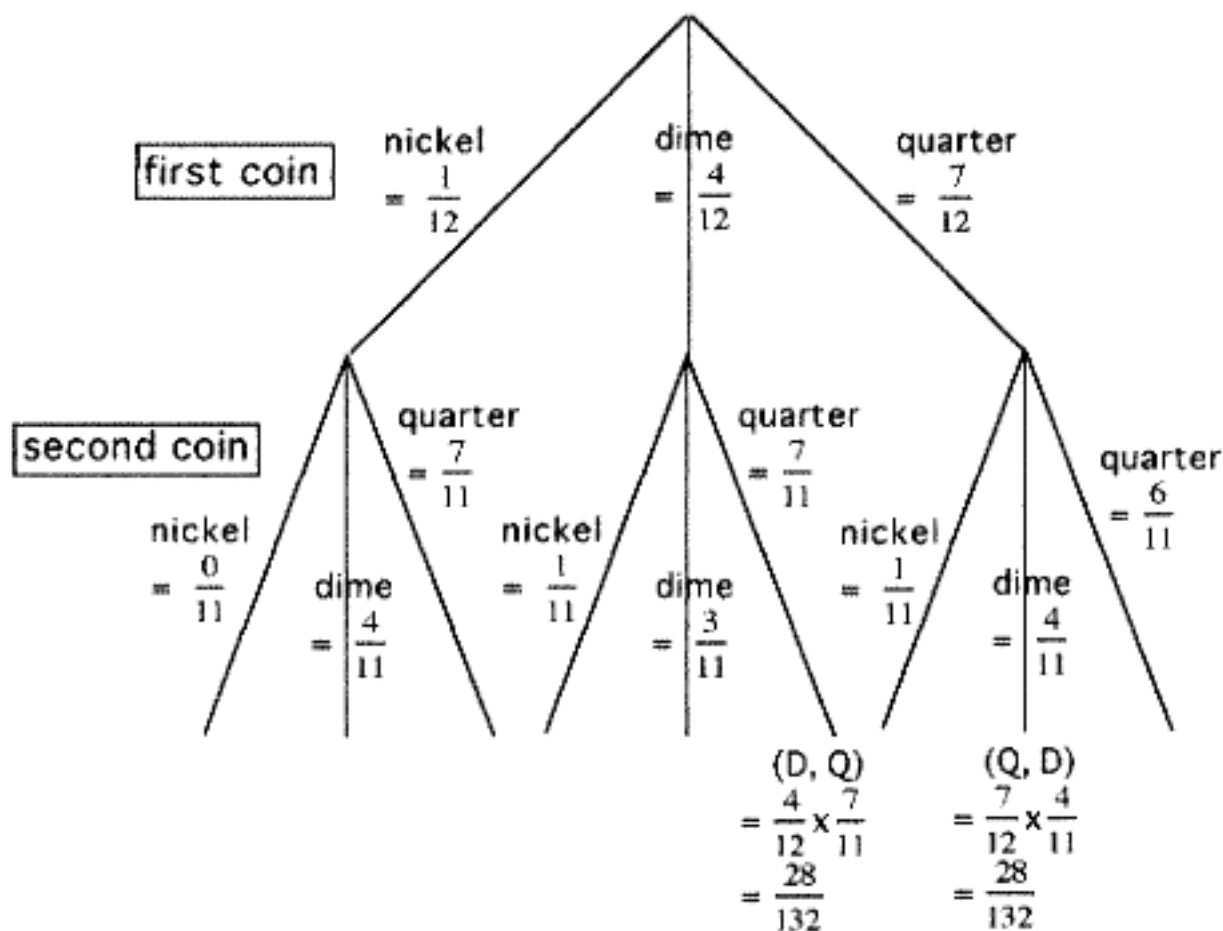
SOLUTIONS - Meet #5 - Category 4

Category 4, continued . . .

There are 17 products out of the 36 which are greater than 10.

$$\frac{17}{36} = 0.472222\dots = 47.2222\dots\% \approx \mathbf{47\%}$$

- 3) The only way that Justin can get 35¢ is if he takes one dime and one quarter. The number of different ways that he can choose one dime and one quarter can be diagrammed as follows:



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SOLUTIONS - Meet #5 - Category 4

Category 4, continued . . .

Since the order that the dime and quarter are chosen does not matter, the probability of choosing a quarter and a dime, any order, is

$$\begin{aligned} & \frac{28}{132} + \frac{28}{132} \\ = & \frac{28+28}{132} \\ = & \frac{56}{132} \\ = & \frac{14}{33} \end{aligned}$$

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Category 5 - Algebra
Meet #5 - March, 2000

- 1) Find the two values of C which make the following equation a true statement:

$$-5C + C^2 = 36$$

- 2) A square and a rectangle have equal areas. The length of the rectangle is B centimeters more than a side of the square. The width of the rectangle is 4 centimeters less than a side of the square. How many centimeters are in the perimeter of the square? Round your answer to the nearest tenth.
- 3) If an object is launched in a vertical direction, it is affected by gravity. If H = the initial position of the object, and R = its initial velocity (rate of speed), and T = the number of seconds since the object was released, then

$$H = RT - 5T^2$$

Cagney stepped off a diving platform which is 20 meters above the water at the same time that Lacey sprang upward, vertically, from a board which is 6 meters above the water. Lacey's initial velocity was 13 meters per second. Two questions (both must be answered correctly for you to receive credit):

- A. Who hit the water first?
B. How many seconds sooner did she hit the water than did the other diver?

ANSWERS

1) { _____, _____ }

2) _____ centimeters

3) A. _____

B. _____ seconds

SOLUTIONS - Meet #5 - Category 5

ANSWERS

CATEGORY 5

ALGEBRA

1) - 4, 9
(any order)

2) 32

3) Cagney
1

(Both answers must be correct for credit to be given.)

$$1) \quad -5C + C^2 = 36$$

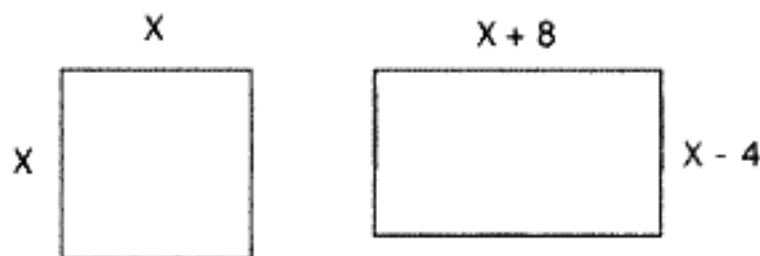
$$C^2 - C - 36 = 0$$

$$(C - 9)(C + 4) = 0$$

$$C - 9 = 0 \quad \text{or} \quad C + 4 = 0$$

$$C = 9 \quad \text{or} \quad C = -4$$

2) Let X = the length of a side of the square



$$\text{Area of square} = \text{Area of rectangle}$$

$$(\text{side})^2 = (\text{length})(\text{width})$$

$$X^2 = (X + 8)(X - 4)$$

$$X^2 = X^2 + 8X - 4X - 32$$

$$X^2 + (-X^2) = X^2 + (-X^2) + 8X - 4X - 32$$

$$0 = 4X - 32$$

$$0 + 32 = 4X - 32 + 32$$

$$32 = 4X$$

$$32 \div 4 = 4X \div 4$$

$$8 = X$$

$$\begin{aligned} \text{The perimeter of the square is} &= 4(X) \\ &= 4(8) \\ &= 32 \text{ cm.} \end{aligned}$$

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SOLUTIONS - Meet #5 - Category 5

Category 5, continued . . .

- 3) Cagney steps off the platform, so her initial velocity is 0 (zero) meters per second. The amount of time it takes for her to hit the water can be found by solving the following equation for T:

$$H = RT - 5T^2$$

$$-20 = (0)T - 5T^2$$

$$-20 = 0 - 5T^2$$

$$-20 = -5T^2$$

$$-20 + 5 = -5T^2 + 5$$

$$4 = T^2$$

$$2 = T \quad \text{or} \quad -2 = T$$

Since time in this case is positive, it takes Cagney 2 seconds to hit the water.

Lacey's time can be figured as follows:

$$H = RT - 5T^2$$

$$-6 = 13T - 5T^2$$

$$5T^2 - 13T - 6 = 0$$

$$(5T + 2)(T - 3) = 0$$

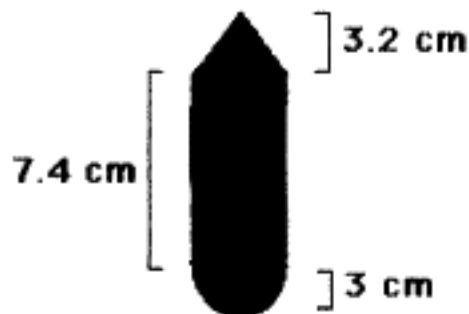
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Category 6 - Team Questions
Meet #5 - March, 2000

- 1) A plumb bob, whose front view is pictured below, is the union of a cylinder, a cone, and a hemisphere. It is made of iron whose density is 212 grams per cubic centimeter. How many grams is the weight of the plumb bob? Use $\pi = 3.14$. Round your final answer to the nearest tenth.



- 2) A pyramid with a square base and an altitude of 72 meters has a volume which is 6893 cubic meters greater than the volume of a cube whose surface area is 11,094 square meters. How many square meters are in the total surface area of the pyramid?
- 3) Lucky picked a positive whole number N , at random, such that $150 \leq N \leq 300$, and N is a multiple of 12. Fortunato picked a positive whole number, P , such that $126 \leq P \leq 324$, and P is divisible by 18. $\frac{X}{Y}$ is the probability that Lucky and Fortunato picked the same number. If X and Y are relatively prime, then what is the value of Y ?
- 4) The positive difference between the squares of two consecutive even integers, F and G , is 508. $F < G$. What is the value of G ?

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Category 6 - continued . . .

- 5) One diagonal of a rhombus is 30 inches long, and the other diagonal is 40 inches long. A circle is inscribed in the rhombus. To the nearest tenth of a square inch, how many square inches are outside the circle, but inside the rhombus? Use $\pi \approx 3.14$.



- 6) Evaluate the expression below, using the answers to questions #1-5 as values for A, B, C, D, and E, respectively. Express your answer in scientific notation.

$$\left(\frac{D}{C-7}\right)\left[(100)(A-E)-5B-191,130\right]$$

ANSWERS

- 1) _____ = A
2) _____ = B
3) _____ = C
4) _____ = D
5) _____ = E
6) _____

SOLUTIONS - Meet #5 - Category 6

ANSWERS

CATEGORY 6 TEAM QUESTIONS

- 1) 62,707.1
2) 12,960
3) 39
4) 128
5) 147.8
6) 2.4×10^7

1) Volume of cone + cylinder + hemisphere

$$\begin{aligned} &= \frac{1}{3}(\text{base})(\text{height}) + (\text{base})(\text{height}) \\ &\quad + \frac{4}{3}\pi R^3 \\ &= \frac{1}{3}(\pi R^2)(3.2) + (\pi R^2)(7.4) \\ &\quad + \frac{4}{3}(\pi)(3^3)(0.5) \\ &= \frac{1}{3}(3.14)(3^2)(3.2) + (3.14)(3^2)(7.4) \\ &\quad + \frac{4}{3}(3.14)(27)(0.5) \\ &= \frac{1}{3}(3.14)(9)(3.2) + (3.14)(9)(7.4) \\ &\quad + \frac{4}{3}(84.78)(0.5) \\ &= \frac{1}{3}(90.432) + 209.124 + 113.04(0.5) \\ &= 30.144 + 209.124 + 56.52 \\ &= 295.788. \end{aligned}$$

Then, Mass = Volume x Density
= (295.788 cu. cm) x (212 gm / cu.cm)
= 62,707.056 grams
= **62,707.1** grams (nearest tenth)

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SOLUTIONS - Meet #5 - Category 6

Category 6, continued . . .

- 2) Surface area of cube = 11,094.
Area of one surface of cube = $11,094 \div 6 = 1849$.
Length of one side of cube = $\sqrt{1849} = 43$.
Volume of cube = $(43)(43)(43) = 79,507$.

Volume of cube + 6893 = volume of pyramid.

$$79,507 + 6893 = \frac{1}{3}(\text{area of base})(\text{height})$$

$$86,400 = \frac{1}{3}(\text{area of base})(72)$$

$$86,400 = 24(\text{area of base})$$

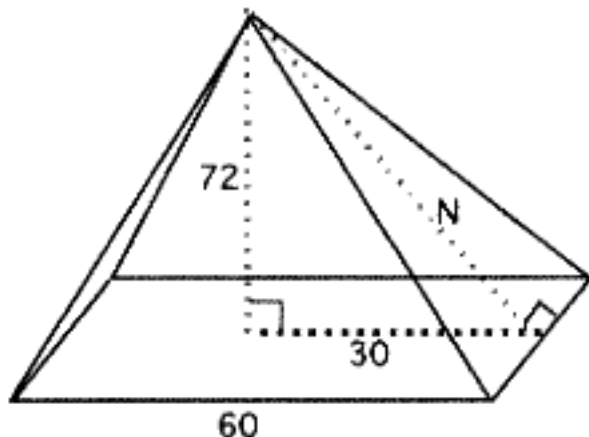
$$86,400 \div 24 = 24(\text{area of base}) \div 24$$

$$3600 = \text{area of base}$$

$$\sqrt{3600} = \text{length of one side of base}$$

$$60 = \text{length of one side of base}$$

To find the area of one triangular surface of the pyramid, it is necessary to know its altitude. The altitude of each surface-triangle is the hypotenuse of an "interior" right triangle whose base is half the length of one side of the square base, and whose height is the height of the pyramid itself. Refer to the 3-dimensional diagram below:



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SOLUTIONS - Meet #5 - Category 6

Category 6, continued . . .

Use the Pythagorean Theorem to find the length marked N:

$$A^2 + B^2 = C^2$$

$$30^2 + 72^2 = C^2$$

$$900 + 5184 = C^2$$

$$6084 = C^2$$

$$78 = C$$

Each surface triangle of the pyramid has an altitude of 78.

To find the total surface area of the pyramid, use this formula:

$$\begin{aligned} & (\text{area of base}) + 4 (\text{area of one triangle}) \\ & (\text{length}) (\text{width}) + 4 (0.5)(\text{base}) (\text{altitude}) \\ = & (60) (60) + 4 (0.5)(60) (78) \\ = & 3600 + 9360 \\ = & 12,960 \text{ square meters.} \end{aligned}$$

- 3) The set $N = \{156, 168, 180, 192, 204, 216, 228, 240, 252, 264, 276, 288, 300\}$

The set $P = \{126, 144, 162, 180, 198, 216, 234, 252, 270, 288, 306, 324\}$

There are 13 values for N, and 12 for P.

There are $(13)(12) \div 2$, or 78 ways that a pair of numbers can be chosen - one from N and one from P (order does not matter). Of those 78 ways, the only "matches" are 180, 216, 252, and 288.

4 out of 78 reduces to 2 out of 39.

Therefore, $Y = 39$.

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SOLUTIONS - Meet #5 - Category 6

Category 6, continued . . .

- 4) Let X = the value of F.
Let $X + 2$ = the value of G

$$(X + 2)^2 - X^2 = 508$$

$$X^2 + 4X + 4 - X^2 = 508$$

$$4X + 4 = 508$$

$$4X + 4 + (-4) = 508 + (-4)$$

$$4X = 504$$

$$4X + 4 = 504 + 4$$

$$X = 126$$

$$X + 2 = 128$$

Therefore, $G = 128$.

Check: $126^2 = 15,876$.

$$128^2 = 16,384$$

$$16,384 - 15,876 = 508 \quad \text{Check!}$$

- 5) To find the area of the rhombus, take half the product of the diagonals. Or else, find the area of one of the four small congruent triangles, and multiply by 4.

$$\text{Area of rhombus} = \frac{1}{2} (40) (30)$$

$$= 600 \text{ square feet.}$$

To find the area of the circle, we need to find the length of its radius (or its diameter). We will need to work backwards by

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SOLUTIONS - Meet #5 - Category 6

Category 6, Question 5 continued . . .

- * finding the length of one side of the rhombus, and using that length as the base of the rhombus,
- * dividing the area of the rhombus by the base to get the height of the rhombus, which happens to be the diameter of the circle!

Use the Pythagorean Theorem to find the length of one side of the rhombus:

$$A^2 + B^2 = C^2$$

$$15^2 + 20^2 = C^2$$

$$225 + 400 = C^2$$

$$625 = C^2$$

$$\sqrt{625} = C$$

$$25 = C$$

So, one side of the rhombus is 25 inches. Since the area of the rhombus is known (600), divide the area by the length of one side to find the height of the rhombus:

$$\begin{aligned} \text{Area} \div \text{base} &= \text{height} \\ 600 \div 25 &= 24. \end{aligned}$$

The height of the rhombus is the same as the diameter of the circle. If the diameter of the circle is 24, then the radius is half of 24, or 12 inches.

$$\begin{aligned} \text{Area of circle} &= \pi R^2 \\ &\approx (3.14) (12^2) \\ &\approx (3.14) (144) \\ &\approx 452.16 \end{aligned}$$

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SOLUTIONS - Meet #5 - Category 6

Category 6, continued . . .

The area outside the circle but inside the rhombus is:

$$\begin{aligned} & \text{(area of rhombus) - (area of circle)} \\ \approx & 600 - 452.16 \\ \approx & 147.84 \\ \approx & \mathbf{147.8} \text{ square inches (rounded to the} \\ & \text{nearest tenth).} \end{aligned}$$

$$\begin{aligned} 6) \quad & \left(\frac{D}{C-7}\right)[(100)(A-E) - 5B - 191,130] \\ = & \left(\frac{128}{39-7}\right)[(100)(62,707.1 - 147.8) - 5(12,960) - 191,130] \\ = & \left(\frac{128}{39-7}\right)[(100)(62,559.3) - 64,800 - 191,130] \\ = & (4)[6,255,930 - 64,800 - 191,130] \\ = & (4)[6,000,000] \\ = & 24,000,000 \\ = & \mathbf{2.4 \times 10^7} \end{aligned}$$